On Interference Management in the Cellular Downlink

Srikrishna Bhashyam

IIT Madras

NCC 2020

Feb 22, 2020

Acknowledgment: M. Vishnu Narayanan

Challenges in Wireless Communication

Fading
(time-variations)

Interference

Slow: Learn and Adapt
Fast: Adapt to the average

This talk

Interference management in Cellular Systems

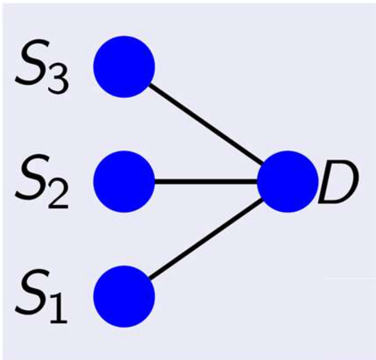
Early
Cellular
Networks
(1G, 2G)

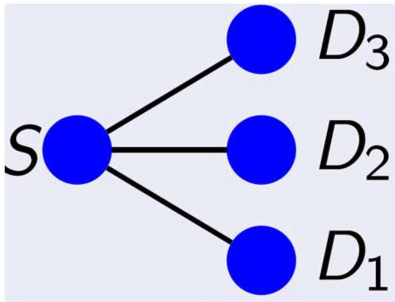
- Avoid interference
  - Frequency planning
  - Time-division and frequency-division
- Network of point-to-point links
  - Basic building block: point-to-point link

Interference management in 3G

3G
Cellular
Networks

- Power control
  - Higher frequency reuse
  - Treat interference as noise
- Network of basic building blocks
  - Multiple access channels
  - Broadcast channels





Interference management in 4G

Modern Cellular Networks

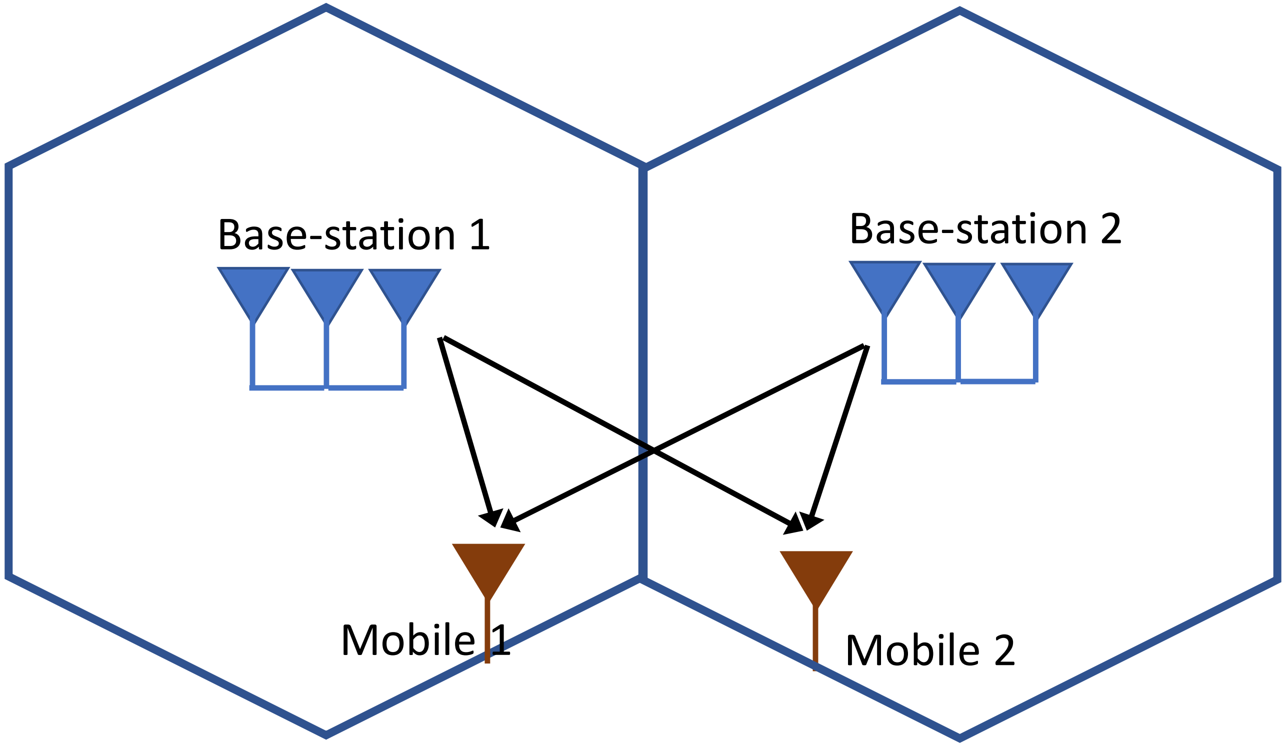
- Interference cancellation
- Interference coordination
- Network of basic building blocks
  - Multiple access channels
  - Broadcast channels
- Link adaptation
  - Point-to-point links

