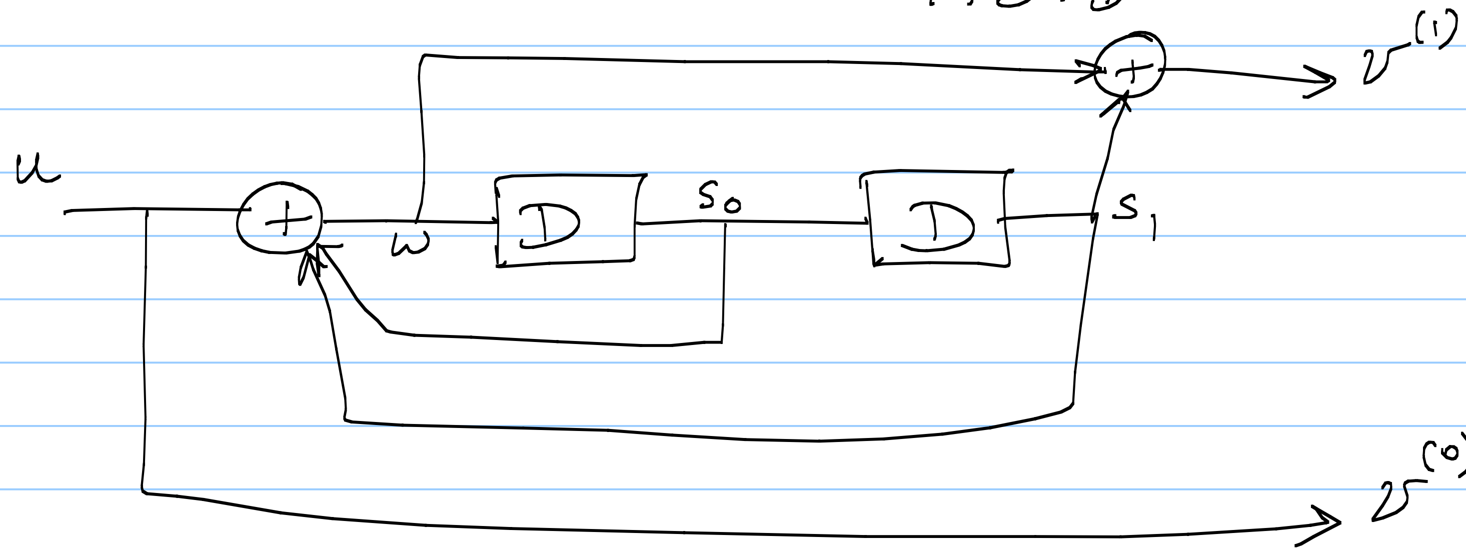
Lecture 32

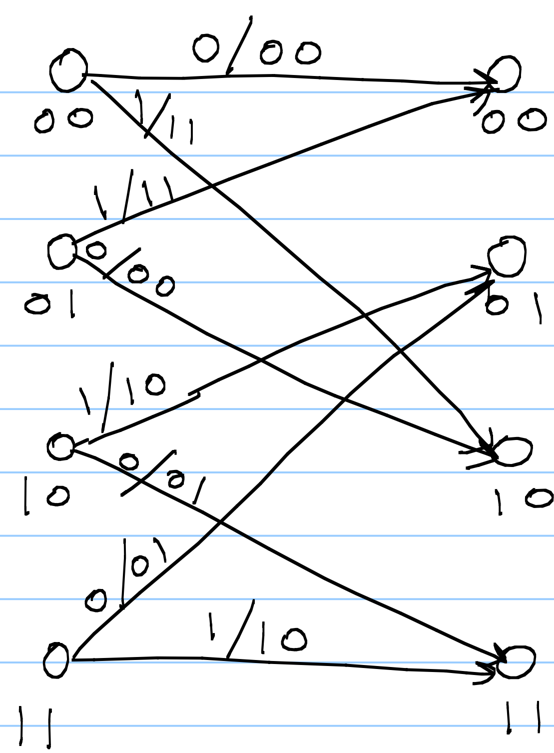
Note Title 4/2/2008

Recap: $G(D) = \left[\, 1 \quad \frac{1+D^2}{1+D+D^2} \,\right]$



$$v_n^{(0)} = u_n$$

$$v_n^{(1)} = w_n + s_1 = u_n + s_0$$



$\underline{s} = [s_0 \quad s_1]$

$v_n^{(0)} = u_n$

$v_n^{(1)} = u_n + s_0$

$s_0^{(n)} = u_n + s_0 + s_1$

$s_1^{(n)} = s_0$

Generator matrix: $G(D) = \left[1 \quad \dfrac{1+D^2}{1+D+D^2} \right]$

all-zero state $\longrightarrow$ k message bits $\longrightarrow$ m termination bits (not all zero) $\longrightarrow$ all-zero state

k-bit message $\underline{u}$ $\longrightarrow$ rate - 1/2 Conv. Encoder $\longrightarrow$ $2(k+m)$ codeword bits.

