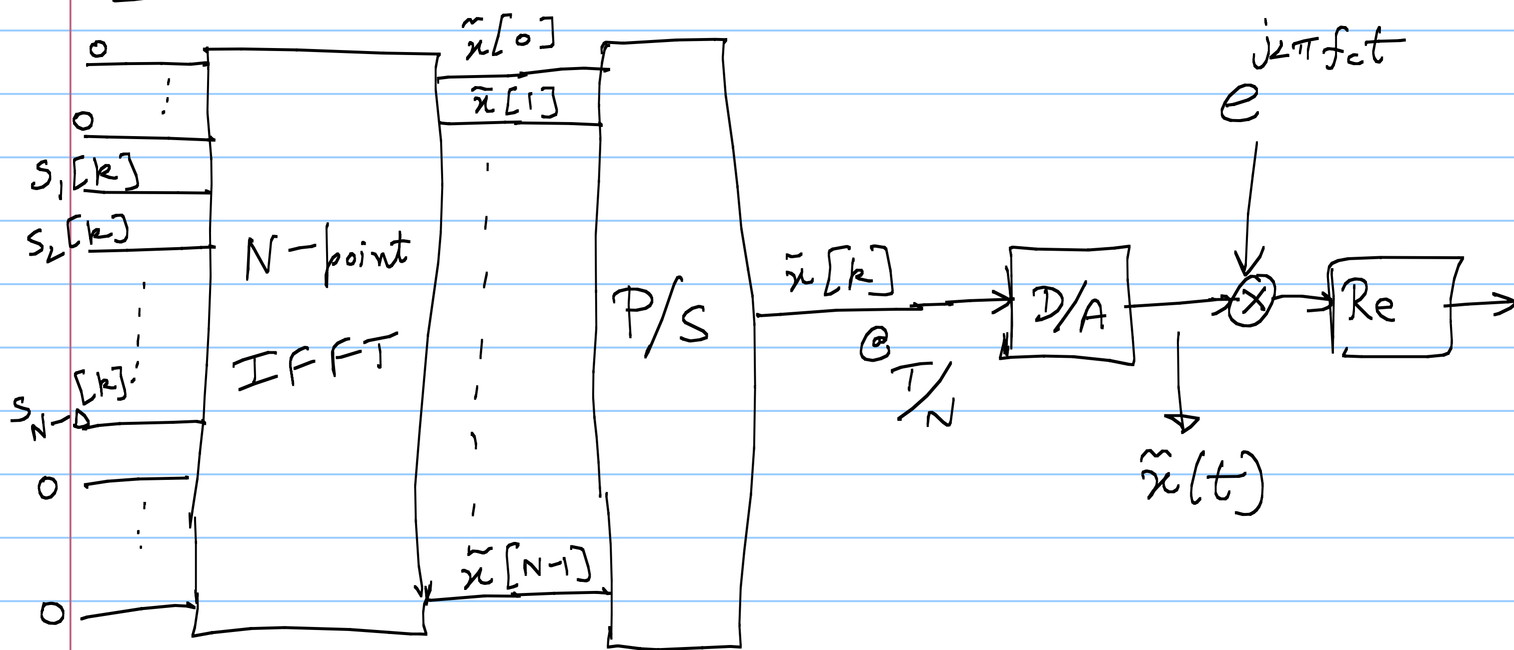
Note Title 10/29/2008

Lecture 40

OFDM:



→ Block modulation scheme

→ Signals are orthogonal.