Adaptive Interference Management

Future Cellular Networks

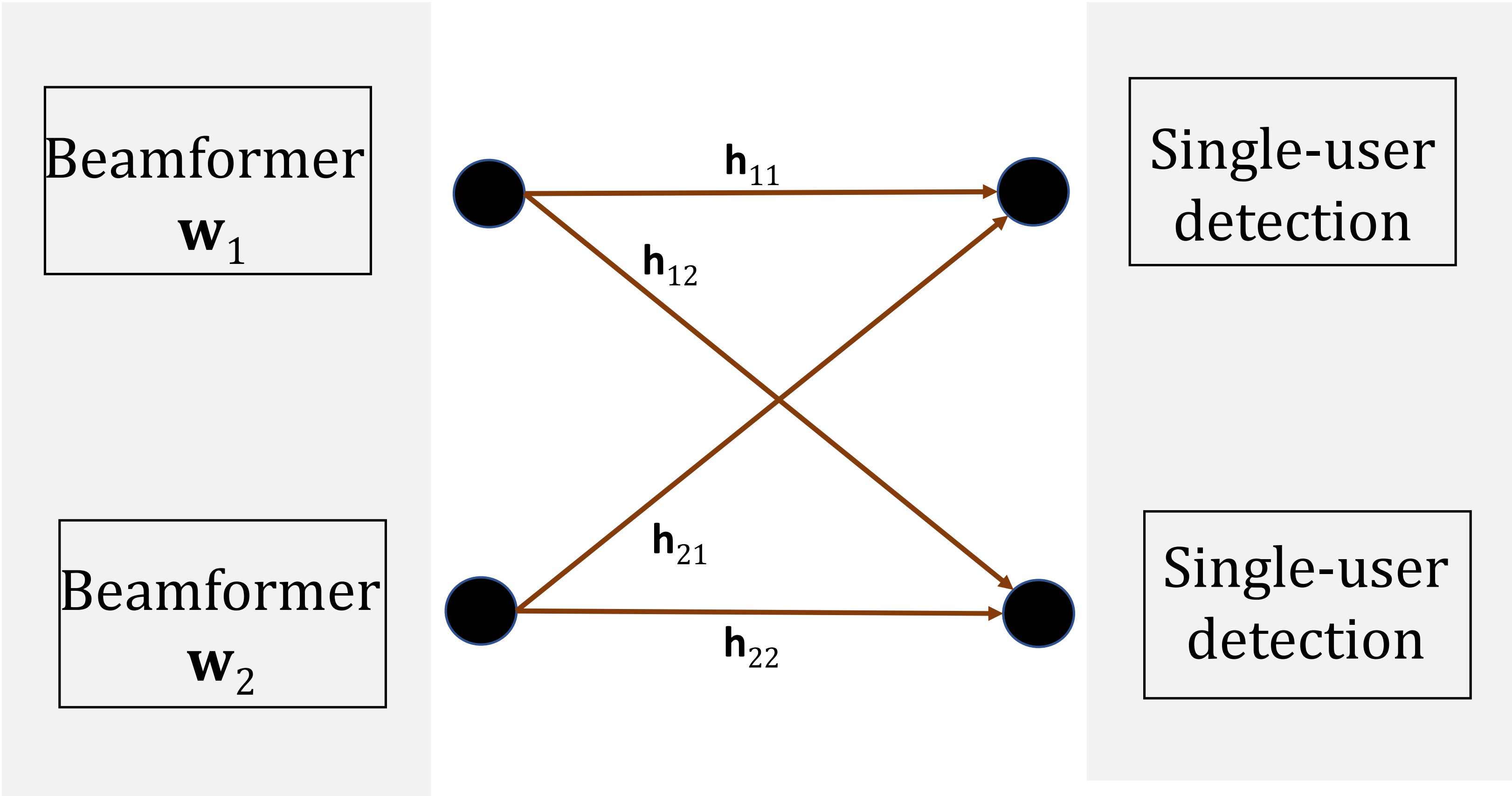
- Adaptive interference management
  - Choose between various schemes
  - Based on channel conditions
- Network of basic building blocks
  - Interference channels

