Note Title 8/14/2008

Lecture 11

Detection: $\underset{\in \mathcal{X}}{\underset{T_x}{\underline{X}}} \xrightarrow[f_{\underline{Y}|\underline{X}}(\underline{y}|\underline{x})]{\text{joint pdf}} \underset{R_x}{\underline{Y}} \longrightarrow \hat{\underline{X}}$

function of $\underline{Y}$

$\hat{\underline{X}}$: estimate of $\underline{X}$

MAP

$$\hat{\underline{X}} = \arg \max_{\underline{x} \in \mathcal{X}} \Pr(\underline{X} = \underline{x} \mid \underline{Y} = \underline{y})$$

ML

$$\hat{\underline{X}} = \arg \max_{\underline{x} \in \mathcal{X}} f_{\underline{Y}|\underline{X}}(\underline{Y} = \underline{y} \mid \underline{X} = \underline{x})$$

Minimum Distance

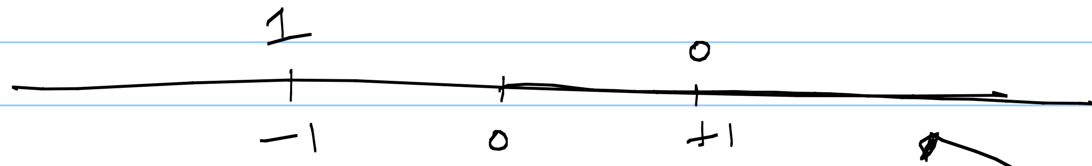
$$\hat{\underline{X}} = \arg \min_{\underline{x} \in \mathcal{X}} \|\underline{y} - \underline{x}\|^2$$

$\hat{\underline{X}}$: function of $\underline{Y}$

Decision Regions:

$\underline{Y} \in$ Signal space

→ but not a point on the constellation.



Decision region for +1: $\{y \in \mathbb{R} : \hat{X}(y) = +1\}$

General: Decision region for $\underline{x} \in \mathcal{X}$: $\{\underline{y} \in \mathbb{R}^M : \hat{\underline{X}}(\underline{y}) = \underline{x}\} = D_{\underline{x}}$

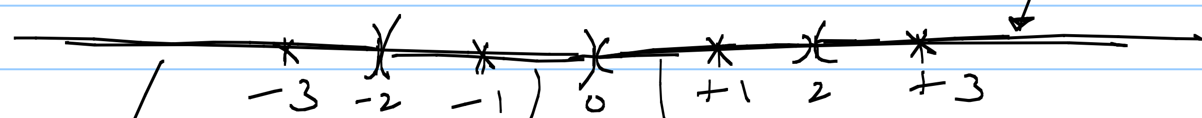
$$\Pr(\hat{\underline{X}} = \underline{x} \mid \underline{X} = \underline{x}) = \Pr(\underline{y} \in D_{\underline{x}} \mid \underline{X} = \underline{x})$$

$$= \int_{\underline{y} \in D_{\underline{x}}} f_{\underline{Y}|\underline{X}}(\underline{y}|\underline{x})\, d\underline{y}$$

