Optimized Codes for Bidirectional Relaying

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***Abstract*—In this article, we study coding strategies and code optimization for bidirectional relaying using Low-Density Parity-Check codes. We attempt to achieve extreme points in the rate region using density-evolution-based optimization and a combination of nesting and shortening of codes. The proposed method with specific choice of codes achieves rates close to capacity outer bounds.**

I. INTRODUCTION

The bidirectional (or two-way) relay problem has attracted significant attention in the past few years.

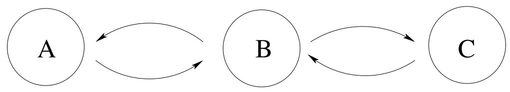

Fig. 1. Bidirectional relaying problem.

The setting is shown in Fig. 1, where node A wishes to send a k_1-bit message $\mathbf{m_1}$ to C, while node C has to send a k_2-bit message $\mathbf{m_2}$ to A. The relay node B facilitates this exchange of information. We assume that there is no direct link between the two communication nodes A and C. Nodes are half-duplex, average power limited with receiver noise variance of σ^2. We consider the channels BA and BC to be corrupted by Additive White Gaussian Noise (AWGN). Let h_1 and h_2 be the gains of channels AB (also, BA) and CB (also, BC), respectively.

Most schemes for bidirectional relaying work in two phases. In the multiple access (MAC) phase, nodes A and C transmit to the relay node B in n_1 time units or channel uses. Node B processes the received information, and transmits in n_2 channel uses to both A and C in the broadcast phase. The processing done by the relay node is called the relaying strategy. The rate at which the message $\mathbf{m_1}$ flows from A to C is $R_{AC} = k_1/(n_1 + n_2)$, and the rate of $\mathbf{m_2}$ is $R_{CA} = k_2/(n_1+n_2)$. The goal of the design is to optimize the rates R_{AC} and R_{CA} for given channel gains.

In [1]–[8], bidirectional relaying was studied from an information theoretic perspective and achievable rates were determined for various protocols. In [9]–[12], specific coding techniques were suggested and analyzed. A relaying strategy that emerges from these studies is XOR decoding during the MAC phase. Specific code constructions using repeat-accumulate and LDPC codes were provided in [9], [11], [12].

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In this work, the goal is to demonstrate low frame-error rates with finite block lengths at rates that are close to the capacity outer bound. To accomplish this objective, we use a Low-Density Parity-Check (LDPC) code that is optimized for XOR decoding of the transmitted codewords at the relay. In the second phase, the estimated XOR is transmitted in a broadcast-with-side-information channel [3] from the relay to the communicating nodes. Data rates and codes in the two phases are optimized to approach the capacity outer bound. A single code (and its nested subcode) are used for achieving different points in the rate region by shortening the code.

Compared to the prior work, novel aspects in our work include the following: (1) jointly optimized design of codes and data rates to approach the capacity outer bound in both the MAC and broadcast phases, (2) LDPC code optimization in the MAC phase for XOR decoding using density-evolution with *iid* channel adapters and nesting conditions.

II. CODING STRATEGY

Let us assume that $|h_2| \geq |h_1|$. Let C_1 and C_2 be two linear block codes of same block length n and dimensions k_1 and k_2, respectively. Let G_1 and G_2 be the generator matrices of codes C_1 and C_2, respectively. Communication between nodes A and C happens in two phases - Multiple access and Broadcast. The choice of rates and codes for these two phases is described next.

Multiple access phase: Nodes A and C encode the messages $\mathbf{m_1}$ and $\mathbf{m_2}$ into codewords $\mathbf{c_1} = \mathbf{m_1}G_1$ and $\mathbf{c_2} = \mathbf{m_2}G_2$, respectively. Nodes A and C simultaneously transmit these codewords, after suitable modulation, to the relay node B.

The relay node attempts to estimate the XOR of the codewords $\mathbf{x} = \mathbf{c_1} \oplus \mathbf{c_2}$. The XOR vector $\mathbf{x}$ belongs to the sum $D = C_1 \oplus C_2 = \{\mathbf{a}+\mathbf{b} : \mathbf{a} \,\epsilon\, C_1, \mathbf{b} \,\epsilon\, C_2\}$, which is another linear block code. We optimize the code D for decoding the XOR of codewords in the MAC-phase.

