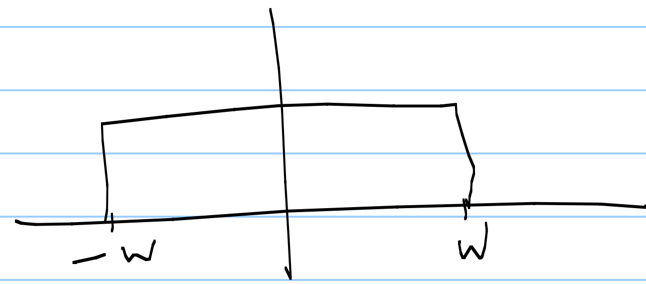
Lecture 17

Note Title 9/2/2008

Summary (Ideal channel)

Baseband PAM



Avoid ISI by choosing a "Nyquist" pulse shape :

$$\frac{1}{T} = \frac{2W}{1+\beta} \qquad 0 < \beta < 1$$

"symbol rate".

Real constellation $\mathcal{X}_r$ $\quad R = \log_2 |\mathcal{X}_r|$ bits/dim

Bit rate $R_b = \dfrac{\log_2 |\mathcal{X}_r|}{T} = \dfrac{2W \log_2 |\mathcal{X}_r|}{1+\beta}$

Spectral efficiency

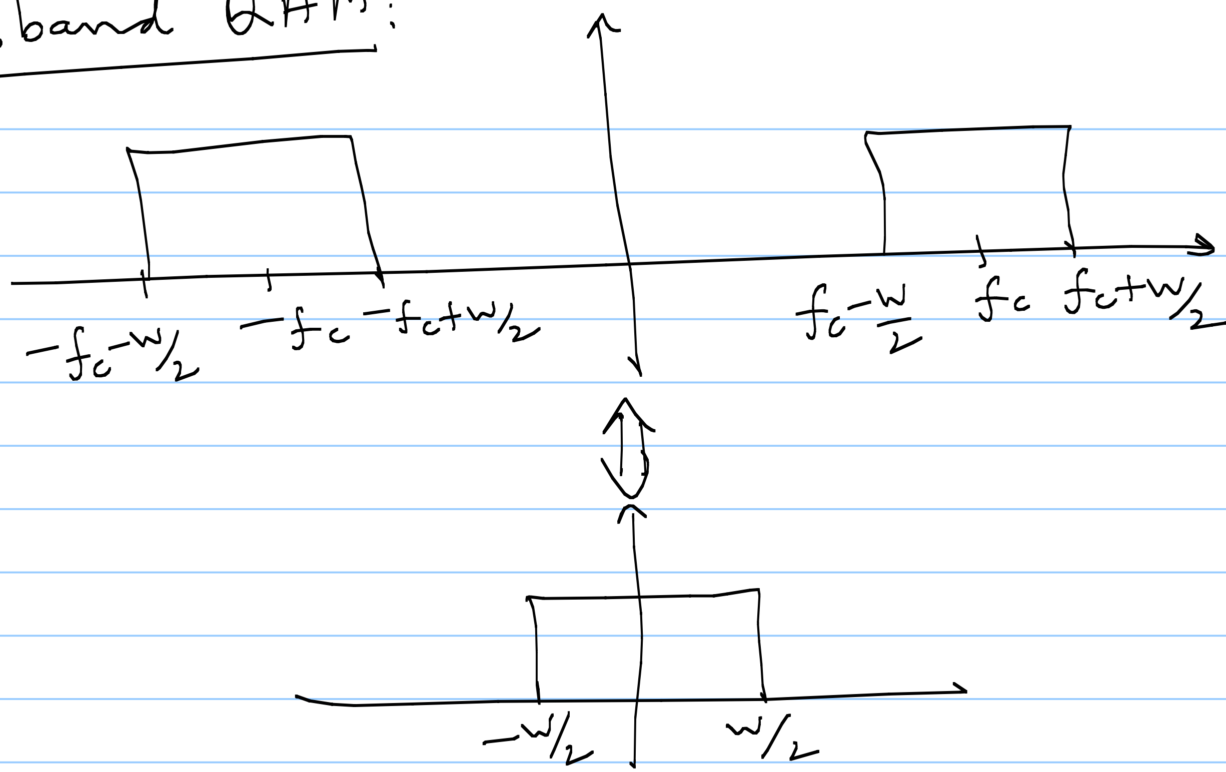
$$\boxed{\nu = \frac{R_b}{W} = \frac{2\log_2 |X_r|}{1+\beta}}$$

$$SNR = \frac{P}{N_0 W} = \frac{E_s}{N_0/2} \quad ; \quad \frac{E_b}{N_0} = \frac{SNR}{2R}$$

M-PAM:

$$P_e \cong 2\,Q\left(\sqrt{\frac{6\log_2 M}{M^2-1}\,\frac{E_b}{N_0}}\right)$$

Passband QAM:



Avoid ISI: $\frac{W}{2} = \frac{1+\beta}{2T}$

Symbol rate $\frac{1}{T} = \frac{W}{1+\beta}$

Complex constellation $\mathcal{X}_c \Rightarrow R = \frac{1}{2}\log_2|\mathcal{X}_c|$ bits/dimension

