EE611 Problem Set 2

1. One of three equally likely messages is communicated over a vector channel which adds a (different) statistically independent zero-mean Gaussian random variable with variance $N_0/2$ to each transmitted vector component. Assume that the transmitter uses the signal vectors $\underline{s}_k = (\cos\theta_k \quad \sin\theta_k)$, where θ_k takes values 0, $2\pi/3$, and $4\pi/3$ for the three messages.

   (a) Determine the minimum distance between any two points in the signal constellation.

   (b) Sketch the decision regions for the optimum receiver that minimizes the probability of symbol error.

   (c) Express the average probability of symbol error, P_e, in terms of the conditional error probabilities given the message index k, $P_{e|k}$.

   (d) Show that the probability of symbol error is upper bounded by $2Q\left(\sqrt{\frac{3}{2N_0}}\right)$.

2. Consider a communication system model where two received outputs, r_1 and r_2, are available for decision making, as shown below:

$$r_1 = s + n_1 \quad \text{and} \quad r_2 = n_1 + n_2.$$

   Assume n_1, n_2 are i. i. d. Gaussian random vectors with each element having variance $\frac{N_0}{2}$. Assume equally likely symbols $s_0 = \sqrt{E_s}$ and $s_1 = -\sqrt{E_s}$ are transmitted. Assume that s, n_1 and n_2 are independent.

   (i) Determine whether r_2 is irrelevant.

   (ii) Determine the optimum decision rule and the probability of symbol error.

   (iii) Repeat part (ii), if

$$r_1 = s + n_1 \quad \text{and} \quad r_2 = 2s + n_2.$$

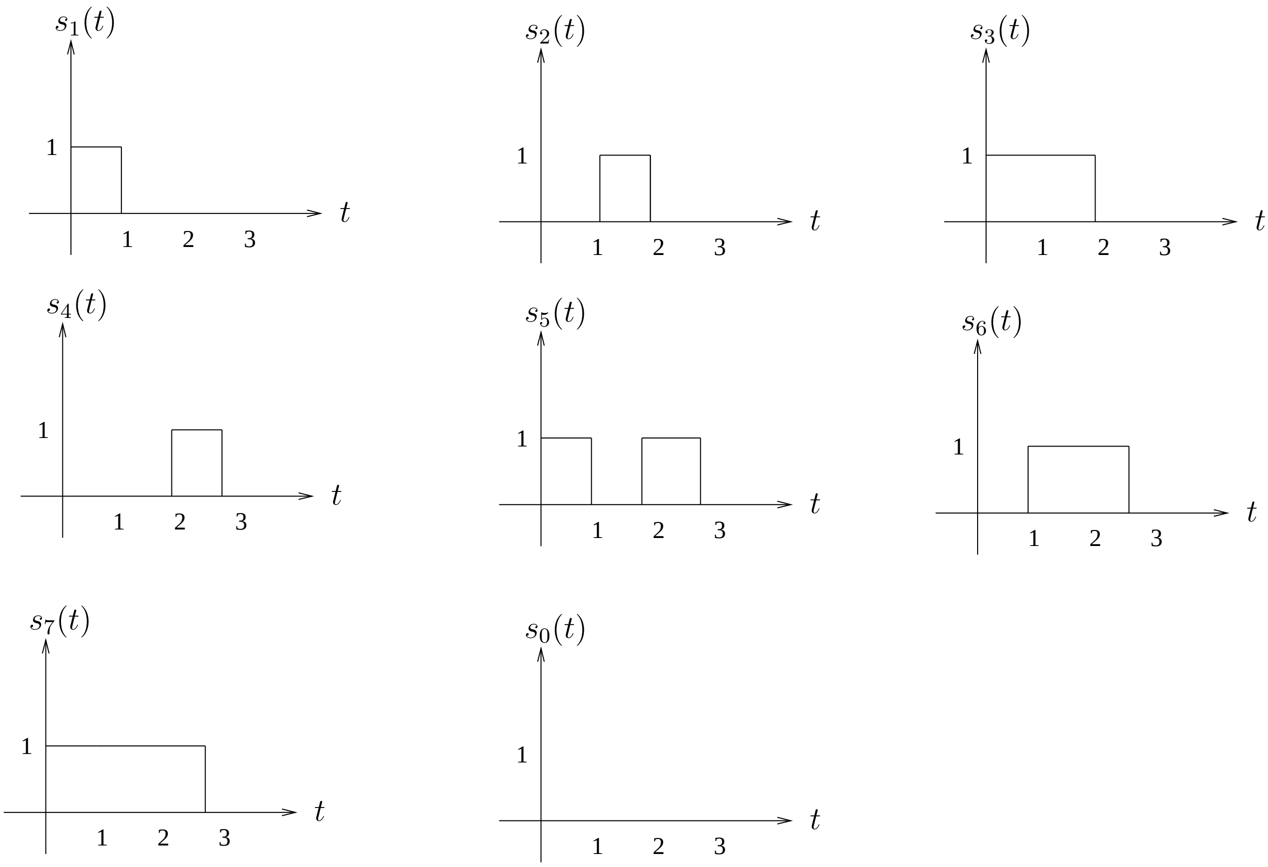
3. Let $r = s + n$, where r is the received signal, s is the transmitted signal, and n is Gaussian, with zero mean. If one of two equally likely messages is transmitted using $s_0 = -2$, and $s_1 = 2$, the optimum receiver yields $P[\varepsilon] = 0.01$.

   (i) What is the minimum attainable probability of error $P[\varepsilon]_{min.}$, when (a) three equally likely messages are transmitted using the signals $s_0 = -4, s_1 = 0, s_2 = 4$? (b) when four equally likely messages are transmitted using the signals $s_0 = -4, s_1 = 0, s_2 = 4, s_3 = 8$?

   (ii) How do the answers to part (i) change if it is known that $E[n] = 1$ rather than 0?

4. One of the four equally likely messages is to be communicated over a vector channel which adds a (different) statsitically independent zero-mean Gaussian random variable with variance $\frac{N_0}{2}$ to each transmitted vector component. Assume that the transmitter uses the signal vectors $\mathbf{s}_i = \left(\cos[\frac{\pi}{4}(2i+1)], \ \sin[\frac{\pi}{4}(2i+1)]\right)$, for $i \in \{0, 1, 2, 3\}$ and express the probability of error, $P[\varepsilon]$, produced by an optimum receiver in terms of the function $Q(\alpha) = \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_\alpha^\infty e^{-\frac{x^2}{2}} dx$.

5. Express the minimum probability of error, $P[\varepsilon]_{min}$, in terms of $Q(\alpha)$ when the signal set given in Figure 1 is used to communicate one of eight equally likely messages over a channel disturbed by additive white Gaussian noise with power spectral density $\frac{N_0}{2}$.



s1(t)
1
t
1
2
3
s2(t)
1
t
1
2
3
s3(t)
1
t
1
2
3
s4(t)
1
t
1
2
3
s5(t)
1
t
1
2
3
s6(t)
1
t
1
2
3
s7(t)
1
t
1
2
3
s0(t)
1
t
1
2
3

Figure 1: