Lecture 20

Note Title 2/28/2008

BPSK over AWGN:

Capacity, $C = f(SNR)$

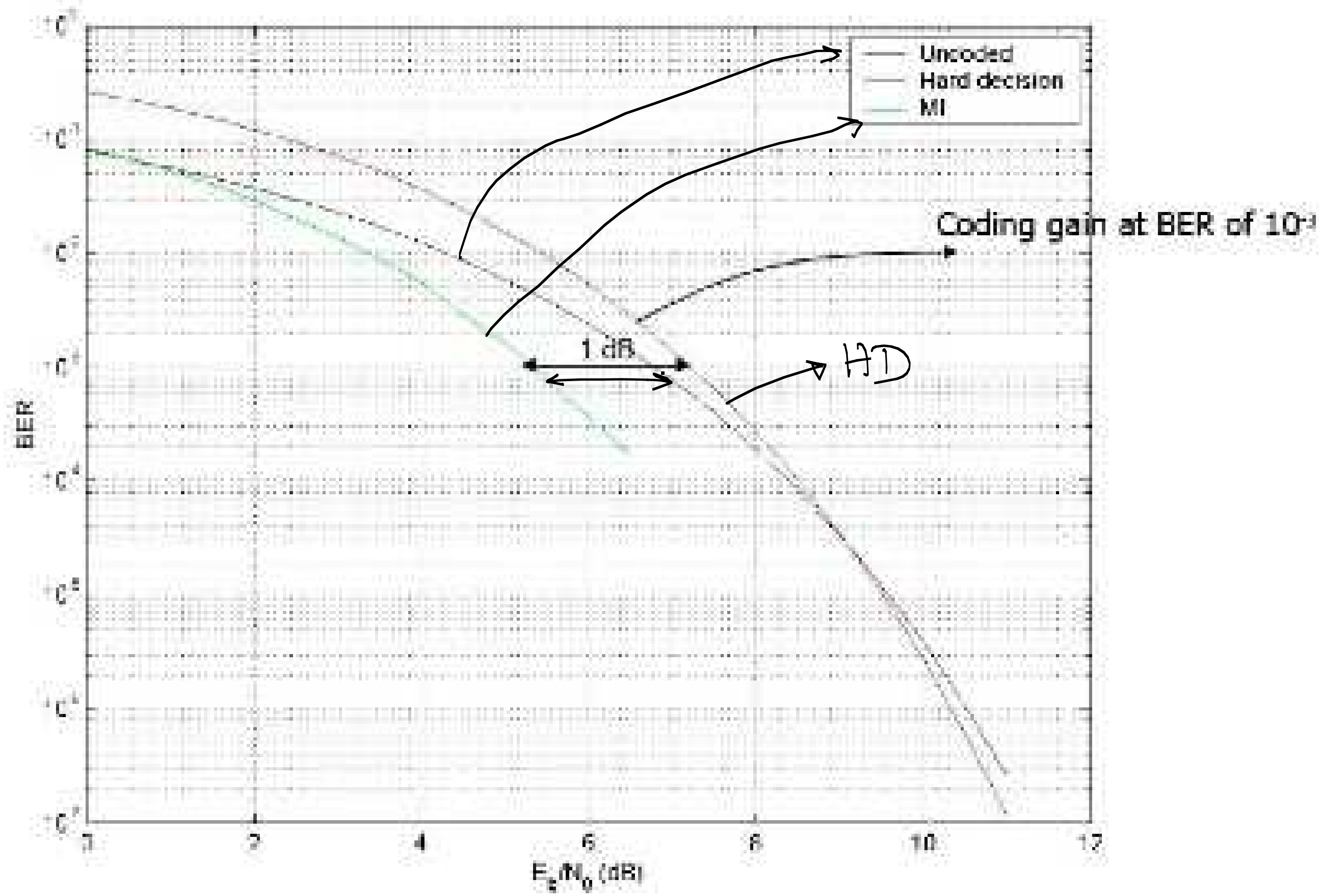
Capacity-achieving $SNR = f^{-1}(C)$

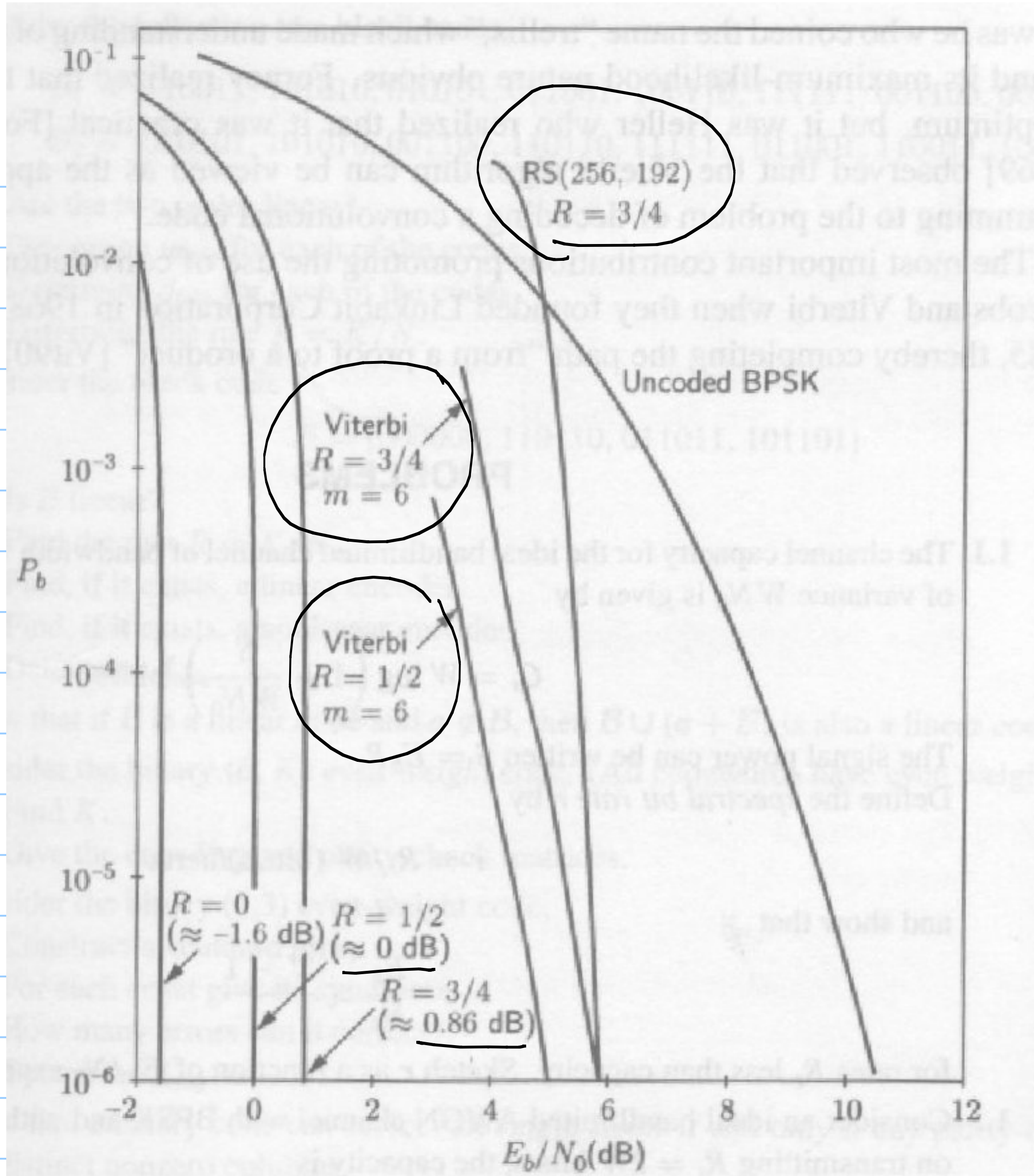
$\rightarrow C = 1/2 \Rightarrow SNR \geq 0.2\,dB$

Capacity-achieving $E_b/N_0 = \frac{1}{2C} f^{-1}(C)$

$\rightarrow$ Uncoded, $E_b/N_0 \approx 11\,dB$ for 10^{-6} BER

Rate-$1/2$, $E_b/N_0 \geq 0.2\,dB$ for "


Uncoded
Hard decision
MI
Coding gain at BER of 10-3
1 dB
HD
BER
E_b/N_0 (dB)



→ from "Fundamentals of Convolutional codes".

→ Proof of capacity involves random codes.

→ "Many" random codes can "approach" capacity at large blocklengths.

→ Problem with random codes → decoding

→ Modern codes: have a random part in their construction.

→ decoding is "efficient".

Low-density Parity-check Codes:

(LDPC)

→ A linear code with a sparse parity-check matrix is said to be an LDPC code.

Regular LDPC codes: n : blocklength
(PC matrix H)

w_c : weight of each column of H.

w_r : weight of each row of H

$$\#\text{ of rows of } H = \frac{n w_c}{w_r} \quad \text{(has to be integral)}$$

$$\text{Rate of the code} \geq \frac{n - \frac{n w_c}{w_r}}{n} = 1 - \frac{w_c}{w_r}$$

Examples: $w_c = 3$

designed rate, $R = 1/2 \Rightarrow w_r = 6$

Notation: $(3,6)$-regular code

$(3,4)$-code $\longrightarrow R = 1/4$

$(3,5)$-code $\longrightarrow R = 2/5$

$(3,30)$-code $\longrightarrow R = 0.9$

choose an n

$\rightarrow \frac{w_c n}{w_r} \in \mathbb{Z}$.

$\rightarrow n$: large enough.

$\longrightarrow$ introduced by Gallager in 60s

Gallager's construction: $w_r \mid n$

(n, w_c, w_r) - LDPC matrix

$l = n/w_r$