Rest of this talk:
Multicell Downlink Beamforming



- Problem: Design beamforming vectors at each BS
- Distributed solution with limited exchange of information

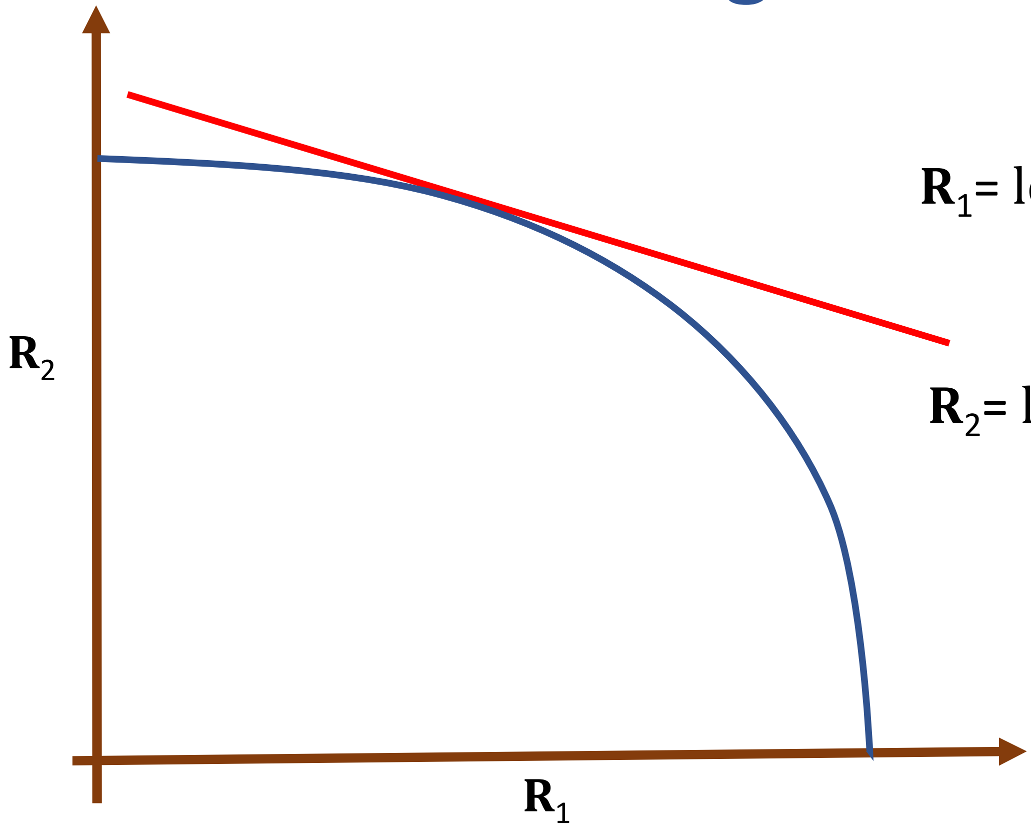
MISO Interference Channel Model



Beamforming optimal under Gaussian codebooks + single-user detection

Zhang & Cui 2010, Shang, Chen & Poor 2009

Achievable Rate Region



$$\mathbf{R}_1 = \log\left(1 + \frac{\left|\mathbf{h}_{11}^H \mathbf{w}_1\right|^2}{\left|\mathbf{h}_{21}^H \mathbf{w}_2\right|^2 + \sigma_1^2}\right)$$

$$\mathbf{R}_2 = \log\left(1 + \frac{\left|\mathbf{h}_{22}^H \mathbf{w}_2\right|^2}{\left|\mathbf{h}_{12}^H \mathbf{w}_1\right|^2 + \sigma_2^2}\right)$$

- Can be non-convex
- Boundary points to be determined
  - Pareto optimal rate vector: Not possible to improve any component without decreasing at least one other component

