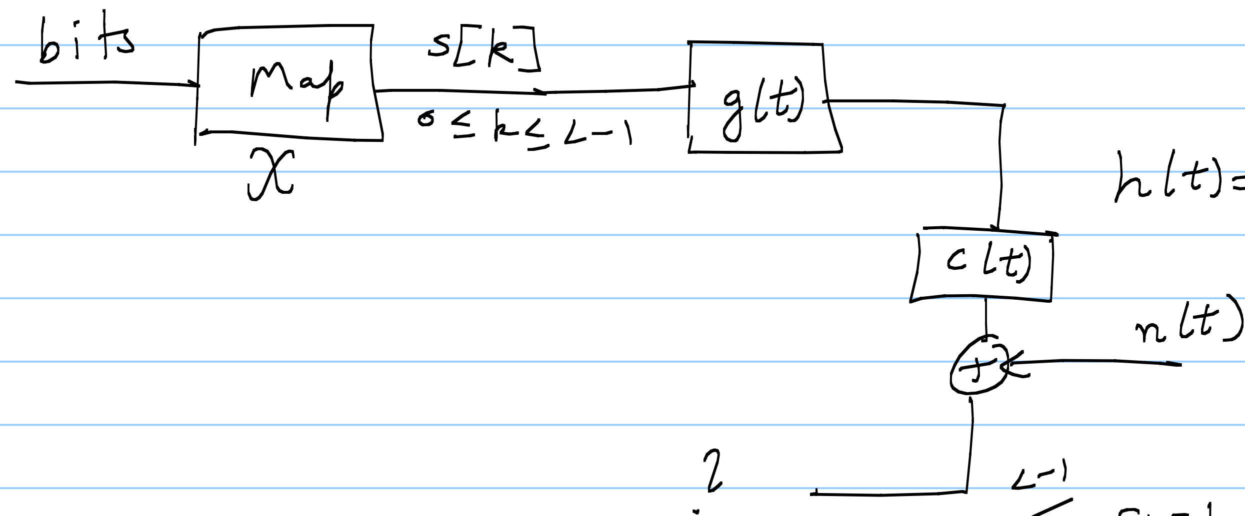
Lecture 18

Note Title 9/3/2008



$$h(t) = g(t) * c(t)$$

$$r(t) = \sum_{k=0}^{L-1} s[k]\, h(t-kT) + n(t)$$

$$\underline{a} \in \mathcal{X}^L$$

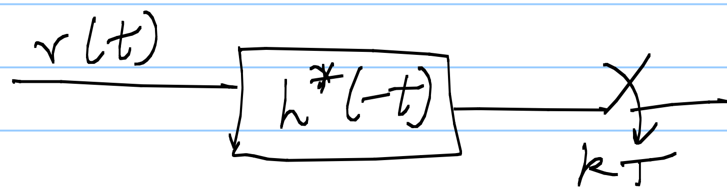
$$J_{\underline{a}} = \text{distance}\left\{ r(t), \sum_{k=0}^{L-1} a[k]\, h(t-kT) \right\}$$

$$\hat{\underline{s}} = \arg \min_{\underline{a} \in \mathcal{X}^L} J_{\underline{a}}$$

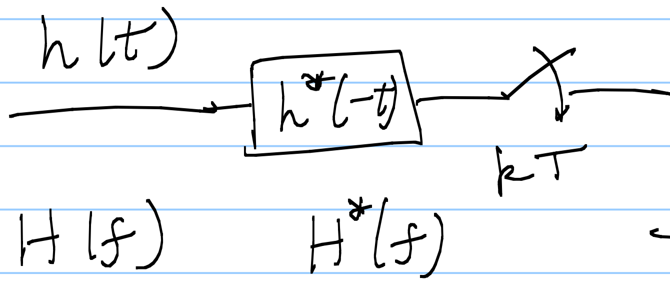
$$J_{\underline{a}} = E_r - 2\,\mathrm{Re}\left\{ \sum_{k=0}^{L-1} a^*[k]\, y[k] \right\} \quad \sim L \text{ computations}$$

