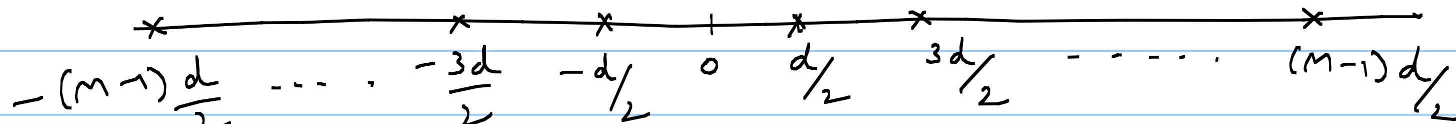
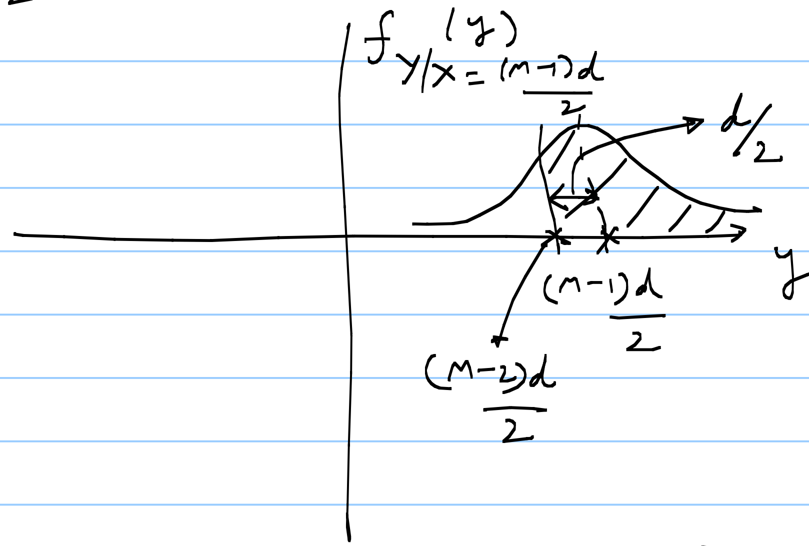
Lecture 12

Note Title 8/18/2008



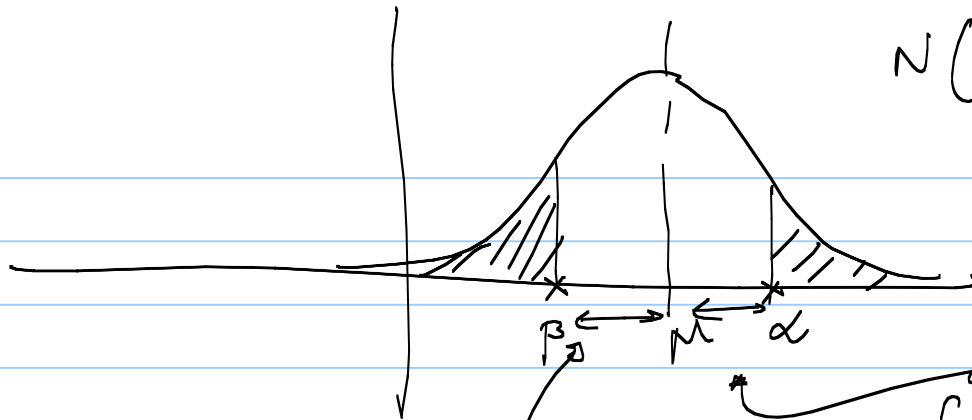
$$x = \frac{(M-1)d}{2}$$



$$\Pr\left(\text{Error} \,\middle/\, x = \frac{(M-1)d}{2}\right)$$

$$= \int_{-\infty}^{\frac{(M-2)d}{2}} N\left(\frac{(M-1)d}{2}, \frac{N_0}{2}\right) = Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$N(\mu, \sigma^2)$



$$\int_{\alpha}^{\infty} N(\mu, \sigma^2) = Q\left(\frac{\alpha - \mu}{\sigma}\right)$$

$$\int_{-\infty}^{\beta} N(\mu, \sigma^2) = 1 - \int_{\beta}^{\infty} N(\mu, \sigma^2)$$

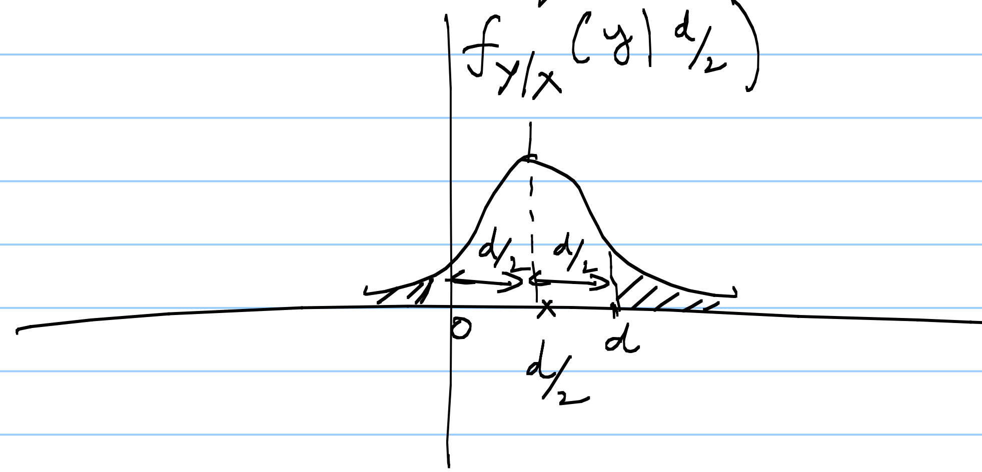
$$= Q\left(\frac{\mu - \beta}{\sigma}\right)$$