After D/A, (ideal sinc interpolation)

$$\tilde{x}(t) = \sum_{n=0}^{N-1} \tilde{x}[n]\, p\left(t - \frac{nT}{N}\right)$$

$$p(t) = \sqrt{\frac{T}{N}}\,\frac{\sin \frac{\pi N t}{T}}{\pi t} \xleftrightarrow{FT} \underset{\left[-\frac{N}{T}, \frac{N}{T}\right]}{\mathrm{rect}}(f)$$

$$\tilde{x}(t) = \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} \underset{\substack{\updownarrow \\ s[l]}}{s_l[k]}\, e^{j\frac{2\pi l n}{N}}\, p\left(t - \frac{nT}{N}\right)$$

$$= \sum_{l=0}^{N-1} s[l] \underbrace{\left[\sum_{n=0}^{N-1} e^{j\frac{2\pi l n}{N}}\, p\left(t - \frac{nT}{N}\right)\right]}_{= \tilde{g}_l(t)}$$

$\{\tilde{g}_\ell(t),\ 0 \le \ell \le N-1\}$: orthonormal

$\downarrow$ basis of the signal space.

$$\tilde{g}_\ell(t) = \sum_{n=0}^{N-1} e^{j\frac{2\pi \ell n}{N}}\, p\left(t - \frac{nT}{N}\right) \longrightarrow \text{sinusoidal between 0 and } T$$

$\uparrow$?

$$\left(\underline{\underline{g_\ell(t)}} = e^{j\frac{2\pi \ell t}{T}} \left(u(t) - u(t-T)\right) \right)$$

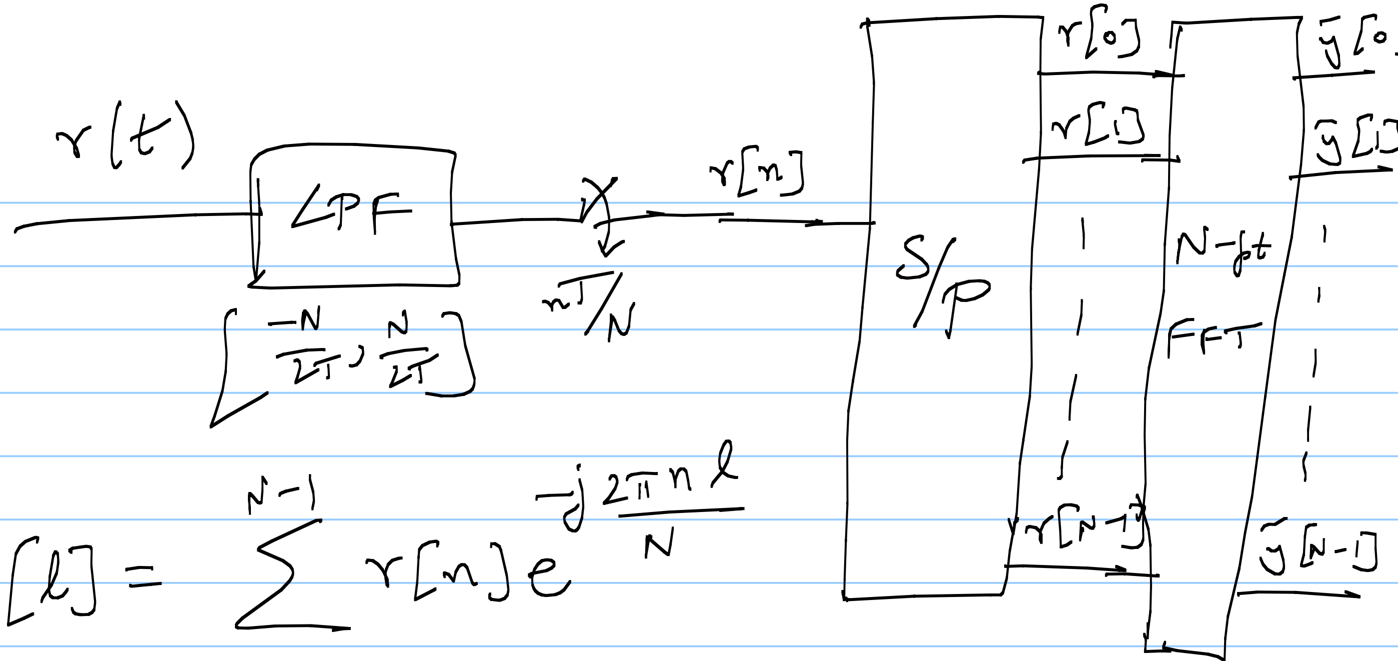
Receiver (No ISI): $r(t) = \tilde{x}(t) + n(t)$