Virtual Channel: Since the relay attempts to decode the XOR of codewords in the MAC-phase, we can consider a virtual XOR-Channel (Fig. 2) with the XOR of messages $\mathbf{m_1}$ and $\mathbf{m_2'}$ as input, where $\mathbf{m_2'}$ is the message $\mathbf{m_2}$ with zero padding such that $\mathbf{m_2'}$ and $\mathbf{m_1}$ have the same lengths. The XOR of messages $\mathbf{m_1} \oplus \mathbf{m_2'}$ is encoded into the codeword $\mathbf{c_{XOR}}$ with code D. If we consider BPSK modulation $\{0, 1\} \rightarrow \{1, -1\}$ at nodes A and C, the constellation of received symbol at relay y_i in MAC-phase is $\{-h_1 - h_2, h_1 - h_2, -h_1 + h_2, h_1 + h_2\}$ (Fig. 3). Points $-h_1 - h_2, h_1 + h_2$ correspond to XOR zero and points $h_1 - h_2, -h_1 + h_2$ correspond to XOR one. Hence, the constellation mapper in the virtual channel is defined with the mapping:

$$x_{XORi} = \begin{cases} (h_1 - h_2) \text{ or } (h_2 - h_1) & \\ \text{with equal probability}, & c_{XORi} = 1, \\ & \\ (h_1 + h_2) \text{ or } (-h_2 - h_1) & \\ \text{with equal probability}, & c_{XORi} = 0, \end{cases} \tag{1}$$

where $\{x_{XORi} : i = 1, 2, \cdots n\}$ is the output symbol of the constellation mapper and $\{c_{XORi} : i = 1, 2, \cdots n\}$ is the input bit to the constellation mapper. The received value over the virtual channel is

$$y_{XORi} = x_{XORi} + z_i, \tag{2}$$

where $\{z_i : i \in 1, 2, \cdots n\}$ is additive Gaussian noise

The virtual XOR-Channel has a capacity $\beta_{XOR} = I(X; Y)$, where X is the random variable corresponding to the transmitted XORed bit and Y is the random variable corresponding to the received symbol of the virtual XOR-Channel. Hence, for reliable decoding of $\mathbf{c_1} \oplus \mathbf{c_2}$ at the relay, the code D needs to have rate $R_{XOR} \leq \beta_{XOR}$.

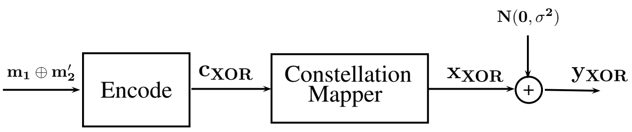

Fig. 2. Virtual XOR-Channel

Broadcast phase: At the end of broadcast phase, the nodes A and C need to be able to successfully decode the messages $\mathbf{m_2}$ and $\mathbf{m_1}$, respectively. Since these nodes already know their own transmitted messages $\mathbf{m_1}$ and $\mathbf{m_2}$, this is a broadcast-with-side-information scenario. The capacity region of the Binary Input Additive White Gaussian (BI-AWGN) broadcast channel with side information is expressed as

$$\mathcal{C}_{BC} = \{r_x, r_y : 0 \leq r_x \leq C(SNR_1), 0 \leq r_y \leq C(SNR_2)\}, \tag{3}$$

where $SNR_1 = \frac{|h_1|^2}{\sigma^2}$, $SNR_2 = \frac{|h_2|^2}{\sigma^2}$ and $C(SNR)$ is the BI-AWGN capacity [3]. In the broadcast phase, we attempt to achieve the point $(C(SNR_1), C(SNR_2))$. If the relay knows the messages $\mathbf{m_1}$ and $\mathbf{m_2}$, this can be accomplished by encoding to $\mathbf{m_1} G_{BC} \oplus \mathbf{m_2} G_{BA}$, where G_{BC} and G_{BA} are generator matrices of capacity-approaching codes for the point-to-point links BC and BA, respectively. However, at the relay we decode the XOR $\mathbf{c_1} \oplus \mathbf{c_2} = \mathbf{m_1} G_1 \oplus \mathbf{m_2} G_2$, and the individual messages $\mathbf{m_1}$ and $\mathbf{m_2}$ are not available. To resolve this problem, we design the codes C_1 and C_2, so that they perform well at rates close to the capacities of the point-to-point links BC and BA, respectively.

