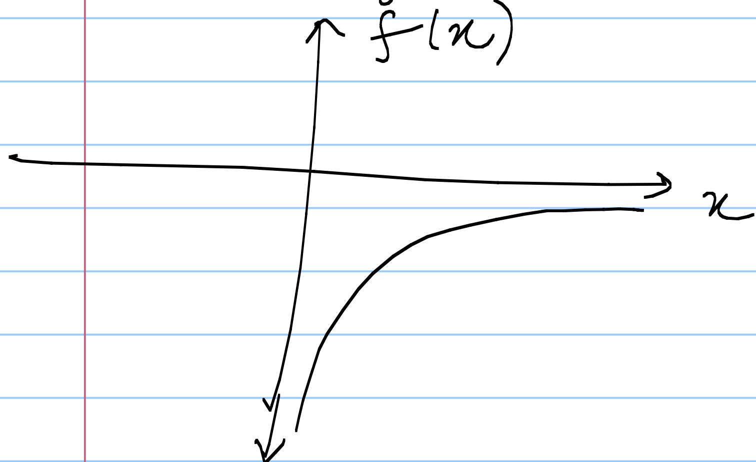
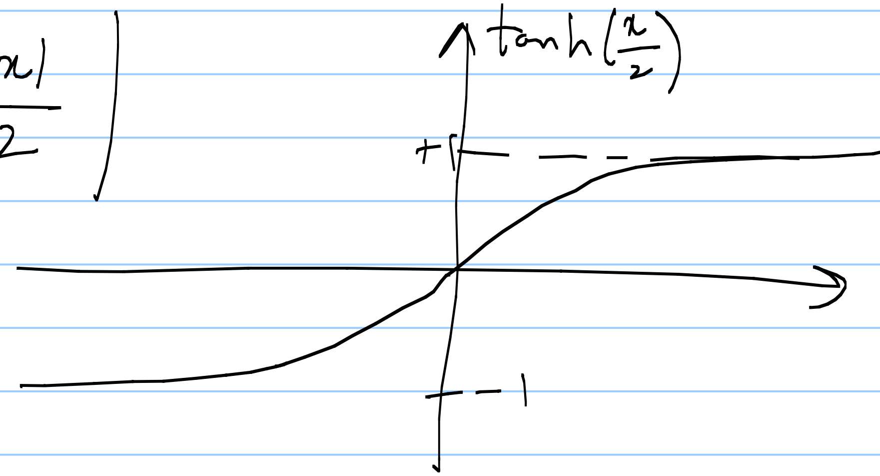
Lecture 27

Note Title 3/14/2008

$$f(x) = \left| \log \tanh \frac{|x|}{2} \right|$$

magnitude





$$f(x) = \log \tanh \frac{|x|}{2}$$

$$f(f(x)) = x$$

Recap:

$$X = Y_1 + Y_2 + \dots + Y_d$$

$$p_{i0} = \Pr(Y_i = 0)$$

$$y_i = \log \frac{p_{i0}}{p_{i1}}$$

$$\mathrm{sign}(x) = \begin{cases} +1, & x > 0 \\ -1, & x < 0 \end{cases}$$

$$\mathrm{mag}(y_x) = f\left(\sum_{i=1}^{d} f(y_i)\right)$$

$$\overline{\mathrm{sign}}(x) = \begin{cases} 0, & x > 0 \\ 1, & x < 0 \end{cases}$$

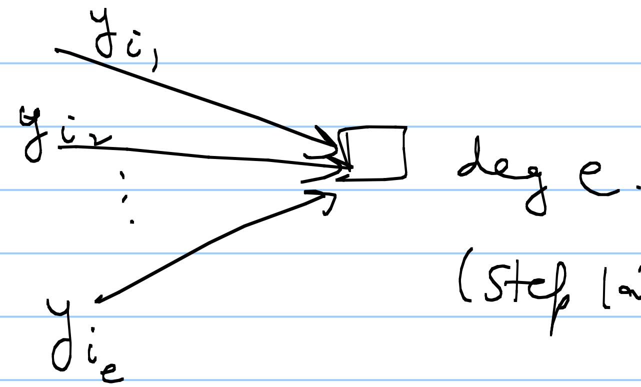
$$\mathrm{sign}(y_x) = \prod_{i=1}^{d} \mathrm{sign}(y_i)$$

$$\mathrm{sign}(x) = (-1)^{\overline{\mathrm{sign}}(x)}$$

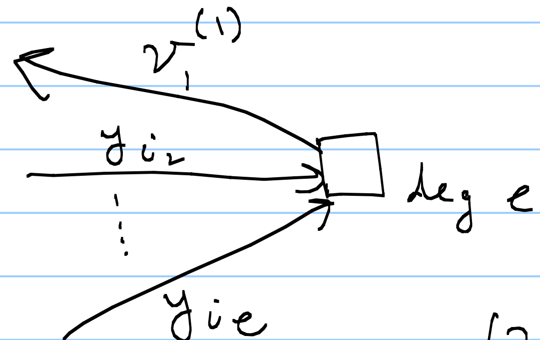
$$\longrightarrow \overline{\mathrm{sign}}(y_x) = \overline{\mathrm{sign}}(y_1) + \overline{\mathrm{sign}}(y_2) + \dots + \overline{\mathrm{sign}}(y_d)$$

