Solutions to Problem Set 8

EE419: Digital Communication Systems

Check the solutions for possible bugs!

1. (a) The DTFT of $\phi[m] = \alpha^{|m|}$ is the PSD $S(e^{j\omega})$ given by

$$S(e^{j\omega}) = \frac{1-\alpha^2}{1+\alpha^2-2\alpha\cos\omega}$$

assuming $|\alpha| < 1$. The minimum and maximum values of $S(e^{j\omega})$ are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1+\alpha}{1-\alpha}$ assuming $0 \leq \alpha \leq 1$.

(b) For a third order filter, the matrix ϕ is given by

$$\phi = \begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}.$$

The eigenvalues when $\alpha = 0.5$ are 1.84, 0.75 and 0.41. The range of step size for convergence is $0 \leq \beta \leq 2/1.84 = 1.08$. As order becomes larger, the maximum eigen value tends to $\frac{1+\alpha}{1-\alpha}$. Hence, we get

$$0 \leq \beta \leq \frac{2(1-\alpha)}{1+\alpha}.$$

2. Construct the matrix ϕ and computes its eigenvalues. If $\lambda_{\max}$ is the largest eigenvalue, then

$$0 \leq \beta \leq 2/\lambda_{\max}.$$

For asymptotics, consider the minimum and maximum values of $S_z(e^{j\omega})$, which is the PSD of $z[k]$.

3. For a DFE with precursor $C(z) = \sum_{i=-P}^{0} c[i]z^{-i}$ and postcursor $D(z) = \sum_{i=1}^{P'} d[i]z^{-i}$, the update is as follows. Set a joint coefficient vector $\underline{v} = [c_{-P} \ \cdots \ c[0] \ -d[1] \ \cdots \ -d[P']]$ and the input signal vector $\underline{z}_k = [z[k+P] \ \cdots \ z[k] \ s[k-1] \ \cdots \ s[k-P']]$ (in training mode). Note that the input signal vector becomes $\underline{z}_k = [z[k+P] \ \cdots \ z[k] \ \hat{s}[k-1] \ \cdots \ \hat{s}[k-P']]$ in decision-directed mode. The slicer error can be written as

$$e[k] = \hat{s}[k] - \underline{v}_k^T \underline{z}[k],$$

where $\underline{v}_k$ is the coefficient vector at time k. The update rule for the coefficient vector becomes

$$\underline{v}_{k+1} = \underline{v}_k + \beta e[k]\underline{z}^*[k].$$

For an FSE, the update is skipped for every second k.