Finding the beamforming vectors

- Weighted sum rate maximization

$$\beta_1 \log\left(1 + \frac{\left|\mathbf{h}_{11}^H \mathbf{w}_1\right|^2}{|\mathbf{h}_{21}^H \mathbf{w}_2|^2 + \sigma_1^2}\right) + \beta_2 \log\left(1 + \frac{\left|\mathbf{h}_{22}^H \mathbf{w}_2\right|^2}{|\mathbf{h}_{12}^H \mathbf{w}_1|^2 + \sigma_2^2}\right)$$

- Power constraints

$$\|\mathbf{w}_1\|^2 \leq P_1$$
$$\|\mathbf{w}_2\|^2 \leq P_2$$

- Centralized solution

Distributed solution with limited coordination

BS 1

$$\log\left(1 + \frac{\left|\mathbf{h}_{11}^{H}\mathbf{w}_1\right|^2}{\Gamma_{21} + \sigma_1^2}\right)$$

$$\left|\mathbf{h}_{12}^{H}\mathbf{w}_1\right|^2 \leq \Gamma_{12}$$

$$\|\mathbf{w}_1\|^2 \leq P_1$$

$$\xrightarrow{\Gamma_{21}}$$

$$\xleftarrow{\Gamma_{12}}$$

BS 2

$$\log\left(1 + \frac{\left|\mathbf{h}_{22}^{H}\mathbf{w}_2\right|^2}{\Gamma_{12} + \sigma_2^2}\right)$$

$$\left|\mathbf{h}_{21}^{H}\mathbf{w}_2\right|^2 \leq \Gamma_{21}$$

$$\|\mathbf{w}_2\|^2 \leq P_2$$

- There exist interference thresholds corresponding to each boundary point
- Local channel information

Zhang & Cui 2010

Solution for given interference thresholds

$$\max_{\gamma_1,\delta_1,\theta_1,\phi_1} \gamma_1$$

$$\gamma_1^2 + \delta_1^2 \leq P_1$$

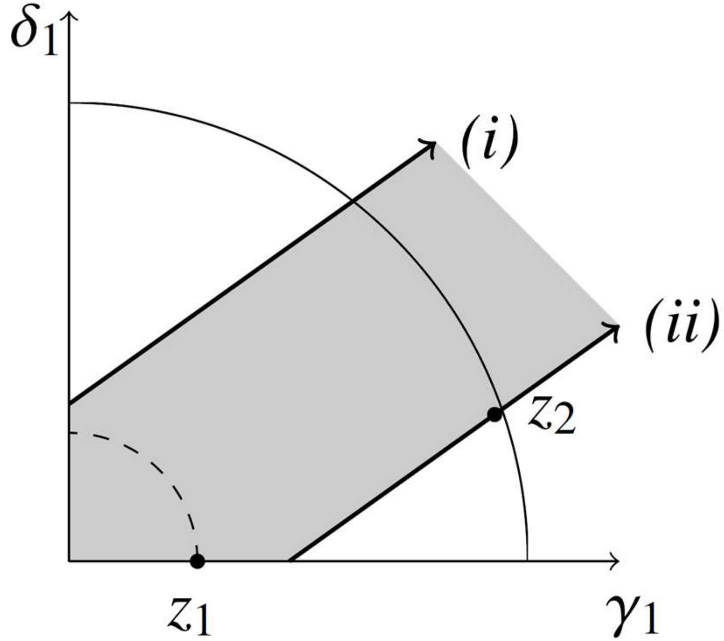
$$a\,\gamma_1^2 + b\delta_1^2 + 2ab\gamma_1\delta_1\cos(\theta_1 - \phi_1) \leq \Gamma_{12}$$

- Power along channel direction (γ_1^2) and along orthogonal direction (δ_1^2)

M. Vishnu Narayanan, S. Bhashyam, *Pareto Optimal Distributed Beamforming for the Multi-band Multi-cell Downlink,* Proceedings of IEEE Global Communications Conference (GLOBECOM 2017), Singapore, Dec. 2017.

