# Lecture 44

Note Title 11/5/2008

$(n, k)$ linear block code

$$\underset{k \times n}{G} = \left[ I_k \quad \underset{k \times n-k}{P} \right] \qquad \underset{n-k \times n}{H} = \left[ P^T \quad I_{n-k} \right]$$

$$\text{Code} = \{ \underline{c} = \underline{m} G : \underline{m} \in \{0,1\}^k \}$$

$$= \{ \underline{c} \in \{0,1\}^n : H \underline{c}^T = \underline{0}^T_{n-k} \}$$

$d$ : minimum distance

Code: induces a $n$-D signal constellation.

Ex: (for $d_{min}$)

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$n = 6, \; k = 3$

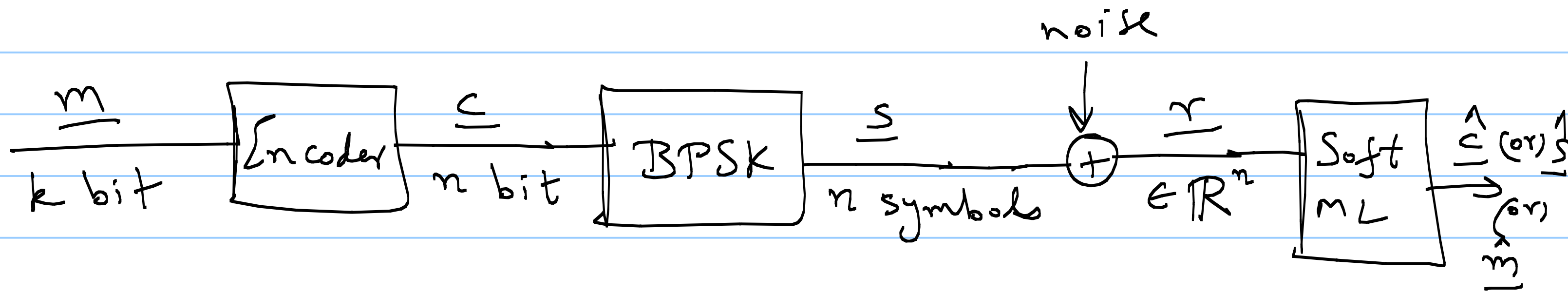
$$C = \{000000,\ 001011,\ 010110,\ 100101,\ 011101,\ 101110,\ 110011,\ 111000\}$$

$d = 3$ , # of min wt codewords $= 4$

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \longrightarrow d = 3$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

# Soft ML decoding




Code: $\mathcal{C}$

$$\begin{array}{ccc} \underline{c} & \in & \mathcal{C} \\ \updownarrow & & \updownarrow \\ \underline{s} & \in & \mathcal{S} \end{array}$$

$\mathcal{S}$, Set of code symbols

$$\mathcal{S} = 1 - 2\mathcal{C}$$

$$(\underline{s} = 1 - 2\underline{c})$$

$$\hat{\underline{c}} = \arg\min_{\underline{c} \in \mathcal{C}} \left\| \underline{r} - (1 - 2\underline{c}) \right\|^2$$

$(n, k, d)$ code: what is the distance between nearest neighbours in $n$-D signal constellation?

$$\downarrow$$
$$2\sqrt{d}$$

$$\underline{u} = [u_1 \;\; u_2 \; \dots \; u_n]$$
$$\underline{v} = [v_1 \;\; v_2 \; \dots \; v_n]$$

$$\|(1-2\underline{u}) - (1-2\underline{v})\|^2 = d_H(\underline{u}, \underline{v}) \cdot 4$$

$K_d$ = # of min-wt non-zero codewords.

$$P_e \approx K_d \, Q\left(\frac{2\sqrt{d}}{2\sigma}\right) = K_d \, Q\left(\frac{\sqrt{d}}{\sigma}\right)$$

$$\frac{E_b}{N_0} = \frac{n}{2k\sigma^2} \qquad \frac{1}{\sigma} = \sqrt{\frac{2k}{n}\frac{E_b}{N_0}}$$

$$P_e \approx K_d \, Q\left(\sqrt{\frac{2kd}{n}\frac{E_b}{N_0}}\right)$$

(coded)

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

(uncoded)

Nominal Coding gain $= 10 \log_{10}\left(\frac{kd}{n}\right)$ dB.