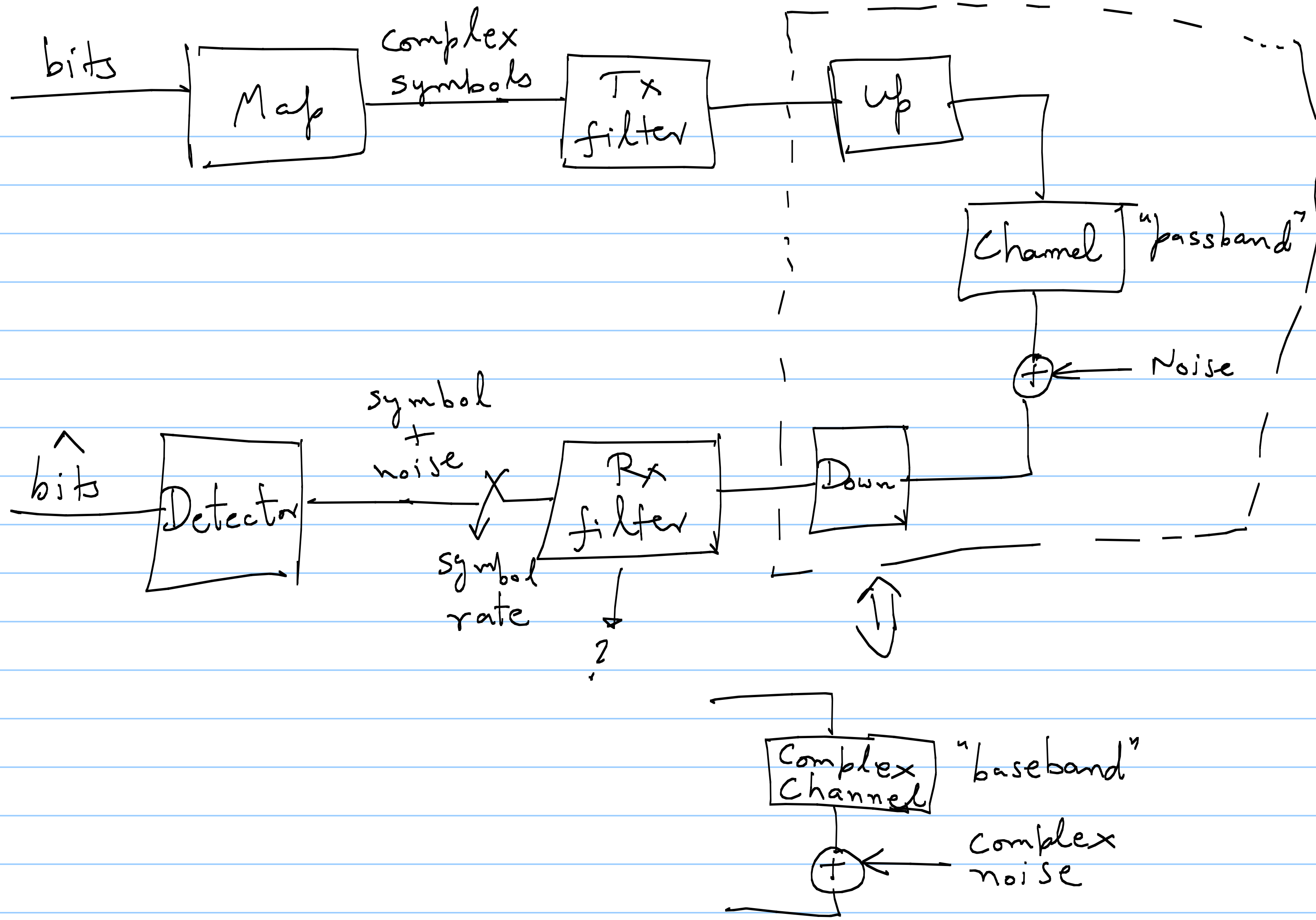
Bit rate $R_b = \dfrac{W \log_2 |\mathcal{X}_c|}{1+\beta}$

Spectral efficiency $\nu = \dfrac{R_b}{W} = \dfrac{\log_2|\mathcal{X}_c|}{1+\beta}$

$$SNR = \frac{P}{N_0 W} = \frac{E_s}{N_0}, \quad \frac{E_b}{N_0} = \frac{SNR}{2R}$$