So, the estimated XOR vector $\hat{\mathbf{x}}$, which is an estimate of $\mathbf{m_1} G_1 \oplus \mathbf{m_2} G_2$ (the encoding necessary to achieve the point $(C(SNR_1), C(SNR_2))$), is broadcast by the relay. The nodes A and C use decoders corresponding to codes C_2 and C_1, respectively, to extract the messages $\mathbf{m_2}$ and $\mathbf{m_1}$, while the codewords $\mathbf{c_1}$ and $\mathbf{c_2}$ are used as side information.

Use of Nested codes: The design criteria on codes considering the multiple-access and broadcast phases are summarized as follows.

1) Code $D = C_1 \oplus C_2$, optimized for virtual XOR-Channel, needs to have rate $R_{XOR} \leq \beta_{XOR}$.
2) Code C_1, optimized for channel BC, needs to have rate $R_1 \leq C(SNR_2)$.
3) Code C_2, optimized for channel BA, needs to have rate $R_2 \leq C(SNR_1)$.

A decoder for the code D can be used to decode the binary XOR $\mathbf{c_1} \oplus \mathbf{c_2}$. The decoding could be further simplified by considering nested codes, $C_2 \subseteq C_1$. Then, $D = C_1$, and decoder for code C_1 could be used for decoding the XOR at the relay. With nested codes, code C_1 is designed for virtual XOR-Channel and code C_2 is designed for channel BA.

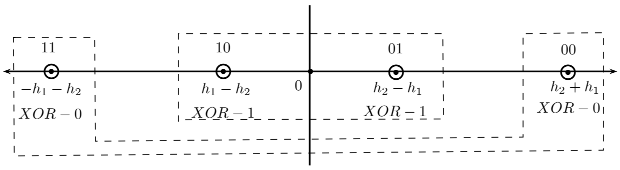

Fig. 3. Received constellation at relay

Estimation at relay B: The received symbol in the relay is

$$y_i = h_1 \mathbf{s}(c_{1i}) + h_2 \mathbf{s}(c_{2i}) + z_{Bi}, \tag{4}$$

where $\{z_{Bi} : i = 1, 2, \cdots n\}$ is additive Gaussian noise and $\mathbf{s} : \{0, 1\} \rightarrow \{1, -1\}$ denotes BPSK modulation done at A and C. While our methods can be extended to an arbitrary QAM constellation, we use BPSK for simplicity of description.

The constellation for the received symbol y_i is $\mathcal{M}_B = \{h_1 + h_2, h_1 - h_2, -h_1 + h_2, -h_1 - h_2\}$. Assuming that a soft decoder for the code D is employed at the relay for estimation of $\mathbf{x} = \mathbf{c_1} \oplus \mathbf{c_2}$, the log-likelihood ratios (LLRs) $l_i = \log \frac{\Pr\{x_i = 0 | y_i\}}{\Pr\{x_i = 1 | y_i\}}$, for the bits in $\mathbf{x} = [x_1 \; x_2 \cdots x_n]$ are computed. The LLRs $[l_1 \; l_2 \cdots l_n]$ are used as input to the soft decoder for the code D at the relay. At the end of soft decoding, we have the final estimate $\hat{\mathbf{x}} = [\hat{x}_1 \; \hat{x}_2 \cdots \hat{x}_n]$ for the vector $\mathbf{x}$.

Decoding at A and C: The estimated XOR vector $\hat{\mathbf{x}}$ is transmitted in the broadcast phase. The received symbols at A and C are

$$y_{Ai} = h_1\hat{x}_i + z_{Ai}, i = 1, 2, \cdots n, \tag{5}$$

$$y_{Ci} = h_2\hat{x}_i + z_{Ci}, i = 1, 2, \cdots n, \tag{6}$$

where $\{z_{Ai} : i = 1, 2 \cdots n\}$ and $\{z_{Ci} : i = 1, 2, \cdots n\}$ are additive Gaussian noise.

