Lecture 19

Note Title 2/27/2008

→ BPSK over AWGN
- → ML $\Leftrightarrow$ Minimum Euclidean distance
- → bitwise MAP
  - ↓ using likelihood ratios

Ex: $n = 6$, $k = 3$

$C = \langle 100101, 010110, 001011 \rangle$

$\underline{r} = [r_1 \ r_2 \ r_3 \ r_4 \ r_5 \ r_6]$

000000	110011
100101	101110
010110	011101
001011	111000

Code

$r_1 + r_2 + r_3 + \dots + r_6$, $-r_1 - r_2 - r_5 - r_6 + r_3 + r_4$

$-r_1 - r_4 - r_6 + r_2 + r_3 + r_5$, $-r_1 - r_3 - r_4 - r_5 + r_2 + r_6$

$r_1 + r_3 + r_6 - r_2 - r_4 - r_5$, $r_1 + r_5 - r_2 - r_3 - r_4 - r_6$

$r_1 + r_2 + r_4 - r_3 - r_5 - r_6$, $-r_1 - r_2 - r_3 + r_4 + r_5 + r_6$

dot products for ML

000000   ✓100011

✓100101   ✓011110

010110   011101

001011   ✓110000

$$l_i = e^{\frac{2r_i}{\sigma^2}}$$

$$L_1 = l_1 \cdot \frac{l_2 l_3 l_4 l_5 l_6 + l_3 l_6 + l_2 l_4 + l_5}{l_2 l_3 l_5 + l_3 l_4 + l_2 l_6 + l_4 l_5 l_6}$$

$$L_2 = l_2 \cdot \frac{l_1 l_3 l_4 l_5 l_6 + l_3 l_5 + l_1 l_4 + l_6}{l_1 l_3 l_6 + l_3 l_4 + l_1 l_5 + l_4 l_5 l_6}$$

$$L_3 = l_3 \cdot \frac{l_1 l_2 l_4 l_5 l_6 + l_2 l_5 + l_1 l_6 + l_4}{l_1 l_2 l_4 + l_2 l_6 + l_1 l_5 + l_4 l_5 l_6}$$

$$ab + ac = a(b+c)$$

Ex: $C = \{000, 111\}$

$\underline{r} = [r_1 \quad r_2 \quad r_3]$

ML: $r_1 + r_2 + r_3 \gtrless -r_1 - r_2 - r_3$

$\Updownarrow$

if $r_1 + r_2 + r_3 > 0$, $\hat{\underline{c}} = 000$

else, $\hat{\underline{c}} = 111$

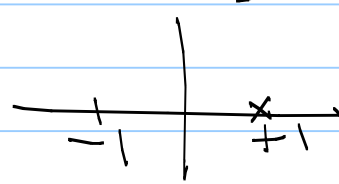
bitwise MAP:

$L_1 = l_1 \cdot l_2 \cdot l_3 = e^{\frac{2}{\sigma^2}(r_1 + r_2 + r_3)}$

$\Updownarrow$

$\underline{\underline{ML}}$

Uncoded:



