Problem Set 4

EE419: Digital Communication Systems

1. A receiver in a digital communication system receives the following signal:

$$r(t) = s(t) + n(t),$$

where $s(t$ is the signal component and $n(t)$ is white Gaussian noise. The signal component $s(t) = 0, 0 \leq t \leq T$ when bit 0 is transmitted and $s(t) = A, 0 \leq t \leq T$ when bit 1 is transmitted. Transmitted bits are 0 or 1 with equal probability.

   (a) Describe an optimal way of processing the received signal.
   (b) Define a suitable SNR and determine the probability of error as a function of SNR.
   (c) Compare with the scenario when $s(t) = -A, 0 \leq t \leq T$ for bit 0.

2. A receiver in a digital communication system receives the following signal:

$$r(t) = s(t) + n(t),$$

where $s(t$ is the signal component and $n(t)$ is white Gaussian noise. The signal component $s(t) = \cos(2\pi t/T), 0 \leq t \leq T$ when bit 0 is transmitted and $s(t) = \cos(4\pi t/T), 0 \leq t \leq T$ when bit 1 is transmitted. Transmitted bits are 0 or 1 with equal probability.

   (a) Describe an optimal way of processing the received signal.
   (b) Define a suitable SNR and determine the probability of error as a function of SNR.
   (c) Compare with the scenario when $s(t) = \cos(\pi t/T), 0 \leq t \leq T$ for bit 0 and $s(t) = \cos(3\pi t/2T), 0 \leq t \leq T$ for bit 1.
   (d) Compare with previous problem.

3. Show that the probability of symbol error of the standard 16-QAM constellation is given exactly by

$$3Q(1/\sigma) - 2.25Q^2(1/\sigma).$$

Can you generalize for 2^{2b}-QAM, $b = 2, 3, \ldots$?

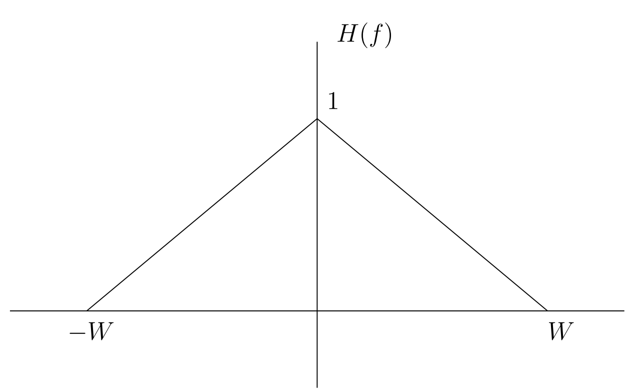
4. Let $h[k]$ be a causal, minimum-phase signal with a rational Z transform $H(z)$. Obtain a signal $g[k]$ whose Z transform is obtained by replacing a zero of $H(z)$ at z_0 with a zero at $1/z_0^*$.

   (a) Show that $|H(e^{j\theta})| = |G(e^{j\theta})|$.
   (b) Show that, for all $N \geq 0$,

$$\sum_{k=0}^{N} |h[k]|^2 \geq \sum_{k=0}^{N} |g[k]|^2.$$

   (c) Conclude from the above that among all (rational Z transform) signals with the same magnitude response, the minimum-phase signal has the most concentration of energy near $k = 0$.

5. Consider a signal $h(t)$ with Fourier transform shown below.

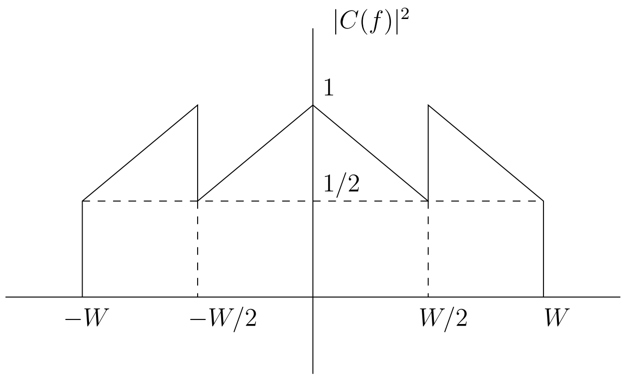


(a) Sketch the spectrum of $\rho_h[k] = h(t) \otimes h^*(-t)|_{kT}$ (called folded spectrum) for various sampling rates $1/T$. Distinguish the cases $W > 1/T$, $W < 1/T$ and so on.

(b) Is there any $1/T$ for which the folded spectrum satisfies the Nyquist criterion?

(c) Find $\rho_h[k]$ for $1/T = W/2$, $1/T = W$ and $1/T = 2W$.

6. Consider QAM transmission over a channel with squared frequency response shown below.



Can you choose a transmit filter with a sinc pulse as its impulse response and satisfy the Nyquist criterion for some symbol rate?

7. Consider QAM transmission over a baseband equivalent channel with frequency response

$$C(f) = \begin{cases} \sin(\pi f/W), & 0 \leq |f| \leq W \\ 0, & \text{else.} \end{cases}$$

Can you choose a transmit filter to satisfy the Nyquist criterion for some symbol rate?

8. Consider QAM transmission over a baseband equivalent channel with frequency response

$$C(f) = \begin{cases} e^{j\phi(f)}, & 0 \leq |f| \leq W \\ 0, & \text{else} \end{cases}$$

for an arbitrary $\phi(f)$.

(a) Can you choose a transmit filter with a sinc pulse as its impulse response and satisfy the Nyquist criterion for some symbol rate?

(b) Study the situation when $\phi(f) = 2\pi f \tau$ (for some τ) is an ideal phase response.

(c) What happens if $\phi(f)$ is some non-ideal phase response?

9. Perform spectral factorization on the folded spectrum given by

$$S_h(e^{j\theta}) = 1 - \cos\theta + \cos^2\theta, \ -\pi \leq \theta \leq \pi.$$

10. Consider a QAM system operating at a symbol rate of $1/T = 2W$ with the transmit filter's impulse response being $g(t) = \frac{\sin(2\pi W t)}{\pi t}$ (sinc pulse with frequency response $\text{rect}_{[-W,W]}(f)$) and channel frequency response (complex baseband) given as

$$C(f) = \begin{cases} \sin(\frac{\pi f}{2W}), & 0 \leq |f| \leq W \\ 0, & \text{else.} \end{cases}$$

Derive the discrete-time equivalent of the transmission.