The LLRs are computed as $l_i^{(A)} = \log \frac{\Pr\{c_{2i}=0|y_{Ai},c_{1i}\}}{\Pr\{c_{2i}=1|y_{Ai},c_{1i}\}}$ for the bits of the codeword $\mathbf{c_2} = [c_{21}\ c_{22} \cdots c_{2n}]$. A soft decoder for the code C_2 uses LLRs $\left[l_1^{(A)}\ l_2^{(A)} \cdots l_n^{(A)}\right]$ to produce an estimate of the codeword $\mathbf{c}_2$ at node A. Similarly, codeword $\mathbf{c}_1$ is decoded at node C.

Design rates: The designed rate from A to C and C to A are given as:

$$R_{AC} = \frac{k_1}{2n}, \qquad R_{CA} = \frac{k_2}{2n}. \tag{7}$$

Power allocation strategy: Let P_1, P_2 be the powers allocated to nodes A and C, respectively, in the MAC-phase. Let P_3 be the power allocated to the broadcast-phase. For a given total power P_T and channel gains, we allocate sufficient power to the broadcast phase to achieve reliable decoding at nodes A and C. Rest of the power, $P_T - P_3$, is allocated to the MAC-phase. In the MAC-phase, we choose P_1, P_2 such that, the rate R_{XOR} is maximized. That is,

$$[P_1, P_2] = \underset{P_1,P_2}{\arg\max}\, I(X;Y), \tag{8}$$

subject to the constraint $P_1 + P_2 = P_T - P_3$.

Fading Channels: For fading channels a Rayleigh fading model is assumed for the links BA and BC. The channel gains are complex Gaussian, $h_1 \sim CN(0, \sigma_1^2)$, $h_2 \sim CN(0, \sigma_2^2)$ and assumed to be constant during the transmission of each codeword block. It is assumed that the channel state information h_1, h_2 are known both to the transmitting and receiving nodes. Let $\mathcal{C} = \{C^{(1)}, C^{(2)}, \cdots C^{(r)}\}$ be a set of LDPC codes with block length n. For the given channel states h_1, h_2, the outer bounds on achievable rates are [8]

$$R_{AC}, R_{CA} \le \frac{C(SNR_1)C(SNR_2)}{C(SNR_1) + C(SNR_2)}, \tag{9}$$

$$R_{AC} + R_{CA} \le C(\min(SNR_1, SNR_2)). \tag{10}$$

For each block, we select a code $C_1 = C^{(i)} \in \mathcal{C}$ based on the channel states h_1, h_2 such that the rate pair is within the outer bounds. C_2 would be the code $C^{(i)}$ shortened by s bits. That is, we need

$$\frac{R_i}{2} < \frac{C(SNR_1)C(SNR_2)}{C(SNR_1) + C(SNR_2)}, \tag{11}$$

$$\frac{R_i}{2} + \frac{R_i'}{2} < C(\min(SNR_1, SNR_2)). \tag{12}$$

where R_i is the rate of code C_1 and $R_i' = R_i - \frac{s}{n}$ is the rate of code C_2. For different ranges of h_1 and h_2, the code $C^{(i)}$ and the parameter s are chosen to achieve the best possible sum rate for the desired frame error rate. This could be done experimentally for the given set of codes for the fading channel and used as a look up table.

III. Capacity region

While the capacity region of a two-way relay is not known, we attempt to characterize the capacity region specifically for the XOR-decoding strategy. We use code $D = C_1 \oplus C_2$ in the MAC-phase. Since we decode the XOR at the relay, the capacity region of MAC-phase can be expressed as

$$\mathcal{C}_{MAC} = \{r_x, r_y : 0 \le r_x, r_y \le \beta_{XOR}\}, \tag{13}$$

where β_{XOR} is the capacity of virtual XOR-channel. In the broadcast-phase, we have a broadcast-with-side-information scenario with capacity as in (3). Let αN be the number of channel uses for MAC-phase and $(1-\alpha)N$ be the number of channel uses for the broadcast-phase, where N is the total number of channel uses and $\alpha \in (0, 1)$ is the time sharing variable. The capacity region for XOR-decoding strategy can be expressed as

$$\begin{aligned}\mathcal{C}_{TWR} = \{R'_{AC}, R'_{CA} : R'_{AC} &\le \min\{\alpha R'_1, (1-\alpha)R'_2\},\\ R'_{CA} &\le \min\{\alpha R'_3, (1-\alpha)R'_4\},\\ (R'_1, R'_3) \in \mathcal{C}_{MAC}&, (R'_2, R'_4) \in \mathcal{C}_{BC}\}.\end{aligned}$$