$$\int_{\beta}^{\alpha} N(\mu, \sigma^2) = ?$$

$$\Pr\left(\text{Error} \,\middle|\, X = -\frac{(M-1)d}{2}\right) = Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$$\Pr\left(\text{Error} \,\middle|\, X = \frac{d}{2}\right)$$

$$= 2Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$



$$\Pr(\text{Error}) = \frac{2}{M} Q\left(\frac{d/2}{\sqrt{N_0/2}}\right) + \frac{(M-2)}{M} 2Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$$= 2\left(1 - \frac{1}{M}\right) Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$\log_2 M$ bits/sym

Signal Energy $E_s = \frac{(M^2-1)d^2}{12}$

$E_N = \frac{N_0}{2}$ → level of PSD.

$$P_r(\text{Error}) = 2\left(1-\frac{1}{M}\right) Q\left(\frac{\sqrt{\frac{3E_s}{M^2-1}}}{\sqrt{E_N}}\right)$$

$$= 2\left(1-\frac{1}{M}\right) Q\left(\sqrt{\frac{3\,(SNR)}{M^2-1}}\right)$$

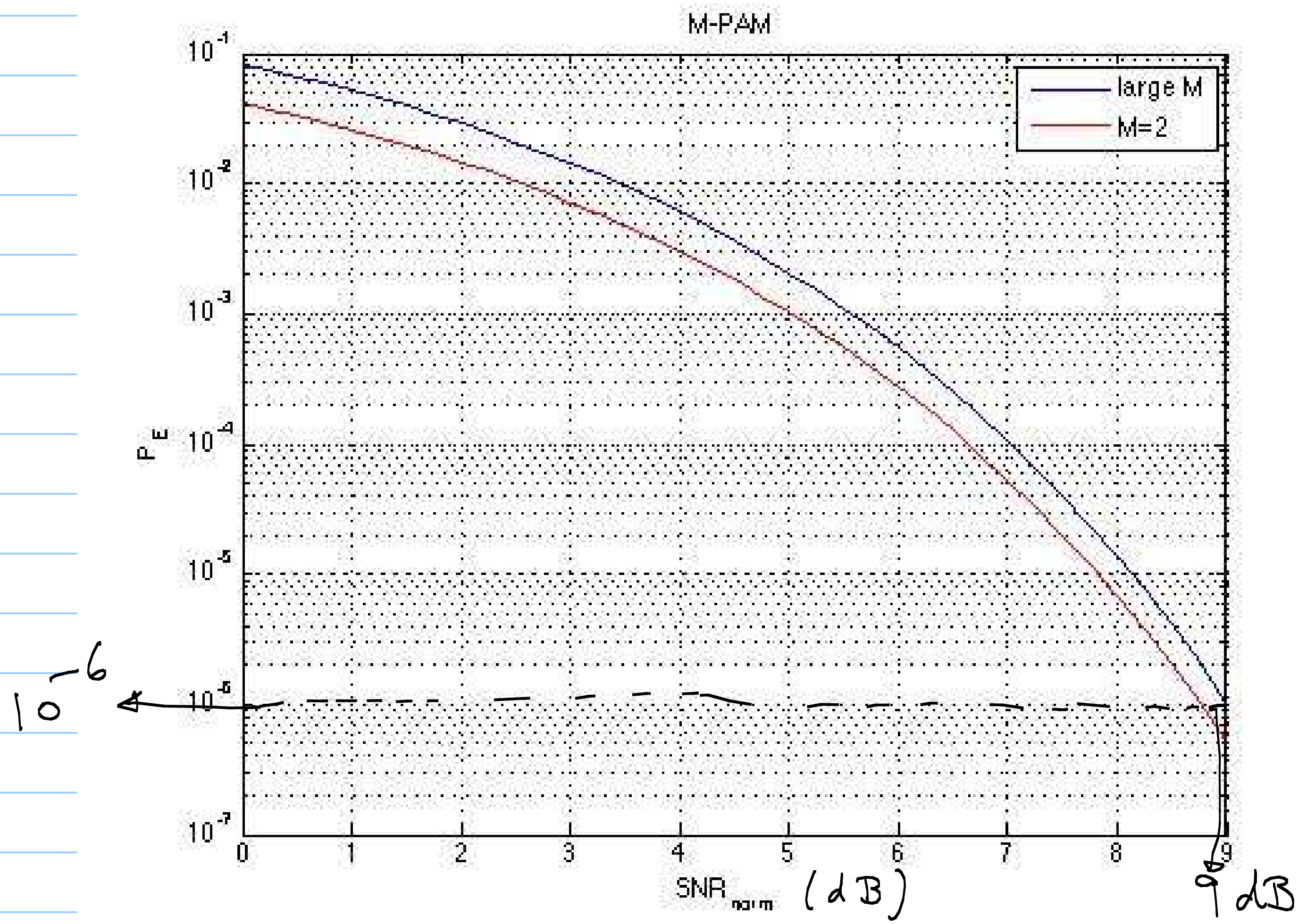
"Signal-to-noise ratio"