$\Pr(\text{Error}) = Q\left(\frac{1}{\sigma}\right)$

$\shortparallel$

$\Pr(-1 \mid +1 \text{ was transmitted})$

$$D = (r_1 + r_2 + r_3)\Big|_{\underline{c} = 000}$$

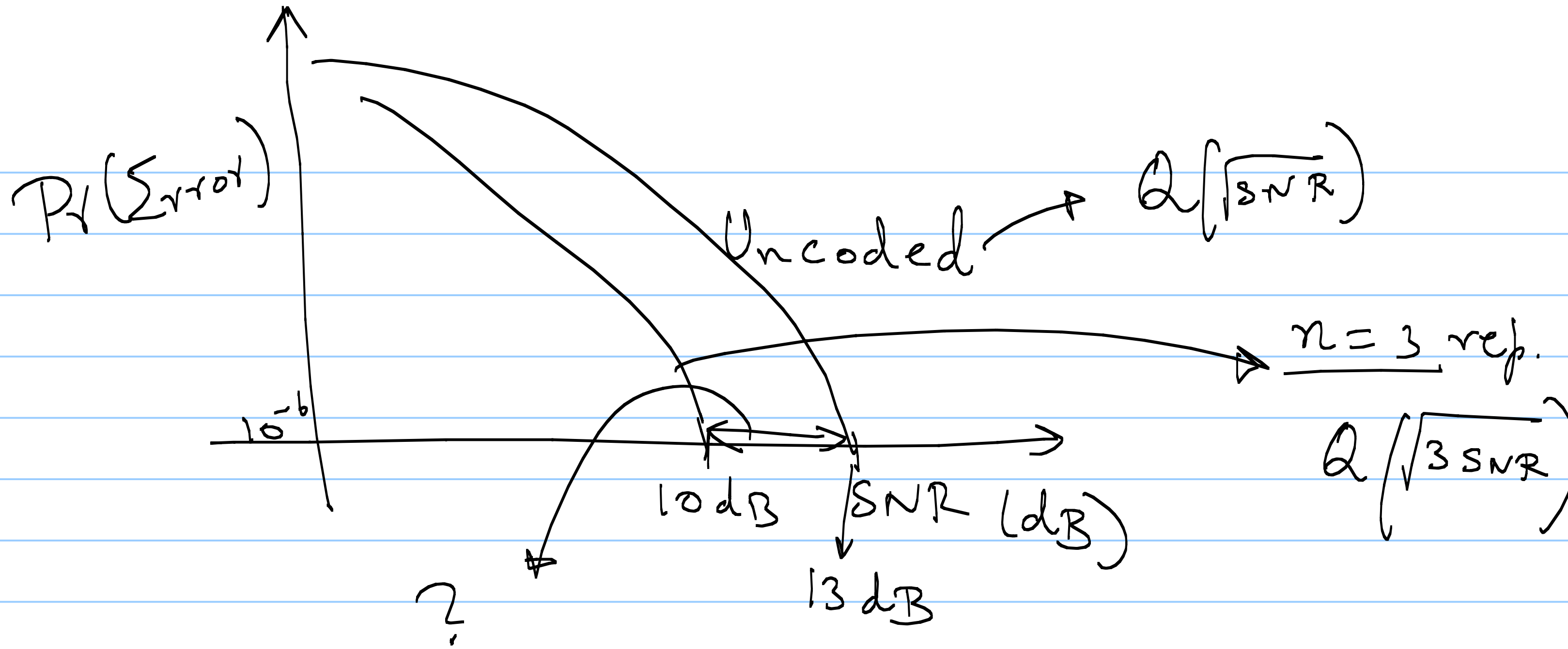
$$D \sim N(3, 3\sigma^2)$$

$$\Pr(D < 0) = \Pr(\text{Error}) = Q\left(\frac{\sqrt{3}}{\sigma}\right)$$

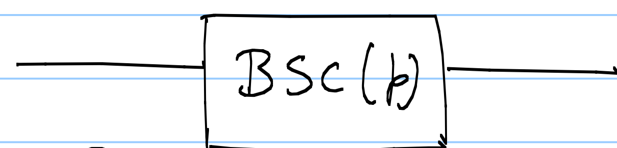
$(n, 1, n)$ repetition code: $\Pr(\text{Error}) = Q\left(\frac{\sqrt{n}}{\sigma}\right)$

$\longrightarrow$ $\Pr(\text{Error})$ can be studied as a function of $SNR = \frac{1}{\sigma^2}$

$$SNR_{(dB)} = 10 \log_{10}\left(\frac{1}{\sigma^2}\right)$$



Capacity:
(for channels)



Arbitrarily low Pr(Error):
Given $\varepsilon > 0$. One can transmit at rate < capacity with $\Pr(\text{Error}) < \varepsilon$.