IV. Design of LDPC codes

LDPC codes are a class of linear block codes with a sparse parity-check matrix.

Threshold of LDPC code: Consider a LDPC code ensemble with an edgewise degree distribution $\lambda(x)$ and $\rho(x)$. Let $p_e^l(\sigma)$ be the average probability of bit error in the message passing decoder at iteration l. The noise threshold of the code ensemble is defined as: $\sigma^* = \sup\{\sigma : \lim_{l\to\infty} P_e^l(\sigma) = 0\}$.

At values of σ below the threshold, reliable decoding is possible with large block lengths and large number of iterations. The design of a LDPC code for a given channel (noise variance σ^2) involves finding the variable node and check node degree distributions $\lambda(x)$ and $\rho(x)$ such that,

1) Rate of the code $r = 1 - \frac{\int_0^1 \rho(x)dx}{\int_0^1 \lambda(x)dx}$ is maximized.
2) Threshold of the code $\sigma^* > \sigma$.

The design and optimization of LDPC codes for bidirectional relaying is explained next.

During MAC phase, the relay needs to decode $x_i = c_{1i} \oplus c_{2i}$ over the virtual XOR-channel of Figs. 2 and 3. We see that $p(y_i|x_i = 0) \ne p(-y_i|x_i = 1)$ (the subscript XOR has been dropped in the notation), and hence the virtual XOR-channel is not symmetric. However, channel symmetry is a desirable property as it simplifies the code design. To force the symmetry on the channel, we employ *iid* adapters introduced in [13]. On the transmitter side, nodes A and C use an *iid* source each and transmit $\tilde{c}_{1i} = c_{1i} \oplus t_{1i}$ and $\tilde{c}_{2i} = c_{2i} \oplus t_{2i}$, where t_{1i} and t_{2i} are equiprobable binary *iid* random variables. On the receiver side, relay node B receives $\tilde{y}_i$, which is given by the expression as (4), but with c_{1i} and c_{2i} replaced by $\tilde{c}_{1i}$ and $\tilde{c}_{2i}$, respectively. A soft decoder tries to estimate $\tilde{x}_i = x_i \oplus t_{1i} \oplus t_{2i}$, whose LLR we denote as $\tilde{l}_i$. As noted before, the distribution of $\tilde{l}_i$ is not symmetric. However, we can compute the distribution of l_i (the LLR for x_i) from $\tilde{l}_i$ as

$$p(l_i) = \frac{1}{2}\left(p(\tilde{l}_i|t_{1i} \oplus t_{2i} = 0) + p(-\tilde{l}_i|t_{1i} \oplus t_{2i} = 1)\right), \tag{14}$$

which is readily seen to be symmetric. So, with the *iid* adapters, density evolution based optimization becomes feasible. We have employed linear programming on EXIT charts for optimization, which is briefly described next. For designing the code, we fix the desired threshold σ_{th}, and the range of bit-node and check-node degrees, $\mathcal{L}$ and $\mathcal{R}$, respectively. We evaluate (14) using simulation for a value of σ slightly greater than σ_{th}. Using $p(l_i)$ as distribution of channel LLRs, we run density evolution with constant bit-node degree $i \in \mathcal{L}$ and various different right distributions (mostly right-regular with different degrees). In each bit-node-update step, mutual information of input and output PDFs (obtained by density evolution) are computed to obtain points on the bit-to-check EXIT curve with bit-node degree i. Similarly, points are obtained on the check-to-bit EXIT curve for each check-node degree $j \in \mathcal{R}$. The entire EXIT curve is obtained by interpolation. The degree distributions, $\lambda(x)$ and $\rho(x)$, are constrained to ensure that ρ_j-linear combination of the check-degree-j EXIT curves lies above the λ_i-linear combination of the bit-degree-i curve. Satisfying this constraint, and considering rate as the objective function, we optimize the degree distribution pairs by performing linear programming over EXIT charts [14].

