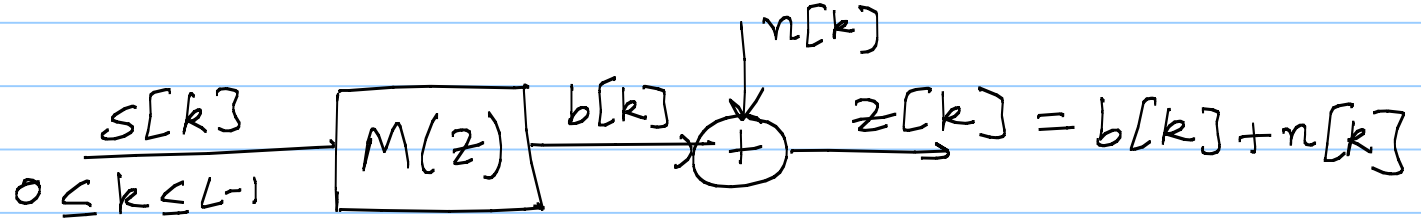
Lecture 21

Note Title 9/10/2008



m[k]: monic,
causal,
min-phase
M-tap filter

b[k]: nonzero
for $0 \le k \le L+M-1$

$$\left(b[k] = \sum_{l=0}^{M} m[l]\, s[k-l] \right.$$

$$\left. k = L+M \;,\quad b[L+M] = 0 \right)$$

Maximum Likelihood Sequence Detection:

$$\hat{\underline{s}} = \arg \min_{\underline{a} \in \mathcal{X}^L} \| \underline{z} - \underline{b} \|^2$$

$$\underline{b} = (a[k] * m[k])$$

$$b[k]$$

$$0 \leq k \leq L+r-1$$

→ brute force : $\sim |\mathcal{X}|^L$ computations.