Solution for given interference thresholds

- Closed form solution



- Power along channel direction (γ_1^2) and along orthogonal direction (δ_1^2)

Weighted sum rate maximization

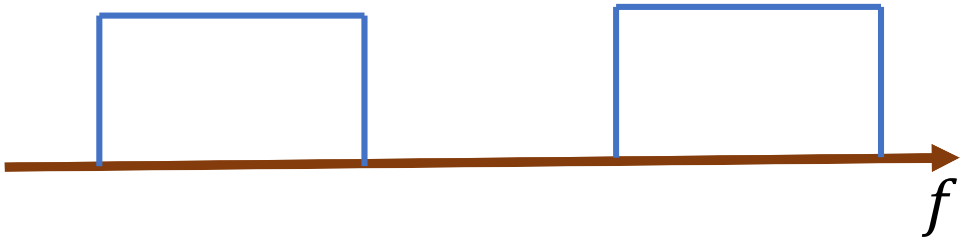
- Initialize $\{\Gamma_{ij}\}$

- Use closed form solution for given thresholds $\{\Gamma_{ij}\}$

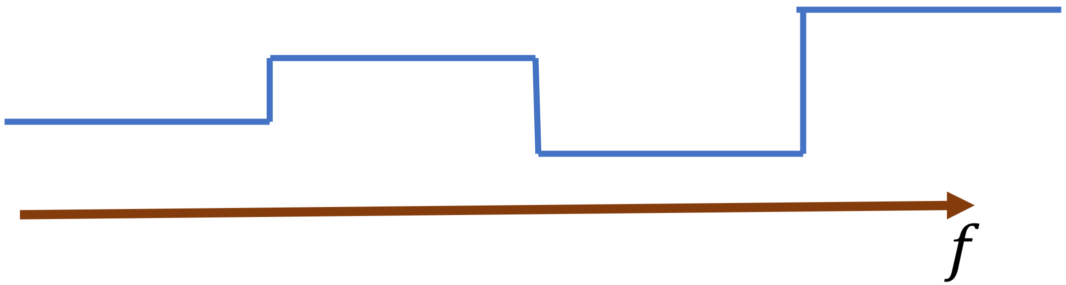
- Update interference thresholds $\{\Gamma_{ij}\}$ using gradient ascent

Multiple band case

Flat fading model so far



Multiple bands or resource blocks



Frequency selective channel

Power allocation + Beamforming

$$\max_{\{\mathbf{w}_{ik}\}} \sum_i \beta_i \sum_k \log\left(1 + \frac{\left|\mathbf{h}_{iik}^H \mathbf{w}_{ik}\right|^2}{\left|\mathbf{h}_{jik}^H \mathbf{w}_{jk}\right|^2 + \sigma_{ik}^2}\right)$$

$$\sum_k \|\mathbf{w}_{ik}\|^2 \le P_i \text{ for all } i$$

- Sum power constraint over all bands
- Beamforming vector for each band

Power allocation + Beamforming

$$\max_{\{\mathbf{w}_{ik}\},\{\alpha_{ik}\}} \sum_i \beta_i \sum_k \log\left(1 + \frac{\left|\mathbf{h}_{iik}^H \mathbf{w}_{ik}\right|^2}{\left|\mathbf{h}_{jik}^H \mathbf{w}_{jk}\right|^2 + \sigma_{ik}^2}\right)$$

$$\|\mathbf{w}_{ik}\|^2 \leq \alpha_{ik} P_i \;\text{ for all } i, k$$

$$\sum_k \alpha_{ik} = 1 \;\text{ for all } i$$

- Introduce variables $\{\alpha_{ik}\}$
  - Power in band k of cell i = $P_{ik} = \alpha_{ik} P_i$
- For a given power allocation $\{\alpha_{ik}\}$, overall multi-band problem reduces to K single-band problems, one for each band

Pareto boundary: *k*-band & 1-band

(R_1, R_2) is Pareto optimal

implies

(R_{1k}, R_{2k}) is Pareto optimal in each band k.

- For a given power allocation, overall multi-band problem reduces to K single-band problems, one for each band

Beamforming for each band

$$\max_{\{\mathbf{w}_{ik}\}} \sum_i \beta_i \log\left(1 + \frac{\left|\mathbf{h}_{iik}^H \mathbf{w}_{ik}\right|^2}{\left|\mathbf{h}_{jik}^H \mathbf{w}_{jk}\right|^2 + \sigma_{ik}^2}\right)$$

$$\|\mathbf{w}_{ik}\|^2 \leq \alpha_{ik} P_i \text{ for all } i$$

- Solve for each band k, for a given $\{\alpha_{ik}\}$

Beamforming: Distributed solution

$$\max_{\mathbf{w}_{ik}} \ \log\left(1 + \frac{\left|\mathbf{h}_{iik}^{H}\mathbf{w}_{ik}\right|^2}{\Gamma_{jik} + \sigma_{ik}^2}\right)$$

$$\|\mathbf{w}_{ik}\|^2 \leq \alpha_{ik} P_i \ \text{ for all } i$$

$$\left|\mathbf{h}_{ijk}^{H}\mathbf{w}_{ik}\right|^2 \leq \Gamma_{ijk} \text{for all } j$$