Design of Nested LDPC codes: In order to design nested LDPC codes C_1, C_2 such that $C_2 \subseteq C_1$, rates $R_1 > R_2$, the code C_1 with degree distribution (edge perspective) $\lambda_1(x)$ and $\rho_1(x)$ is designed first. Let H_1 be the corresponding parity-check matrix. Let n be the block length of code C_1, m be the number of rows in the parity-check matrix H_1. The parity-check matrix H_2 for C_2 is constructed by adding $M = (R_1 - R_2)n$ rows to H_1. The degree distribution $\lambda_2(x)$ and $\rho_2(x)$ of code $C_2 \subseteq C_1$ is designed as in [14], but with extra constraints [15] given below:

$$R_{i,2} \geq \left(\frac{m}{m+M}\right) R_{i,1}, \quad i = 1, 2, \cdots d_{c,1}, \tag{15}$$

$$\sum_{i=k}^{d_{v,2}} L_{i,2} \geq \sum_{i=k}^{d_{v,1}} L_{i,1}, \quad k = 2, 3, \cdots d_{v,1}, \tag{16}$$

where $[L_{1,i} L_{2,i} \cdots L_{d_{v,i},i}]$ and $[R_{1,i} R_{2,i} \cdots R_{d_{c,i},i}]$ are the variable node and check node degree distributions of code C_i in the node perspective. The constraint in (15) ensures that the number of rows of degree i in H_2, would be greater than or equal to that in H_1. The constraint in (16) ensures that the number of columns of degree greater than or equal to k in H_2, would be greater than or equal to that in H_1. The construction of parity-check matrices from degree distributions could be done using modified progressive edge growth algorithm as suggested in [15].

V. Simulation and Results

AWGN Channel:

For illustration, the links AB and BC are assumed to have SNRs of $SNR_1 = 5.3$ dB and $SNR_2 = 7.0$ dB, respectively. The LDPC code design procedure resulted in a code C_1 of rate 0.855 and degree distribution $\lambda_1(x) = 0.1x + 0.4704x^2 + 0.3887x^9 + 0.0409x^{10}$, $\rho_1(x) = 0.41x^{26} + 0.59x^{27}$, which was optimized for XOR decoding in the MAC phase. Code C_2 obtained by nested code design procedure has rate 0.8 and degree distribution $\lambda_2(x) = 0.0461x + 0.3641x^2 + 0.0058x^6 + 0.1848x^9 + 0.3992x^{29}$ and $\rho_2(x) = 0.2903x^{26} + 0.4177x^{27} + 0.2921x^{29}$. The block length considered is $n = 100000$. The parity-check matrix H_1 corresponding to code C_1 is constructed from the degree distribution $\lambda_1(x)$ and $\rho_1(x)$ with $m = 14500$ rows. The parity-check matrix H_2 corresponding to code C_2 is constructed from degree distribution $\lambda_2(x)$ and $\rho_2(x)$ by adding $M = 5500$ rows to the parity-check matrix H_1. The different rate pairs achieved using the codes C_1 and C_2 are shown in Fig. 4. Point A corresponds to the rate pair $R_{AC} = 0.4275, R_{CA} = 0.4$ with code optimized for XOR decoding in MAC channel. Points B, C, D, G are achieved by shortening the code at point A. Point K which has lesser rates compared to point A, is achieved using code design optimized for BI-AWGN point-to-point channel.

For simulations, a frame error rate of 10^{-2} was considered acceptable. For a given total power of $P_T = 3.0$, based on the power allocation strategy explained earlier, the transmitted powers were chosen as $P_1 = 1.2570, P_2 = 0.8772$ for the MAC phase and $P_3 = 1.0$ for the broadcast phase.