Ex: (7, 4) Hamming Code

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

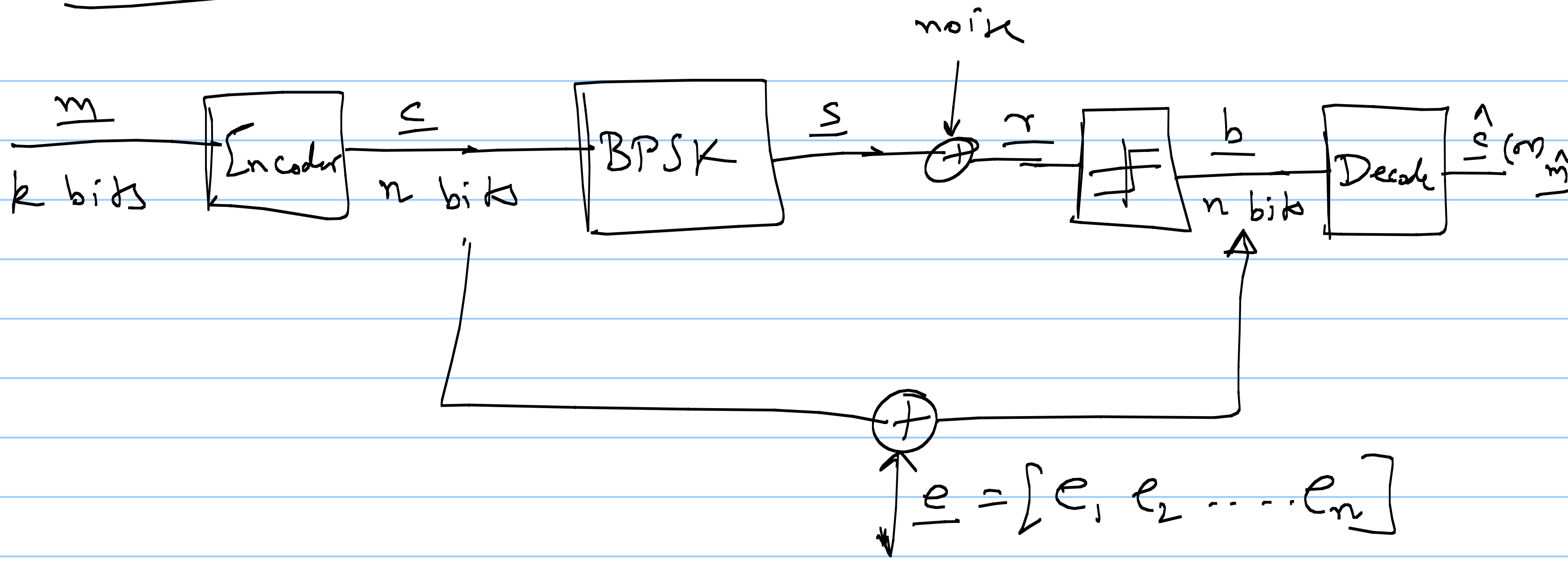
$$\underline{c} = [1 \;\; 0 \;\; 0 \;\; 0 \;\; 1 \;\; 1 \;\; 0]$$

$$d = 3$$

$$K_d = 7$$

$$\text{Nominal Coding gain} = 10 \log_{10} \frac{4 \times 3}{7} \simeq 2.5 \text{ dB}$$

# Hard ML decoder:




$$b_i = c_i \quad \text{with prob} \quad 1 - Q(1/\sigma)$$

$$\neq c_i \quad \text{with prob} \quad p = Q(1/\sigma)$$

$$e_i = \begin{cases} 0 & \text{w.p.} \quad 1-p \\ 1 & \text{w.p.} \quad p \end{cases}$$

(iid)

$$\xrightarrow[k \text{ bits}]{\underline{m}} \boxed{\text{Encoder}} \xrightarrow[n \text{ bits}]{\underline{c}} \boxed{BSC(p)} \xrightarrow[n \text{ bits}]{\underline{b}} \boxed{\text{Decode}} \xrightarrow{\hat{\underline{c}} \text{ (or } \hat{\underline{m}})}$$

$p < 1/2$

$\underline{e}$ : $2^n$ possibilities

$n$ bits

$\underline{c} \in \mathcal{C}$

$\to 2^k$ possibilities

$$\Pr(\underline{e}) = p^{wt(\underline{e})} (1-p)^{n - wt(\underline{e})}$$

$$\hat{\underline{c}} = \arg\min_{\underline{c} \in \mathcal{C}} d_H(\underline{b}, \underline{c})$$