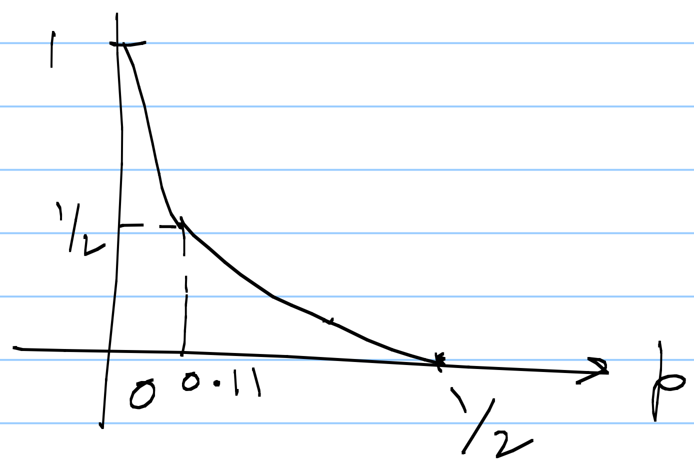
Rate = (# of information bits)/Channel use

Given $\varepsilon > 0$. $\exists$ an (n, k) code for sufficiently large n $(= f(\varepsilon))$ with $k/n \to$ capacity s.t. $P_r(\text{Error}) < \varepsilon$.

(under ML decoding)

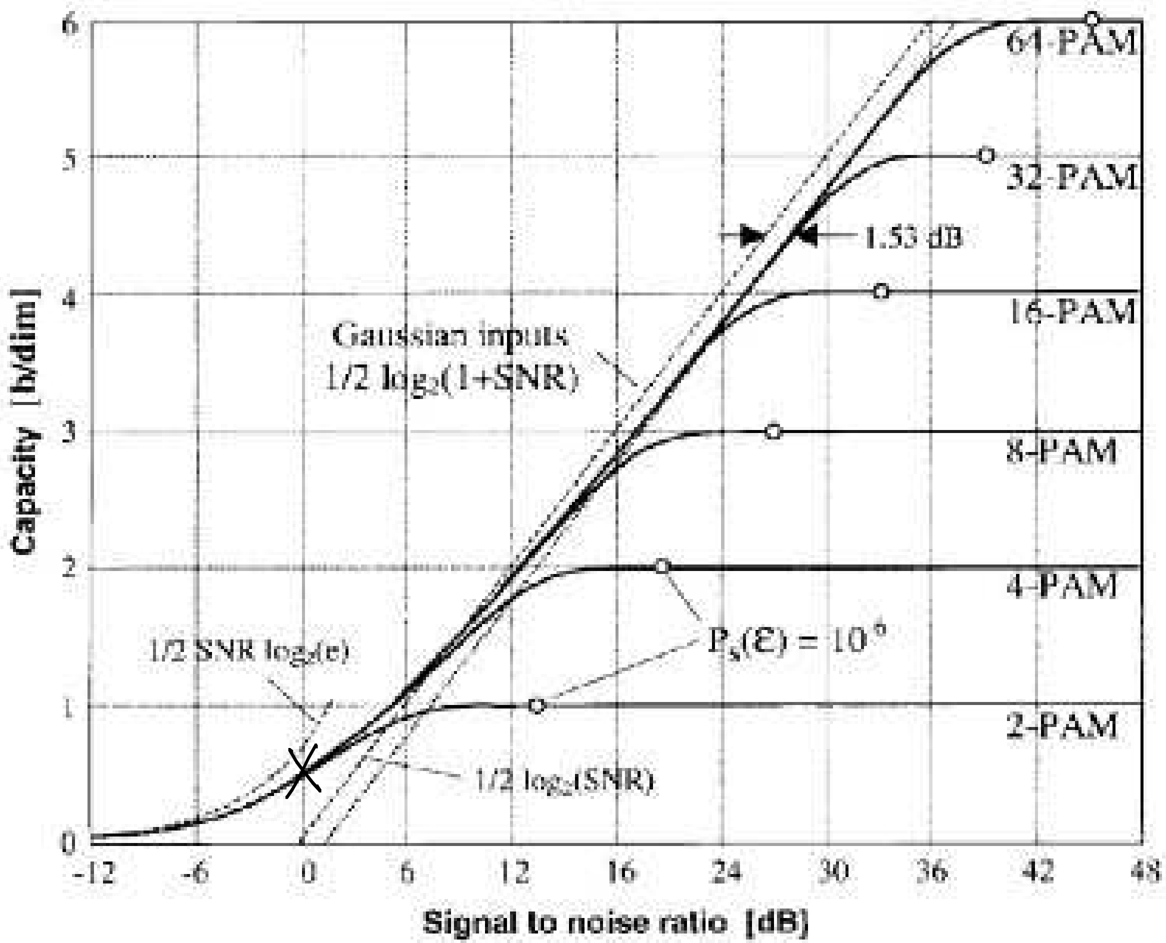
BSC (p): Capacity $= 1 + p \log_2 p + (1-p) \log_2 (1-p)$