→ Viterbi algorithm : $\sim |\mathcal{X}|^{\mu}$ computations

$|\mathcal{X}| = 4$

$\mu = 2$

$$m[k] \longleftrightarrow M(z) = m_0 + m_1 z^{-1} + \cdots + m_\mu z^{-\mu}$$

$$b[k] = \sum_{l=0}^{\mu} m[l]\, s[k-l]$$

$$b[k] = m[0]s[k] + m[1]s[k-1] + \cdots + m[\mu]s[k-\mu]$$

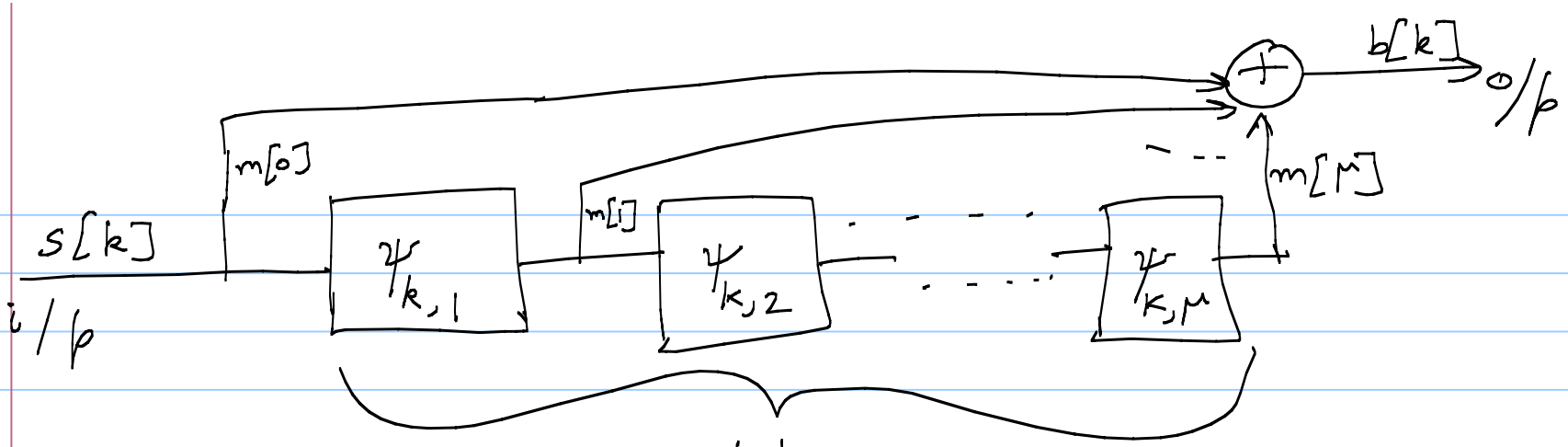
$b[k]$: o/p at time k

$s[k]$: i/p at time k

$s[k-1] + \cdots + m[\mu]s[k-\mu]$: state at time k

$$\psi_k = [s[k-1] \ldots s[k-\mu]] \in \mathcal{X}^\mu$$

$$\psi_{k+1} = [s[k] \quad s[k-1] \ldots s[k-\mu]]$$



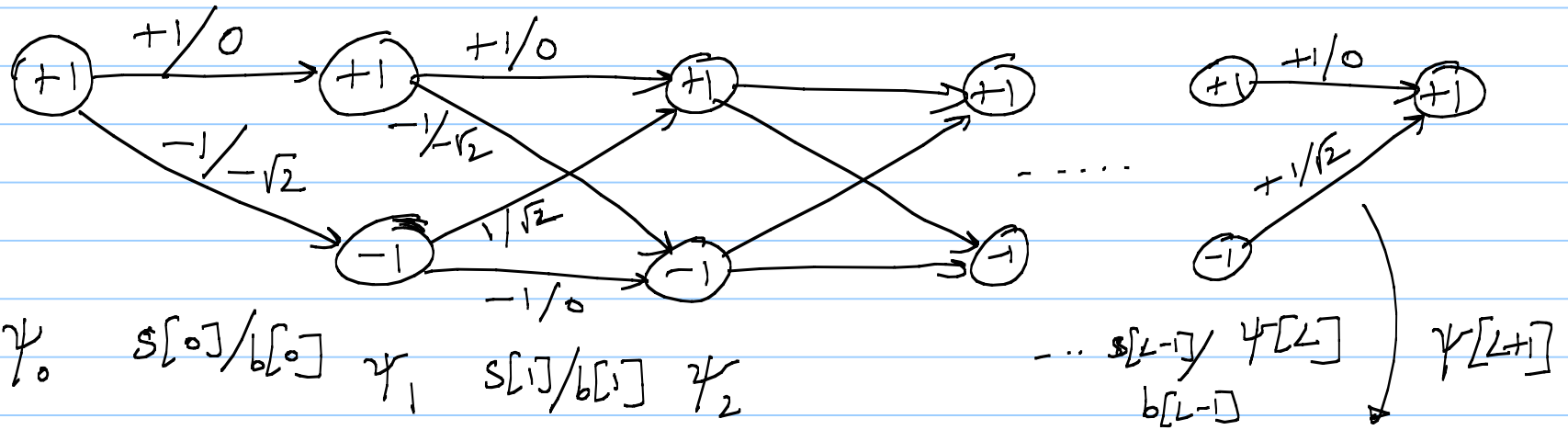
Total # of states $= |\mathcal{X}|^{\mu}$

Trellis:

⟶ State diagram with a time axis.

Ex: $M(z) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} z^{-1}$, $\mathcal{X} = \{\pm 1\}$

$\psi_k = s[k-1]$, $\psi_0 = [+1]$

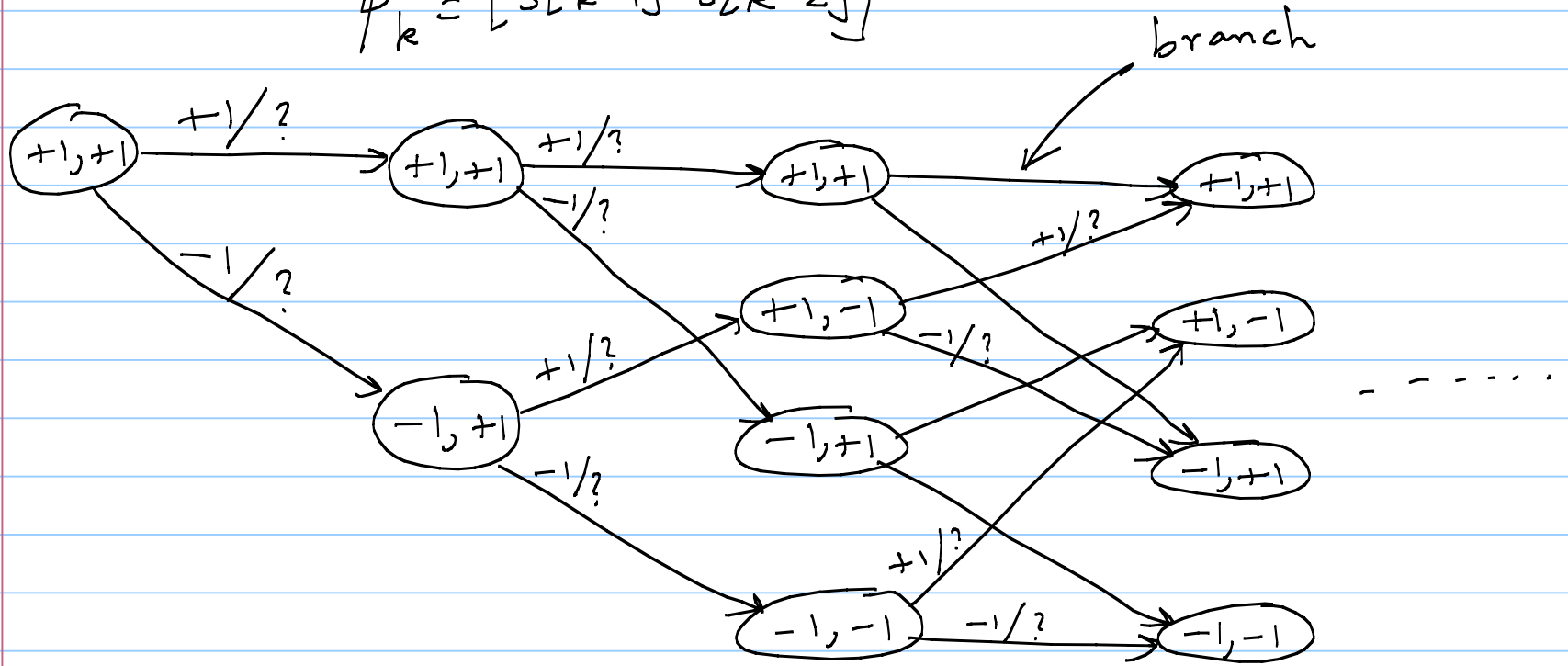


ψ_0, $s[L]$: known at Rx.

Why termination? will be used in MLSD.

Ex: $M(z) = \frac{1}{\sqrt{2}} + \frac{1}{2} z^{-1} + \frac{1}{2} z^{-2}$, $\mathcal{X} = \{\pm 1\}$

$\psi_k = [s[k-1] \; s[k-2]]$



$z[0]$ $z[1]$ $z[2]$ . . . .

ψ_0 $s[0]/b[0]$ ψ_1 $s[1]/b[1]$ ψ_2 $s[2]/b[2]$ ψ_3 . . . .

stage 2