$$E_r = \|r(t)\|^2$$

$$+ \sum_{k=0}^{L-1} \sum_{j=0}^{L-1} a[k]\, a^*[j]\, \rho_h[j-k] \quad \sim L^2$$



$$y[k] = \int_{-\infty}^{\infty} r(t)\, h^*(t-kT)\, dt$$



$$\rho_h[k]$$

$$\updownarrow$$

$$S_h(f) = \frac{1}{T} \sum_{m=-\infty}^{\infty} \left| H\left(f - \frac{m}{T}\right)\right|^2$$

$$\updownarrow$$

$$S_h\left(e^{j2\pi fT}\right)$$

Spectral factorization:

$\rho_h[k] \overset{DTFT}{\longleftrightarrow} S_h(e^{j\theta})$: real, non-negative

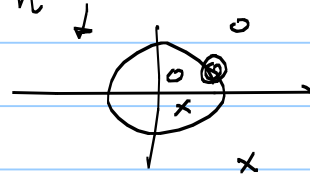
$\Downarrow$ unique

$$\rho_h[k] = \gamma^2 \, m[k] * m^*[-k] \longrightarrow \text{true always.}$$

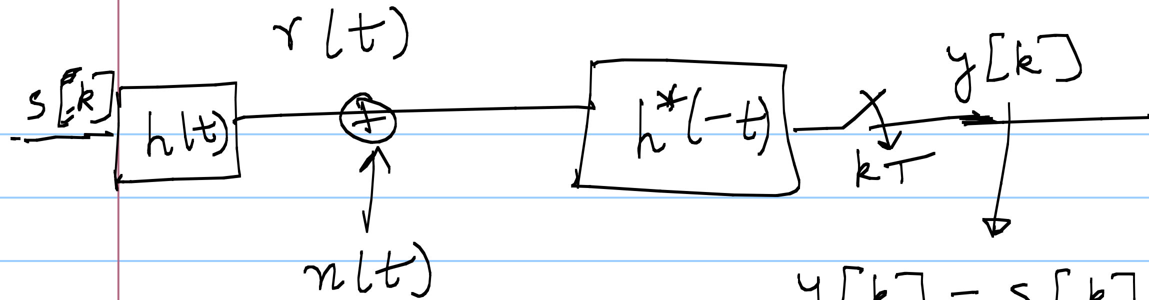
$m[k]$: monic, causal, loosely minimum-phase

loosely: zeros on unit circle is allowed.

Z-domain: $S_h(z) = \gamma^2 M(z) \cdot M^*\left(\frac{1}{z^*}\right)$



"Signal Analysis" A. Papoulis.



$$r^2 * m[k] * m^*[-k]$$

$$y[k] = s[k] * \rho_h[k] + n[k]$$

$$\sum_{l=-\infty}^{\infty} s[l]\,\rho_h[k-l]$$

both

Causal & anti-causal ISI terms

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PSD: $S_h(f)$

$$m^*[-k] \Longleftrightarrow M^*(1/z^*)$$

$$J_{\underline{a}} = E_r - 2\,\mathrm{Re}\left\{ \sum_{k=0}^{L-1} a^*[k]\, y[k] \right\}$$ ~L computations

$\|r(t)\|^2$

$$+ \sum_{k=0}^{L-1} \sum_{j=0}^{L-1} a[k]\, a^*[j]\, \rho_h[j-k]$$ ~L^2

$$r(t) \longrightarrow \boxed{h^*(-t)} \xrightarrow{kT} y[k] \longrightarrow \boxed{\frac{1}{\gamma^2 m^*(1/z^*)}} \longrightarrow z[k]$$

$$\gamma^2 z[k] * m^*[-k] = y[k]$$

$$y[k] = \gamma^2 \sum_{l=k}^{\infty} z[l]\, m^*[l-k]$$

$$J_{\underline{a}} = E_v + \gamma^2 \| z[k] - a[k] * m[k] \|^2 - \gamma^2 \| z[k] \|^2$$