$=$ function of p



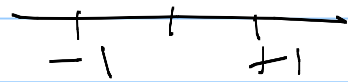
BPSK over AWGN:

Capacity = function of SNR $\left(\frac{1}{\sigma^2}\right)$



E_b/N_0 : rate-normalized SNR.

$$\frac{N_0}{2} = \sigma^2$$



$$E_b = \frac{1}{R} = \frac{E_s}{R}$$

Energy of BPSK

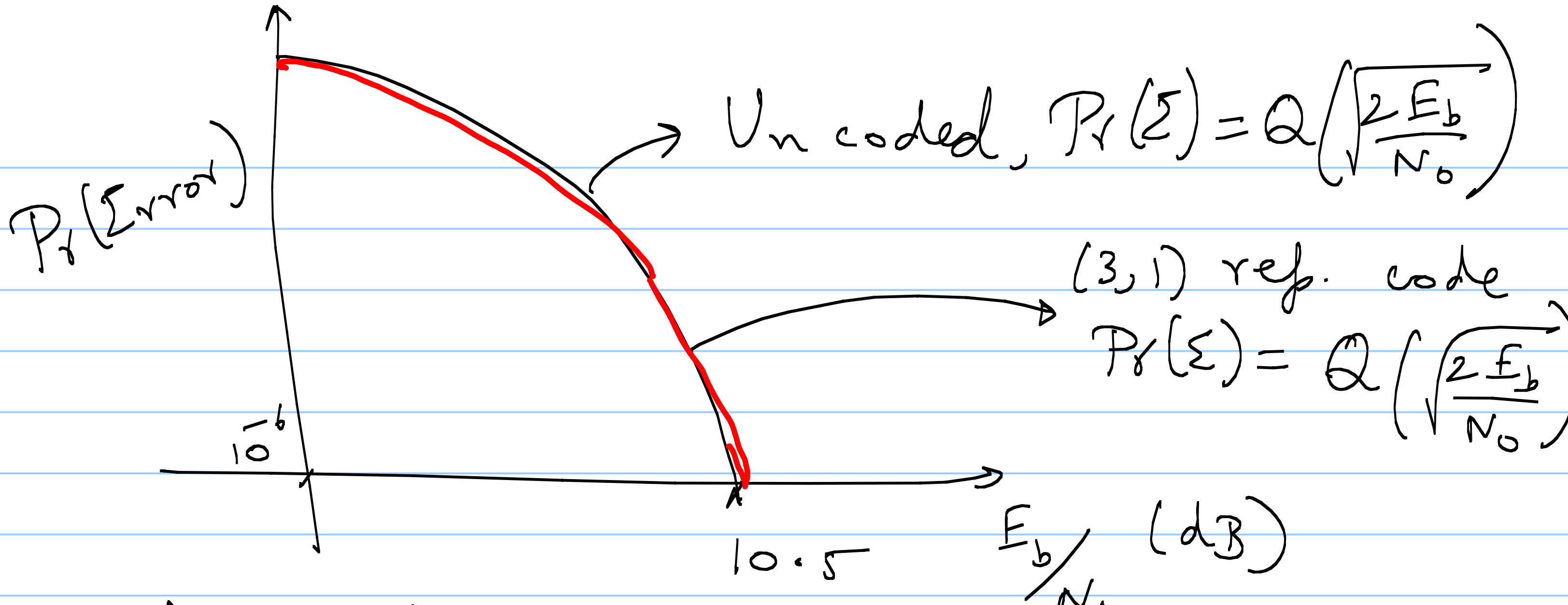
average Energy per modulated symbol

Average Energy per information bit

$$R = \frac{k}{n}$$

$$E_b = \frac{E_s \cdot n}{k}$$

$$\frac{E_b}{N_0} = \frac{1}{2R\sigma^2} \qquad \frac{E_b}{N_0} \text{ (in dB)} = 10\log_{10}\frac{1}{2R\sigma^2}$$



→ No coding gain for repetition code.