Examples of Decision Regions

(2) **4-PAM**

$D_3 = [2, \infty)$

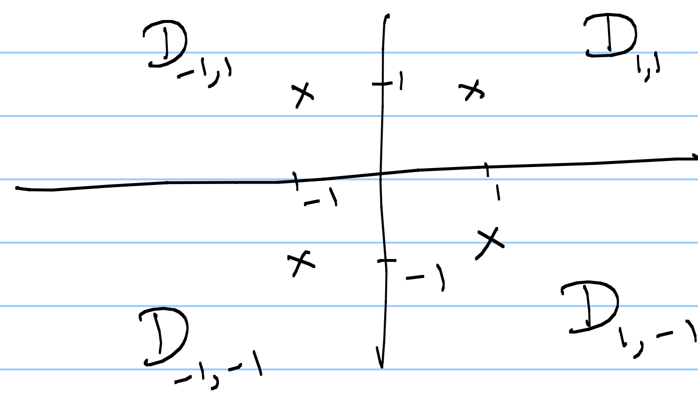


$D_{-3} = (-\infty, -2]$

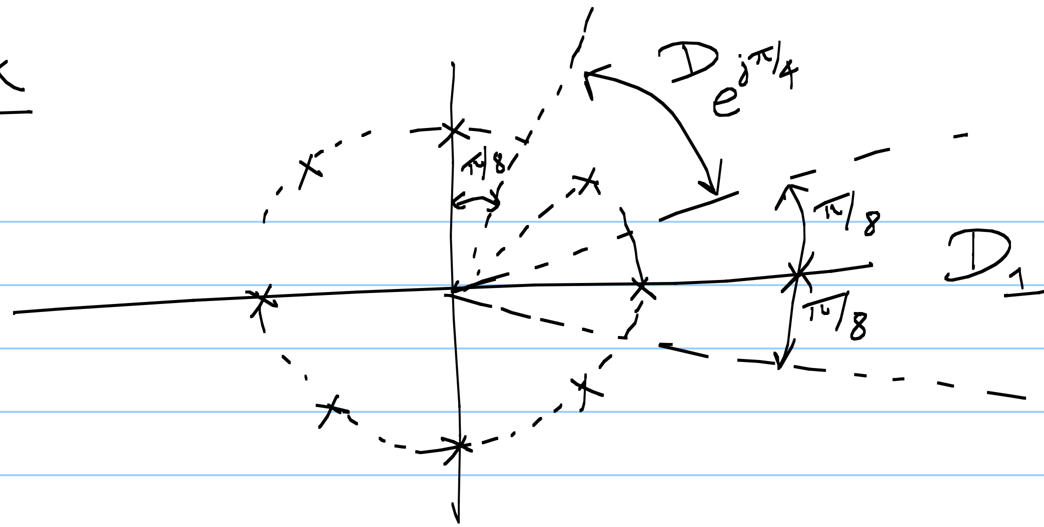
$D_1 = [0, 2]$

$D_{-1} = [-2, 0]$

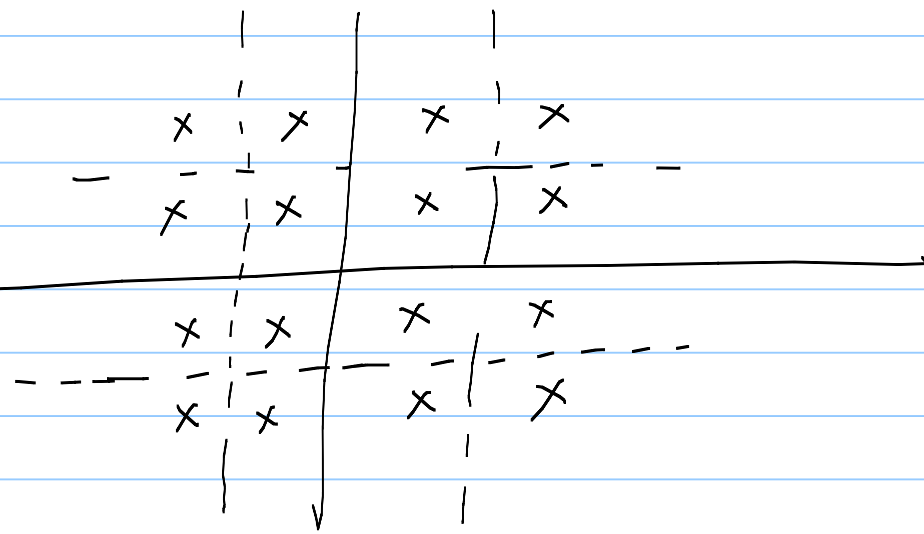
(1) **4-QAM**



④ 8-PSK



⑤ 16-QAM



(6) M-PAM

$-(M-1) \quad \cdots \quad -3 \quad -1 \quad 0 \quad +1 \quad +3 \quad \cdots \quad (M-1)$

$$D_{M-1} = [M-2, \infty)$$

$$D_{-(M-1)} = (-\infty, -(M-2)]$$

$$D_i = [i-1, i+1] \quad (\text{other } i)$$

Probability of error for M-PAM (Minimum distance decoder)

General PAM: $\quad X \sim \text{Uniform}$

$-(M-1)\frac{d}{2} \quad \cdots \quad -\frac{3d}{2} \quad -d/2 \quad 0 \quad d/2 \quad 3d/2 \quad \cdots \quad (M-1)\frac{d}{2}$

Energy in signal $E_s = E[X^2] = E[X(t)^2]$

$$E_s = \frac{(m^2-1)d^2}{12} \quad (\text{or})$$

$$d = \sqrt{\frac{12 E_s}{m^2-1}}$$

$$N \sim N\left(0, \frac{N_0}{2}\right)$$

$$E_N = E[n_1^2(t)] = \frac{N_0}{2}$$

2

Received value $Y = X + N$

$$\Pr\{\text{Error}\} = P_E = \Pr\{\hat{X} \neq X\}$$

$$= \sum_{x \in \mathcal{X}} \underbrace{\Pr(\hat{X} \neq x \mid X = x)} \cdot \underbrace{\Pr(X = x)}_{1/m}$$

$\Pr\{\hat{X} \neq x \mid X = x\}$:

$$x = \pm \frac{(M-1)d}{2}$$

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^{\infty} e^{-t^2/2}\, dt$$

$$\left(x = \frac{(M-1)d}{2}\right)$$

$$\Pr(\hat{X} \neq x \mid X = x) = \int_{-\infty}^{\frac{(M-2)d}{2}} N\left(\frac{(M-1)d}{2}, \frac{N_0}{2}\right)$$

$$x = -\frac{(M-1)d}{2}$$

$$\Pr(\hat{X} \neq x \mid X = x) = \int_{-\frac{(M-2)d}{2}}^{\infty} N\left(-\frac{(M-1)d}{2}, \frac{N_0}{2}\right) \checkmark$$

$$= Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$$x = \pm i \frac{d}{2} \qquad \Pr(\hat{X} \neq x \mid X = x) = 2\,Q\left(\frac{d/2}{\sqrt{N_0/2}}\right)$$

$$i = 1, 3, \ldots, (M-3)$$