$\longrightarrow$ correlation with $\tilde{g}_\ell^*(t)$

$$\tilde{y}[\ell] = \int_{-\infty}^{\infty} r(t)\, \tilde{g}_\ell^*(t)\, dt \quad , 0 \le \ell \le N-1$$

$$= \int_{-\infty}^{\infty} r(t) \sum_{n=0}^{N-1} e^{-j\frac{2\pi n \ell}{N}} p\left(t - \frac{nT}{N}\right) dt$$

$$= \sum_{n=0}^{N-1} e^{-j\frac{2\pi n \ell}{N}} \cdot \underbrace{\int_{-\infty}^{\infty} r(t)\, p\left(t - \frac{nT}{N}\right) dt}_{r[n]}$$



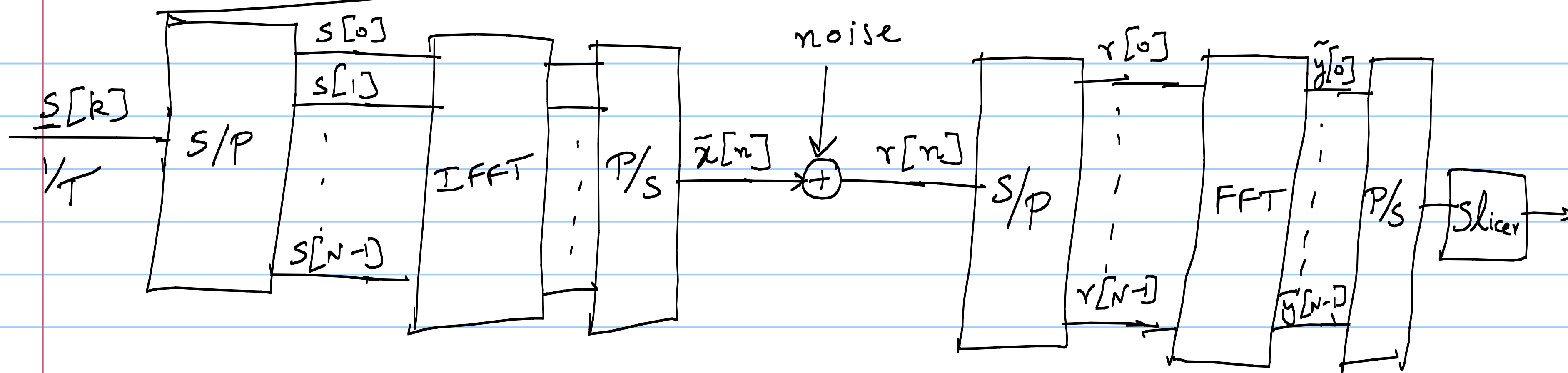
independent slicing

$$\tilde{y}[l] = \sum_{n=0}^{N-1} r[n]\, e^{-j\frac{2\pi n l}{N}}$$

$\downarrow$

FFT of $r[0], \ldots, r[N-1]$
at l

Discrete-time OFDM (No ISI)



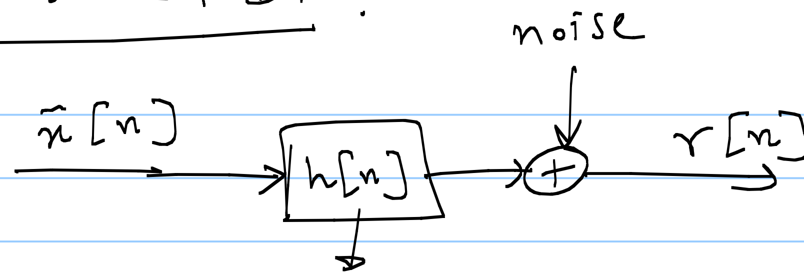
$$\underbrace{\begin{bmatrix}\tilde{x}[0]\\ \tilde{x}[1]\\ \vdots \\ \tilde{x}[1]\end{bmatrix}}_{\underline{\tilde{x}}} = FFT_N^{-1} \underbrace{\begin{bmatrix} s[0]\\ s[1]\\ \vdots\\ s[N-1]\end{bmatrix}}_{\underline{s}}$$

$$\underline{r} = \begin{bmatrix} r[0] \\ \vdots \\ r[N-1] \end{bmatrix} = \underbrace{FFT_N^{-1}}_{\text{unitary}} \underline{s} + \underline{n}$$