- Solve for each band k and each cell i for given $\{\Gamma_{ijk}\}, \{\alpha_{ik}\}$

Power allocation + Beamforming

$$\max_{\{\tilde{\mathbf{w}}_{ik}\},\{\alpha_{ik}\},\{\Gamma_{ijk}\}} \sum_i \beta_i \sum_k \log\left(1 + \frac{\left|\mathbf{h}_{iik}^H \tilde{\mathbf{w}}_{ik}\right|^2 \alpha_{ik} P_i}{\Gamma_{jik} + \sigma_{ik}^2}\right)$$

$$\left|\mathbf{h}_{ijk}^H \mathbf{w}_{ik}\right|^2 \leq \Gamma_{ijk} \text{ for all } i, j, k$$

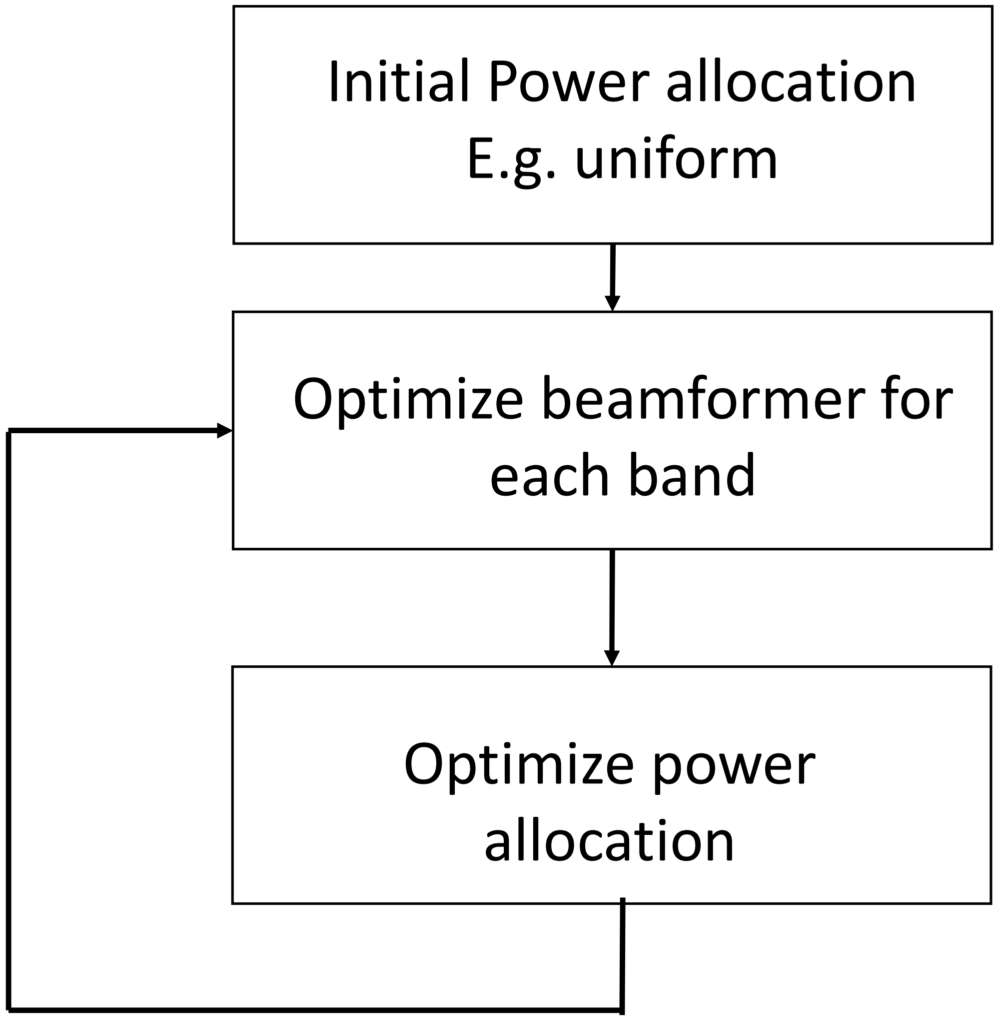
$$\|\tilde{\mathbf{w}}_{ik}\|^2 \leq 1 \text{ for all } i, k$$