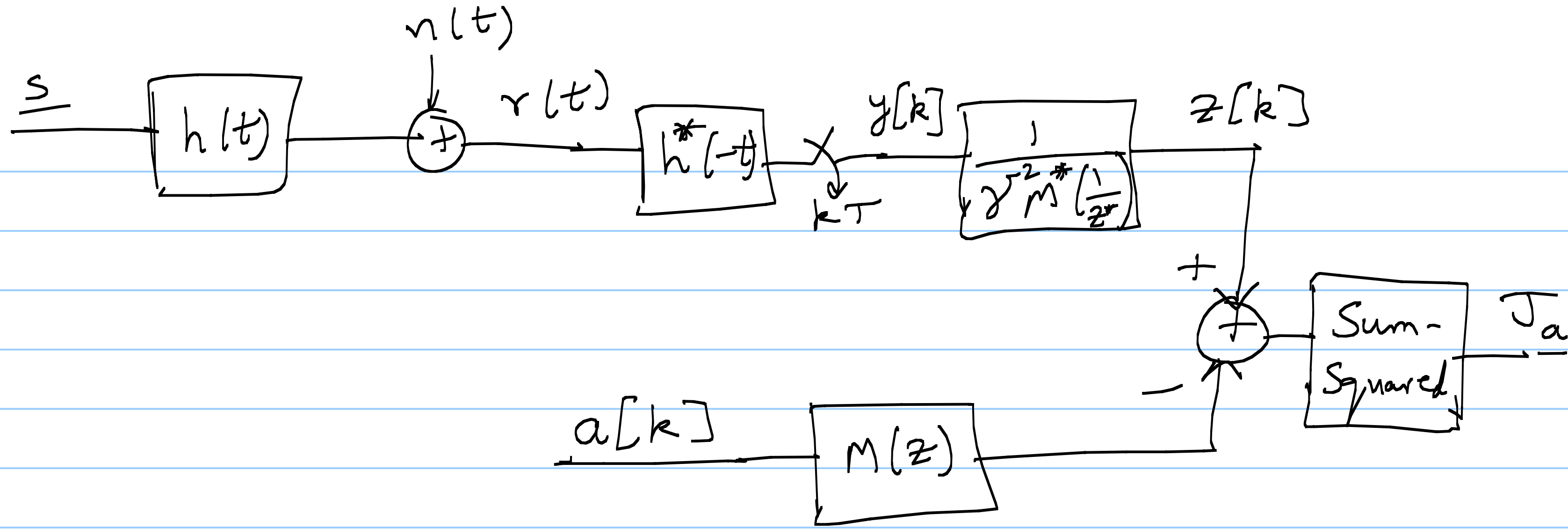
$$\left(\| x[k] \|^2 = \sum_{k=-\infty}^{\infty} | x[k] |^2 \right)$$

$$\sum_{l=0}^{L-1} a[l] m[k-l]$$

$$\underline{\hat{S}} = \arg \min_{\underline{a}} \left\| z[k] - \sum_{l=0}^{L-1} a[l] m[k-l] \right\|^2 \rightarrow J_{\underline{a}}'$$

$$\underline{\hat{s}} = \arg \min_{\underline{a}} \sum_{k=0}^{\infty} \left| z[k] - \sum_{l=0}^{L-1} a[l] m[k-l] \right|^2$$

$\sim L$ computations



$$y[k] = s[k] * \rho_h[k] + n[k]$$

PSD: $S_h(f)$

$$z[k] = s[k] * m[k] + n'[k]$$

PSD: $\frac{N_0}{2\gamma^2}$ white.