Lecture 24

Note Title 3/7/2008

$\longrightarrow$ Gallager A decoder

1) Iterative decoder

   $\hookrightarrow$ Message-passing decoder.

2) $p^{(\ell)} = f_{\omega_r, \omega_c}(p^{(\ell-1)}, p)$

   $\hookrightarrow$ Density evolution.

   (all-zero codeword assumption,
   tree-like neighbourhood assumption)

3) Threshold: p^*

$$p < p^* \implies p^{(\ell)} \to 0$$

$$p > p^* \implies p^{(\ell)} \not\to 0$$

(3,6) - regular LDPC code : Threshold $\underline{\underline{0.04}}$

rate = $1/2$, Capacity = 0.11

$\left(1 - h(0.11) = 1/2\right)$

Things to come:

1) Irregular codes

2) Soft decoding

Irregular LDPC codes:

node perspective

L_i = fraction of columns of weight i

($i = 1, 2, 3, \ldots$)

R_j = fraction of rows of weight j

($j = 1, 2, \ldots$)

Designed Rate, $R = 1 - \dfrac{\sum i L_i}{\sum j R_j} = 1 - \dfrac{L'(1)}{R'(1)}$

$$\left(\begin{array}{l} L(x) = \sum L_i x^i \\ R(x) = \sum R_j x^j \end{array} \right) \qquad L(1) = R(1) = 1$$

Ex: 1) Regular

2) $L_1 = 0$, $L_2 = 0.5$, $L_4 = 0.25$, $L_7 = 0.25$

Rate-1/2 $\qquad R_w \neq 0$, $R_{w+1} \neq 0$

all other $R_j = 0$

$w = 7$, $R_7 = 1/2$, $R_8 = 1/2$

Notation: $(L(x), R(x))$ — LDPC code

$\rightarrow$ (x^3, x^6) — LDPC code

$R = 1/2$

$\rightarrow$ $\left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^7}{4}, \frac{x^7}{2} + \frac{x^8}{2}\right)$ — LDPC code

$\rightarrow$ all $(L(x), R(x))$ s.t. $R = 1/2$

$$1/2 = 1 - \frac{\sum i L_i}{\sum j R_j}$$

$$\boxed{2 \sum i L_i - \sum j R_j = 0}$$

$L_i = 0$ $\quad i > d_l$

$R_\omega \neq 0, R_{\omega+1} \neq 0$

$R_j = 0, \; j \neq \omega$

Edge perspective:

λ_i = fraction of edges connected to degree-i bit nodes.

$i = 1, 2, \ldots$

ρ_j = fraction of edges connected to degree-j check nodes.

block-length $= n$

$(L(x), R(x))$ – LDPC code

$$\lambda_i = \frac{i L_i n}{\sum i L_i n} = \frac{i L_i}{\sum i L_i}$$

$$\rho_j = \frac{j R_j}{\sum j R_j}$$

$$\rho(x) = \sum \rho_j \, x^{j-1}$$

$$\lambda(x) = \sum \lambda_i \, x^{i-1}$$

$$R = 1 - \frac{\sum_j \frac{\rho_j}{j}}{\sum_i \frac{\lambda_i}{i}}$$

$$= 1 - \frac{\int_0^1 \rho(x)\,dx}{\int_0^1 \lambda(x)\,dx}.$$

Ex: $\left(\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^7}{4},\ \frac{x^7}{2} + \frac{x^8}{2}\right) \longrightarrow$ node perspective

$$\lambda(x) = \frac{1}{3.75}x + \frac{1}{3.75}x^3 + \frac{1.75}{3.75}x^6$$

$$\rho(x) = \frac{1.75}{3.75}x^6 + \frac{2}{3.75}x^7$$