$SNR = \frac{E_s}{E_N}$

large-M $\approx 2\,Q\left(\sqrt{\frac{3\,SNR}{M^2-1}}\right)$

Normalized SNR (M-PAM), $SNR_{norm} = \frac{SNR}{M^2-1}$

$$\Pr(\text{Error}) \approx 2\,Q\left(\sqrt{3\,SNR_{norm}}\right)$$



$$\Pr\{\text{Error}\} \simeq 2\, Q\left(\sqrt{\frac{3\ \text{SNR}}{M^2 - 1}}\right)$$

$$\text{rate} = \log_2 M \ \text{bits/symbol}$$

increase rate by 1

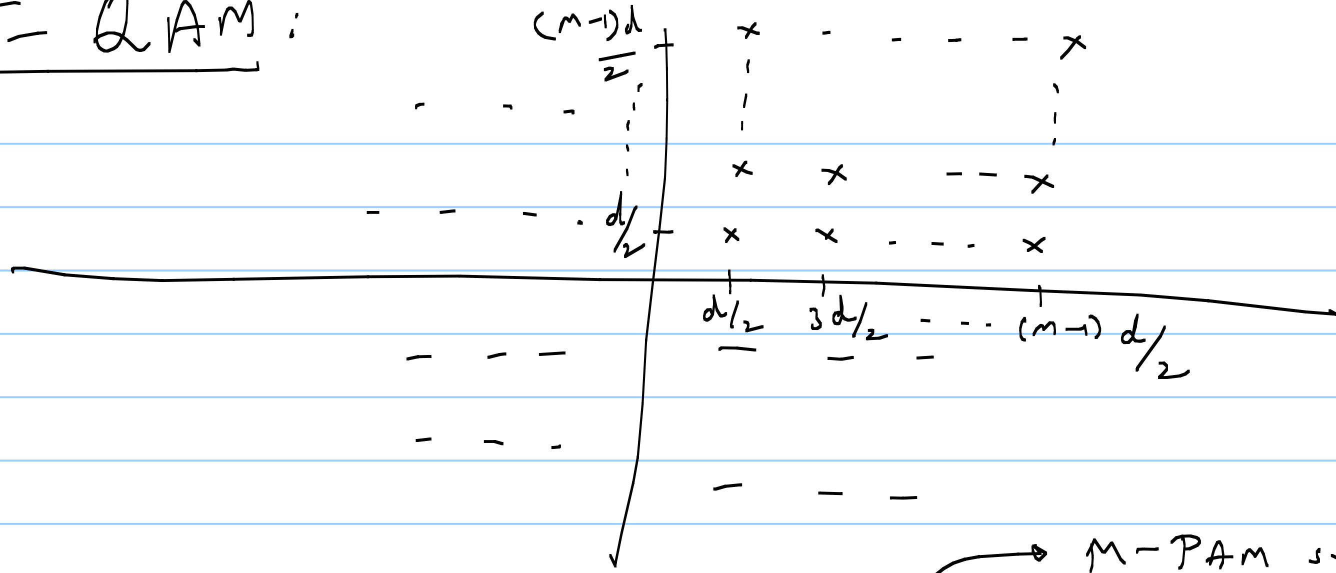
$\Downarrow$

Double M

$\Downarrow$

increase SNR by 6 dB

M^2 – QAM :



M^2 – QAM symbol $\underline{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$

X_1 → M – PAM symbol

X_2 → M – PAM symbol

X_1 and X_2 independent

$$\underbrace{\Pr(\underline{X} \text{ is in error})}_{M^2-\text{QAM}} = 1 - \Big(1 - \underbrace{\Pr(X_1 \text{ is in error})}_{M-\text{PAM}}\Big)^2$$

$$\Pr\{\text{Error}\} = 1 - \left(1 - 2\,Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)\right)^2$$

for m^2 - QAM

$$\simeq 4\,Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$$E_N = \frac{N_0}{2}\,, \qquad E_s = \frac{2\,(m^2-1)\,d^2}{12}$$

$$\text{Rate} = 2\log_2 m \text{ bits/symbol}$$

$$\Pr\{\text{Error}\} = 4\,Q\left(\sqrt{\frac{3}{2}\,\frac{SNR}{(m^2-1)}}\right)$$

for m^2 - QAM

M-PSK:

$$\Pr\{Error\} \approx (?)\, Q\left(\sqrt{SNR \cdot (?)}\right)$$

for M-PSK