Soft Message-passing decoder:

Iteration 1: Step (b)



(Step (a))



$$\mathrm{mag}(v_1^{(1)}) = f\left(f(y_{i_2}) + f(y_{i_3}) + \cdots + f(y_{i_e})\right)$$

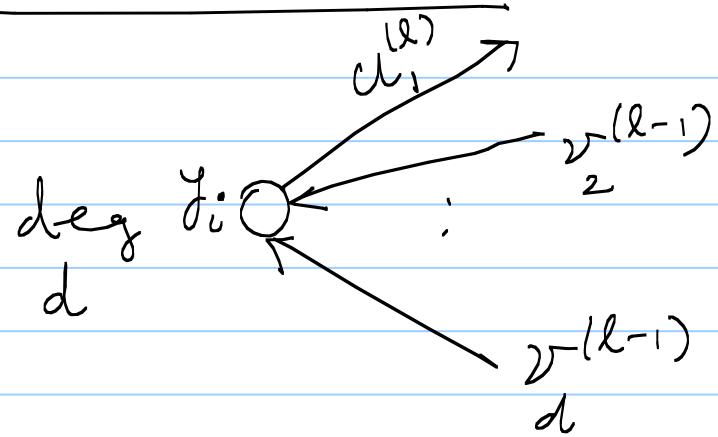
$$\mathrm{Sign}(v_1^{(1)}) = \mathrm{sign}(y_{i_2})\,\mathrm{sign}(y_{i_3})\cdots\mathrm{sign}(y_{i_e})$$

$$\mathrm{mag}(v_2^{(1)}) = f\left(f(y_{i_1}) + f(y_{i_3}) + \cdots + f(y_{i_e})\right)$$

$$\vdots$$

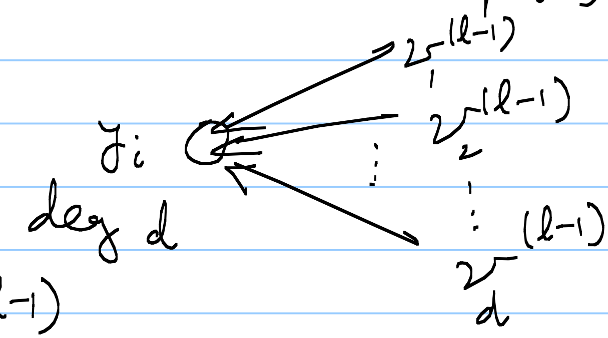
$$\mathrm{mag}(v_e^{(1)}) = f\left(f(y_{i_1}) + f(y_{i_2}) + \cdots + f(y_{i_{e-1}})\right)$$

Iteration ℓ : step (a)



Iteration $\ell-1$

Step (b)



$$u_1^{(\ell)} = y_i + v_2^{(\ell-1)} + \cdots + v_d^{(\ell-1)}$$

$$(c_i = 0 \mid r_i) \qquad (c_i = 0 \mid ?) \to M_2 \qquad (c_i = 0 \mid ?) \to M_d$$

$$c_i = 0 \mid (r_i, M_2, \dots, M_d)$$

Finally, $c_i = 0 \mid \underline{r}$ $\longrightarrow$ can only "approximate" in practice.

$$u_1^{(l)} = y_i + v_2^{(l-1)} + v_3^{(l-1)} + \cdots + v_d^{(l-1)}$$

$$u_2^{(l)} = y_i + v_1^{(l-1)} + v_3^{(l-1)} + \cdots + v_d^{(l-1)}$$

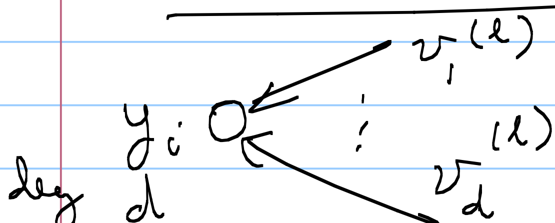
$$\vdots$$

$$u_d^{(l)} = y_i + v_1^{(l-1)} + v_2^{(l-1)} + \cdots + v_{d-1}^{(l-1)}$$

<u>Step (b):</u>

Replace $y_{i_1}, y_{i_2}, \ldots, y_{i_e}$ with $u_1^{(l)}, u_2^{(l)}, \ldots, u_e^{(l)}$ & repeat same steps as iteration 1

<u>Decision after iteration l:</u>



output $= y_i + v_1^{(l)} + v_2^{(l)} + \cdots + v_d^{(l)}$

$\ll R$

$$\hat{c}_i^{(l)} = \begin{cases} 0 & \text{if output}_{LLR} > 0 \\ 1 & \text{, else.} \end{cases}$$

$$\underline{\hat{c}}^{(l)} = [\hat{c}_1^{(l)} \;\; \hat{c}_2^{(l)} \; \cdots\cdots \; \hat{c}_n^{(l)}].$$

Stop condition: $H \underline{\hat{c}}^{(l)T} = 0$

Density Evolution:

- $\rightarrow$ track pdf of messages
- $\rightarrow$ regular case
- $\rightarrow$ all-zero codeword assumption.
- $\rightarrow$ Neighbourhoods are tree-like.

→ Message is in error if it is negative.