EE611 Problem Set 3

1. A transmitter uses the signals $\{s_i(t)\}$ to communicate one of $M > 2$ equally likely messages over an AWGN channel with power spectral density $N_0/2$, where for $i = 0,\ 1,\ 2,\ \cdots,\ M-1$

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_s}{T}} \cos\left(2\pi \dfrac{k}{T} t + \dfrac{2\pi i}{M}\right) & 0 \le t < T, \quad k \text{ an integer} \\ 0 & \text{elsewhere.} \end{cases}$$

   (a) Sketch the signal vectors and optimum decision regions for $M = 5$.

   (b) Show that the minimum attainable $P[\varepsilon]$ is bounded by

$$p \le P[\varepsilon] \le 2p,$$

   where

$$p = Q\left(\sqrt{\frac{2E_s}{N_0}} \sin\frac{\pi}{M}\right).$$

2. Sketch the decision regions for the optimum receiver for the 8-point signal constellation in Figure 1.

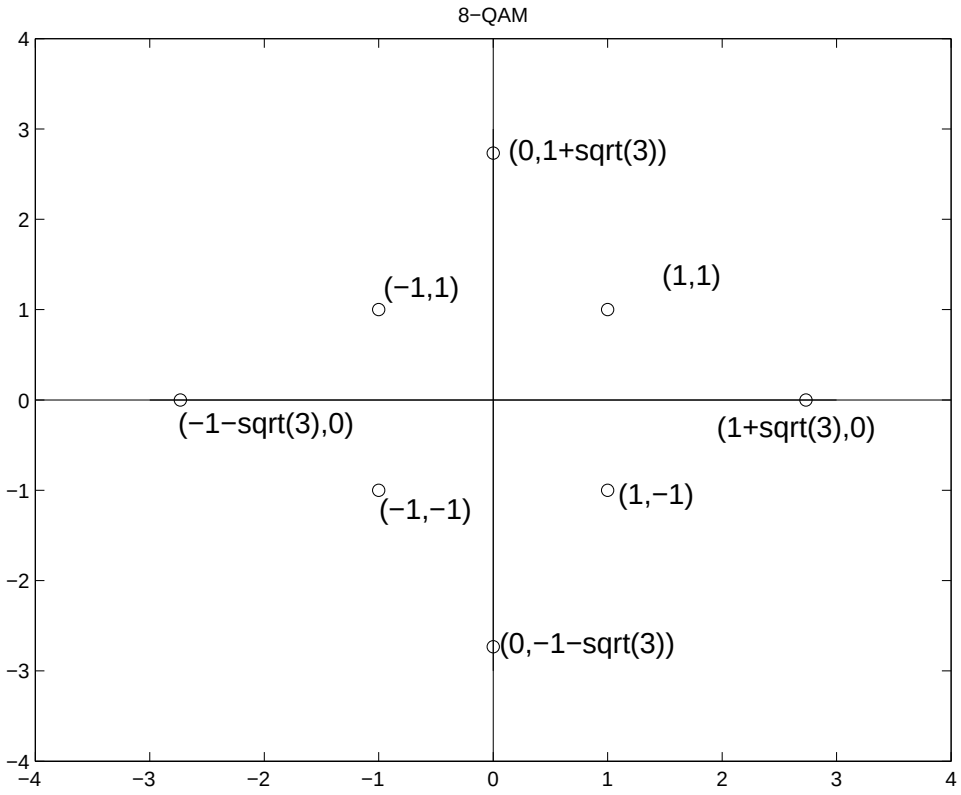

Figure 1: Signal Constellation

3. A voice-band telephone channel has a passband characteristic in the frequency range $300Hz < f < 3000Hz$. Assume that the channel has an ideal frequency response characteristic in this band.

   (a) Select a symbol rate and a power-efficient constellation size to achieve 9600 bps signal transmission.

   (b) If a square-root raised cosine spectrum pulse is used for the transmitter pulse, select the roll-off factor.

4. The Nyquist criterion gives the necessary and sufficient condition for the spectrum $X(f)$ of the pulse $x(t)$ that yields zero ISI. For any pulse (real) that is band-limited to $|f| < 1/T$, the zero-ISI condition is satisfied if $Re[X(f)]$, for $f > 0$, consists of a rectangular function plus an arbitrary odd function around $f = 1/2T$, and $Im[X(f)]$ is any arbitrary even function around $f = 1/2T$.

   (a) Express the raised cosine spectrum in the above form, i.e, as the sum of a rectangular function and an odd function.

   (b) Give another example for $X(f)$ that satisfies the above condition.

5. Suppose a digital communications system employs Gaussian-shaped pulses of the form $x(t) = \exp(-\pi a^2 t^2)$. To reduce the level of ISI to a relatively small amount, we impose the condition that $x(T) = 0.01$, where T is the symbol interval. The bandwidth W of the pulse $x(t)$ is defined as the value of W for which $X(W)/X(0) = 0.01$, where $X(f)$ is the Fourier transform of $x(t)$. Determine the value of W and compare this value to that of the raised-cosine spectrum with 100% rolloff.

6. Show that the raised cosine spectrum, $X_{rc}(f)$, satisfies $\int_{-\infty}^{\infty} X_{rc}(f)df = 1$ for any value of β, the roll-off factor.

7. White noise with power spectral density $N_0/2$ is filtered by a filter with square-root raised cosine frequency response. What is the variance of the filtered noise?

8. A 4 kHz bandpass channel is to be used for transmission of data at a rate of 9600 bps. Design a QAM modulation using a signal pulse with a raised-cosine spectrum having a roll-off factor of at least 0.5.