Rx:

$$FFT_N \, \underline{r} = \underline{s} + \underbrace{FFT_N \, \underline{n}}_{\text{same statistics as } \underline{n}}$$

ISI and OFDM:

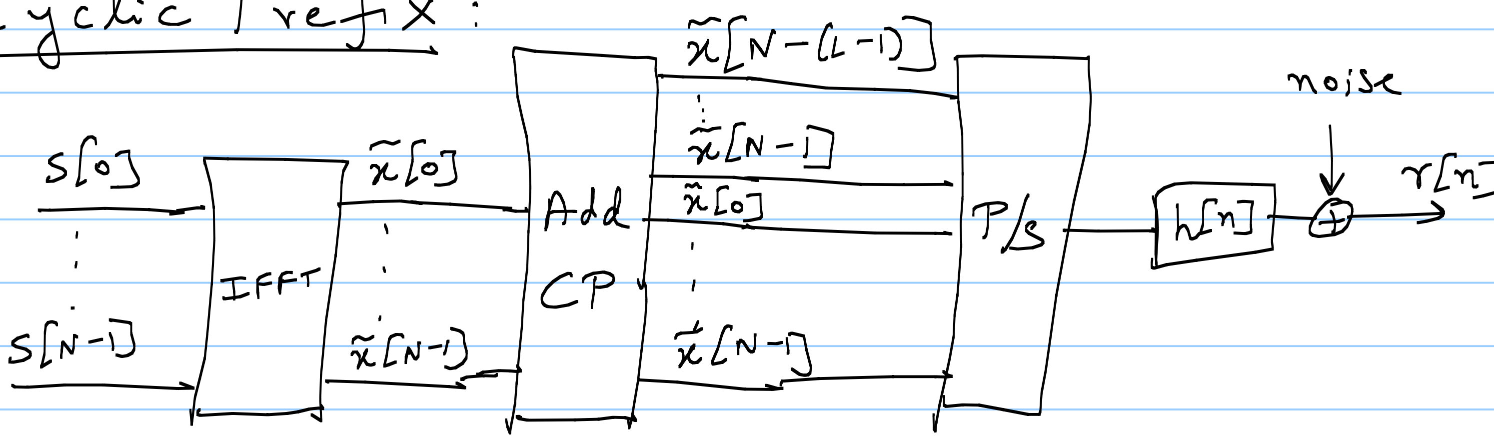


$$h[n] = \big(h[0] \quad h[1] \; \cdots \; h[L-1] \big)$$

L-tap channel.

$$r[n] = \tilde{x}[n] * h[n] + \text{noise}$$

Cyclic Prefix:



$$r[n] = \sum_{m=0}^{L-1} h[m]\, \tilde{x}[n-m] \quad + \text{noise}$$

$$\underline{\tilde{x}} = \left[\tilde{x}[N-L+1] \cdots \tilde{x}[N-1]\ \tilde{x}[0] - - - - \tilde{x}[N-1]\right]$$

$$\underline{r} = \left[\tilde{r}[-L+1] \cdots \tilde{r}[-1]\ \underbrace{\tilde{r}[0] - - - - \tilde{r}[N-1]}\right]$$

$$\begin{bmatrix} \tilde{r}[0] \\ \vdots \\ \vdots \\ \tilde{r}[N-1] \end{bmatrix} = \underset{\substack{\text{Circulant} \\ \text{matrix}}}{?} \begin{bmatrix} \tilde{x}[0] \\ \vdots \\ \vdots \\ \vdots \\ \tilde{x}[N-1] \end{bmatrix}$$