M^2-QAM

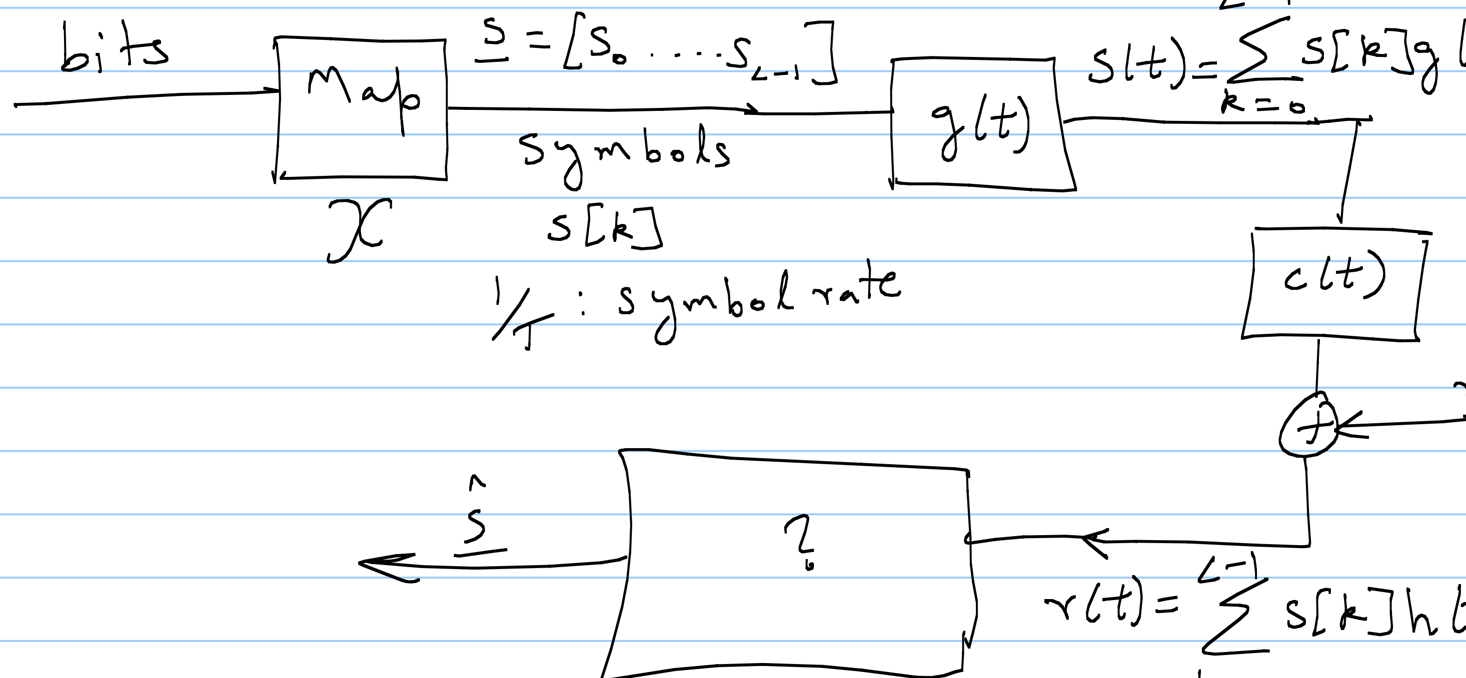
$$P_e \simeq 4\, Q\left(\sqrt{\frac{6\log_2 M}{M^2-1}\,\frac{E_b}{N_0}}\right)$$


bits
Map
complex symbols
Tx filter
Up
Channel
"passband"
+
Noise
Down
Rx filter
symbol + noise
symbol rate
?
Detector
$\hat{\text{bits}}$
Complex Channel
"baseband"
+
complex noise

Further ahead: → non-ideal channels

→ ISI at Rx.

→ Sec 5.4 in Barry, Lee & Messers...



$$s(t) = \sum_{k=0}^{L-1} s[k]\, g(t-kT)$$

$$r(t) = \sum_{k=0}^{L-1} s[k]\, h(t-kT) + n(t)$$

$$h(t) = g(t) * c(t)$$

"heuristic", minimum distance receiver:

$$\hat{\underline{s}} = \arg \min_{\underline{a} \in \mathcal{X}^L} \text{distance}\left(r(t), \sum_{k=0}^{L-1} a[k] h(t-kT)\right)$$

? $|x|^2$

$$J_{\underline{a}} = \int_{-\infty}^{\infty} \left| r(t) - \sum_{k=0}^{L-1} a[k] h(t-kT) \right|^2 dt$$

complex, continuous-time signal

$$= \int_{-\infty}^{\infty} |r(t)|^2 dt - 2 \,\text{Re}\left\{ \sum_{k=0}^{L-1} \left[a^*[k] \int_{-\infty}^{\infty} r(t) h^*(t-kT) dt \right] \right\}$$

$$+ \sum_{k=0}^{L-1} \sum_{j=0}^{L-1} \left[a_k a_j^* \int_{-\infty}^{\infty} h(t-kT) h^*(t-jT) dt \right]$$

$$y[k] = \int_{-\infty}^{\infty} r(t)\, h^*(t-kT)\, dt$$

$$\rho_h[k] = \int_{-\infty}^{\infty} h(t)\, h^*(t-kT)\, dt$$

$$J_{\underline{a}} = \overset{?}{E}_r - 2\,\mathrm{Re}\left\{\sum_{k=0}^{L-1} a_k^*\, y[k]\right\} + \sum_{k=0}^{L-1}\sum_{j=0}^{L-1} a_k a_j^* \rho_h[j-k]$$