$$\underline{v}(D) = \underline{u}(D)\left[\,1 \quad \frac{1+D^2}{1+D+D^2}\,\right]$$

Set $u(D) = (u_0 + u_1 D + \cdots + u_{k-1} D^{k-1})(1+D+D^2)$

message

$$\underline{v}(D) = \left[\,u(D)(1+D+D^2) \quad u(D)(1+D^2)\,\right]$$

exact same as feed-forward case.

To retain systematic nature,

$$u(D) = \underbrace{u_0 + u_1 D + \cdots + u_{k-1} D^{k-1}}_{\text{message bits}} + u_k D^k + u_{k+1} D^{k+1},$$

$$\text{s.t. } (1+D+D^2) \mid u(D).$$

Ex:

$$G(D) = \left[1 \quad \frac{1+D^4}{1+D+D^2+D^3+D^4} \right]$$

$$v(D) = u(D)\,G(D) = \left[u(D) \quad \frac{u(D)(1+D^4)}{1+D+D^2+D^3+D^4} \right]$$

$$u(D) = 1+D^5, \quad v(D) = \left[1+D^5 \quad 1+D^4+D^5+D^9 \right]$$

$$wt = 6$$

$$u(D) = D^l (1+D^5)$$

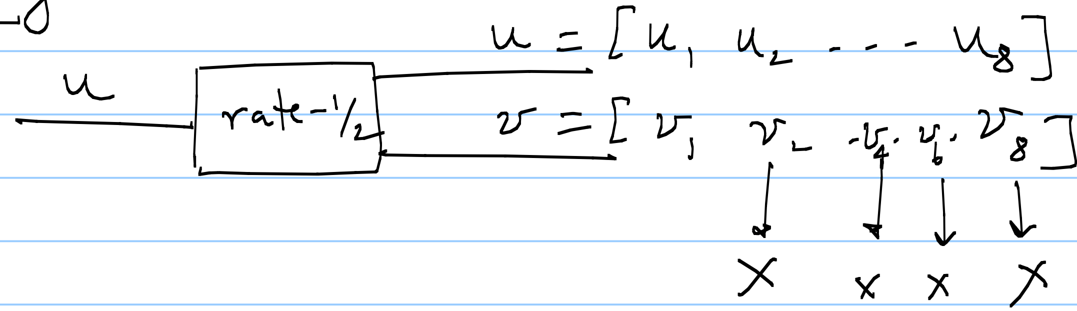
Towards Turbo codes ....