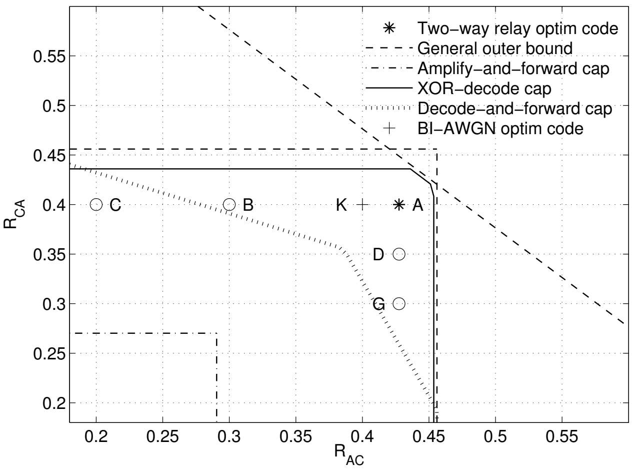

Fig. 4. Rate region for $SNR_1 = 5.3dB$, $SNR_2 = 7.0dB$.

The general outer bounds [8] (legend: General outer bound), capacity outer bound for XOR-decode-and-forward (legend: XOR-decode capacity), decode-and-forward (legend: Decode- and-forward capacity), amplify-and-forward [12] (legend: Amplify-and-forward capacity) are shown in figure.

We see that the capacity region of the proposed XOR-decode and forward strategy is close to the theoretical upper bounds and larger than the capacity region for decode-and-forward and amplify-and-forward protocols. In the decode-and-forward strategy, the relay attempts to decode the individual codewords transmitted in the MAC-phase. The rate pairs achieved by the proposed scheme are better than the theoretical rates that can be achieved by the other protocols.

Fading Channel:

A set of LDPC code pairs (C_1, C_2) of rates $(0.89, 0.89)$, $(0.89, 0.7)$, $(0.8, 0.6)$, $(0.7, 0.5)$, $(0.5, 0.5)$, $(0.5, 0.3)$, $(0.4, 0.2)$, $(0.2, 0.2)$ is considered. The codes C_1 in the set are designed for BI-AWGN channels using EXIT chart analysis as suggested in [14]. We consider block length $n = 20000$. The codes C_2 in the set are designed by shortening the codes C_1. We assume $h_1 \sim CN(0, \sigma_1^2), h_2 \sim CN(0, \sigma_2^2)$. The channel gains are assumed to be constant for one block. We fix $\sigma_1^2 = 4.5$ ($SNR_1 = 10.6dB$) and for different values of σ_2^2 (different average SNR_2 values) simulations were done for 10000 blocks. For each block, based on the (SNR_1, SNR_2) values, a code pair (C_1, C_2) is chosen as explained in Section II. Fig. 5 plots the average sum rate achieved versus the average SNR_2. We consider frame error rates lower than 10^{-2} as acceptable. From the figure, we see that the sum rate achieved approaches the upper bound for a wide range of average SNRs.

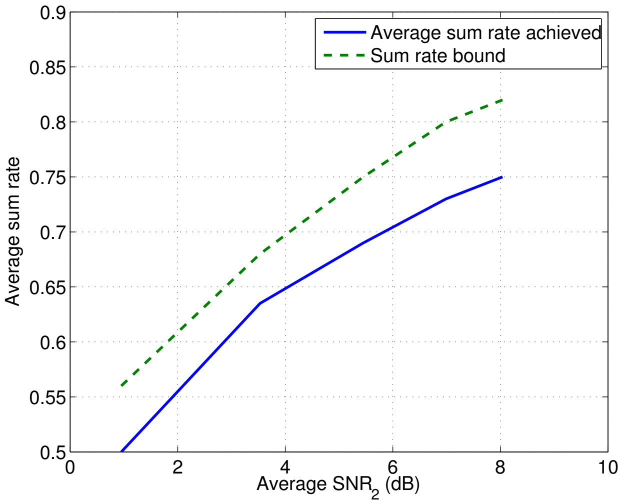

Fig. 5. Performance for Rayleigh fading channels

VI. Conclusion

In this paper, we analyzed the XOR-decode-and-forward strategy for bidirectional relay. We characterize the capacity region for this strategy and propose a specific way of choosing codes to achieve rates close to capacity outer bounds. We also propose a method to optimize codes specifically for XOR-decoding.

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