Majority logic decoding of RM codes

Note Title

Ex: $RM(2,5)$ $d_{min} = 8$

$J = 7$ $EG(5,2)$

3-step is possible.

1-step: $e_0 + e_1$ 1-flat $\longrightarrow$ $e_0 + e_1 + e_2 + e_3$ 2-flats $\longrightarrow$

$e_0 + e_2$ $\quad$ $e_0 + e_1 + e_4 + e_5$

$\vdots$ $\quad$ $\vdots$

$e_0 + e_7$ $\quad$ $e_0 + e_1 + e_{14} + e_{15}$

3-flat

$e_0 + e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 = s_1 = r_0 + r_1 + r_2 + r_3 + r_4 + r_5 + r_6 + r_7$

$e_0 + e_1 + e_2 + e_3 + e_8 + e_9 + e_{10} + e_{11} = s_2$

$\vdots$ $\quad$ $\vdots$

$e_0 + e_1 + e_2 + e_3 + e_{28} + e_{29} + e_{30} + e_{31} = s_7$

$RM(r,m)$: $n = 2^m$

$$k = 1 + \binom{m}{1} + \cdots + \binom{m}{r}$$

$$d = 2^{m-r}$$

③ G & $\overset{?}{H}$ for $RM(2,4)$

dual: $RM(1,4)$

find generator

(4) $C_1 = RM(1,3)$, $C_2 = (8,1)$ repetition code $= RM(0,3)$

$C = \{u|u+v : u \in C_1, v \in C_2\}$

$\to RM(1,4)$ $\quad d = 8$

(5) $f(v_1, v_2, \ldots, v_{m-1})$: Boolean function

Show $\underline{v_m} \overset{xor}{+} f(v_1, v_2, \ldots, v_{m-1})$ takes values 0 & 1 equally often.

(6) $RM(r+1, m) = \{u+v : u \in RM(r,m)$

$v = 0$ (or each term deg $= r+1\}$

(7) (n,k) code C.

<u>1-step maj logic decoding</u>

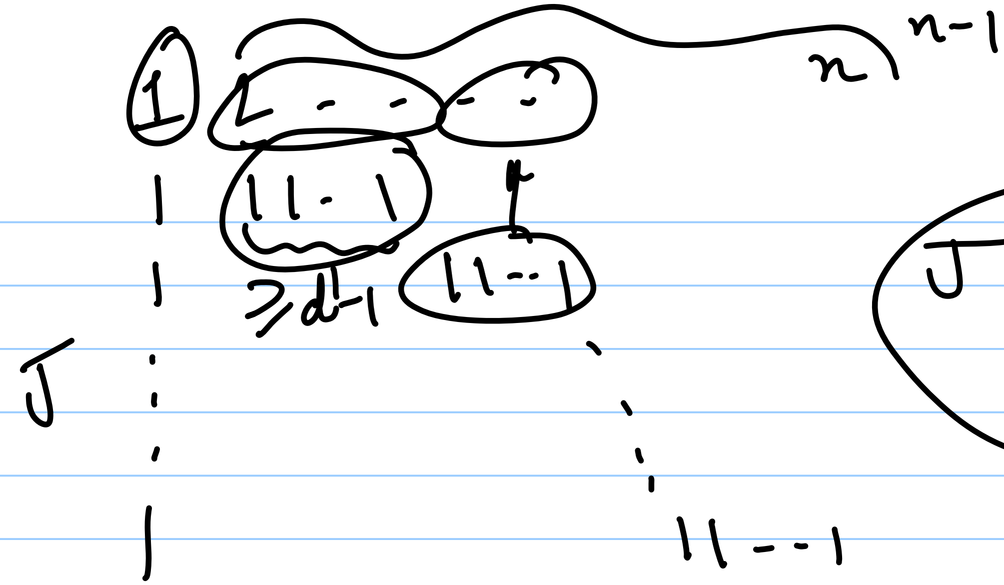
Show error-correcting capability $\leq \dfrac{n-1}{2(d^\perp - 1)}$

RS: $(n, k, n-k+1)$

$\updownarrow$

$(n, n-k, k+1)$

$d^\perp : d_{min}(C^\perp)$



$$J \leq \frac{n-1}{d'-1}$$

(8) L-step maj logic decoding

error-correcting capability $\leq \dfrac{n}{d'} - \dfrac{1}{2}$

(9) C: $\exists$ J parity-checks orthogonal on each coordinate.

$\Rightarrow \quad d_{min} \geqslant J+1$

(10) Devise maj-logic decoders for $(7,4)$ Hamming code

& $(15,7)$ 2-error-correcting BCH code

(11) $RM(r,m) \quad d = 2^{m-r}$

Devise a $(2^{m-r}-1)$-erasure-correcting decoder.

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ x_1 \\ c_3 \end{bmatrix}$$

$H:$

$$H\underline{c}^T = \underline{0} \qquad = \begin{bmatrix} 1 \\ 0 \end{bmatrix} x_1 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_3 \end{bmatrix}$$

$$e \le 2^{n-r} - 1$$

$$\underline{r} = [c_1 \; c_2 \cdots x_1 \cdots x_2 \cdots x_e \cdots c_n]$$

$$H[c_1 \; c_2 \cdots x_1 \cdots x_2 \cdots x_e \cdots c_n]^T = \underline{0}$$

$$\overset{\longleftarrow e \longrightarrow}{\begin{bmatrix} H_x \end{bmatrix}} \begin{bmatrix} x_1 \\ \vdots \\ x_e \end{bmatrix} = \underline{s}$$