(1) LTI is not good

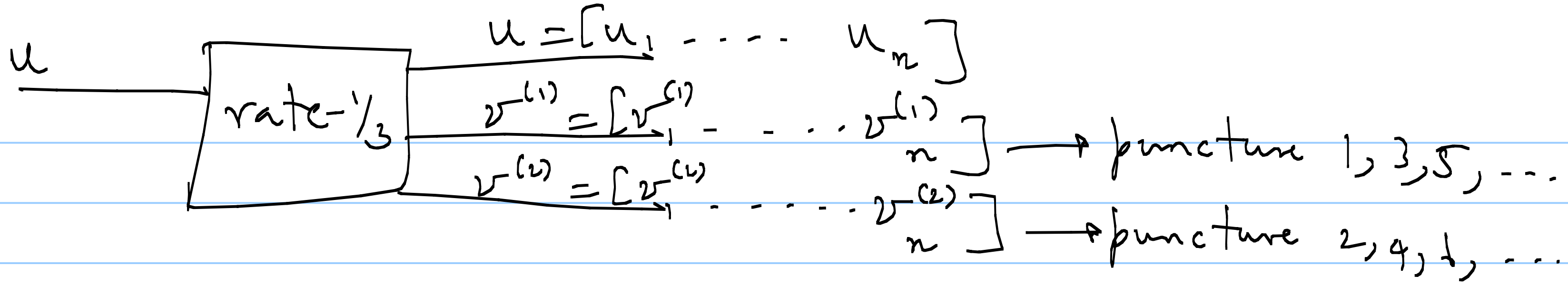
(2) Need some random element

(3) Sub-optimal iterative bitwise-MAP decoder

Puncturing:



Decoder: Set received values for punctured positions to zero. (BPSK case)



After puncturing, rate = 1/2

Bitwise-MAP decoder for convolutional codes

rate-1/2 encoder $\quad \underline{u} \longrightarrow \underline{u}, \underline{v} \xrightarrow[\text{AWGN}]{\text{BPSK}} \underline{r}^{(0)}, \underline{r}^{(1)}$

$$\underline{u} = [u_1 \; u_2 \; \cdots \; u_n]$$

$$\underline{v} = [v_1 \; v_2 \; \cdots \; v_n]$$

$$\underline{r}^{(0)} = [\; - \quad - \quad - \;]$$

$$\underline{r}^{(1)} = [\; - \quad - \quad - \; \cdots \;]$$

Target: $\Pr(u_i = 1 \mid \underline{r}^{(0)}, \underline{r}^{(1)})$

$$L_i = \log \frac{\Pr(u_i = 0 \mid \underline{r}^{(0)}, \underline{r}^{(1)})}{\Pr(u_i = 1 \mid \underline{r}^{(0)}, \underline{r}^{(1)})}$$

$$\Pr(u_i = 0 \mid \underline{r}^{(0)}, \underline{r}^{(1)}) = \frac{f(u_i = 0, \underline{r})}{f(\underline{r})} = \frac{f(u_i = 0, r_i, r_{\neq i})}{f(r_i, r_{\neq i})}$$

$\Big(\underline{r} = [\underline{r}^{(0)} \ \underline{r}^{(1)}]$

$r_i = [r_i^{(0)} \ r_i^{(1)}]$

$\underline{r}_{\neq i} = [r_1 \ r_2 \dots r_{i-1} \ r_{i+1} \dots r_n] \Big)$

$= f(u_i = 0$ . . . . .

$$\boxed{L_i = \frac{2}{\sigma^2} \cdot r_i^{(0)} + \log \frac{\Pr(u_i = 0 \mid \underline{r}_{\neq i})}{\Pr(u_i = 1 \mid \underline{r}_{\neq i})}}$$

intrinsic

extrinsic.

$$\underline{r}^{(0)}, \underline{r}^{(1)} \longrightarrow \boxed{\text{bitwise-MAP}} \longrightarrow \underline{L} = [L_1 \; L_2 \; \ldots \; L_n]$$

$$L_i = \frac{2}{\sigma^2} r_i^{(0)} + L_{ext}.$$

Turbo codes : (Parallel concatenated)