$$\sum_k \alpha_{ik} = 1 \text{ for all } i, k$$

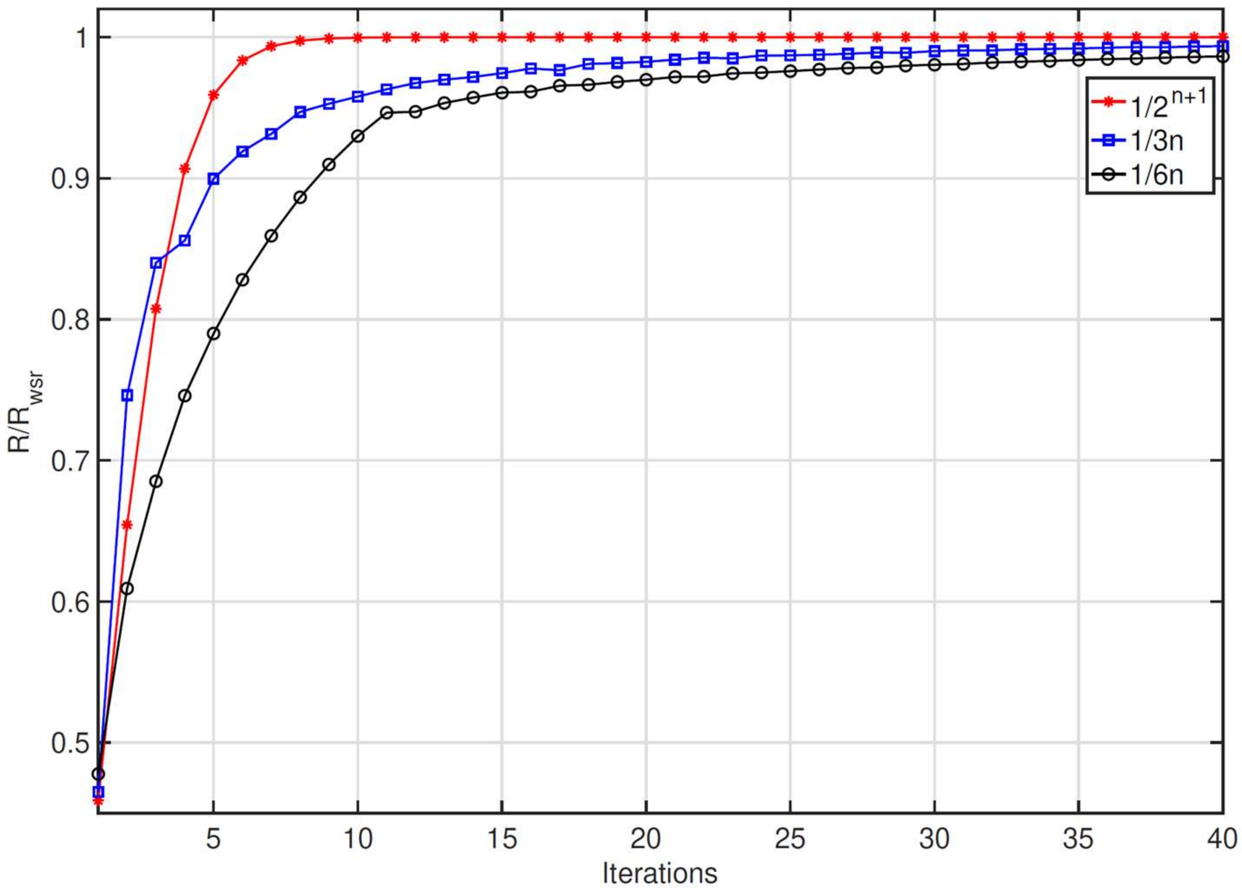
$$\mathbf{w}_{ik} = \tilde{\mathbf{w}}_{ik}\sqrt{\alpha_{ik} P_i}$$

- Power allocation step
  - Waterfilling

Alternating Maximization

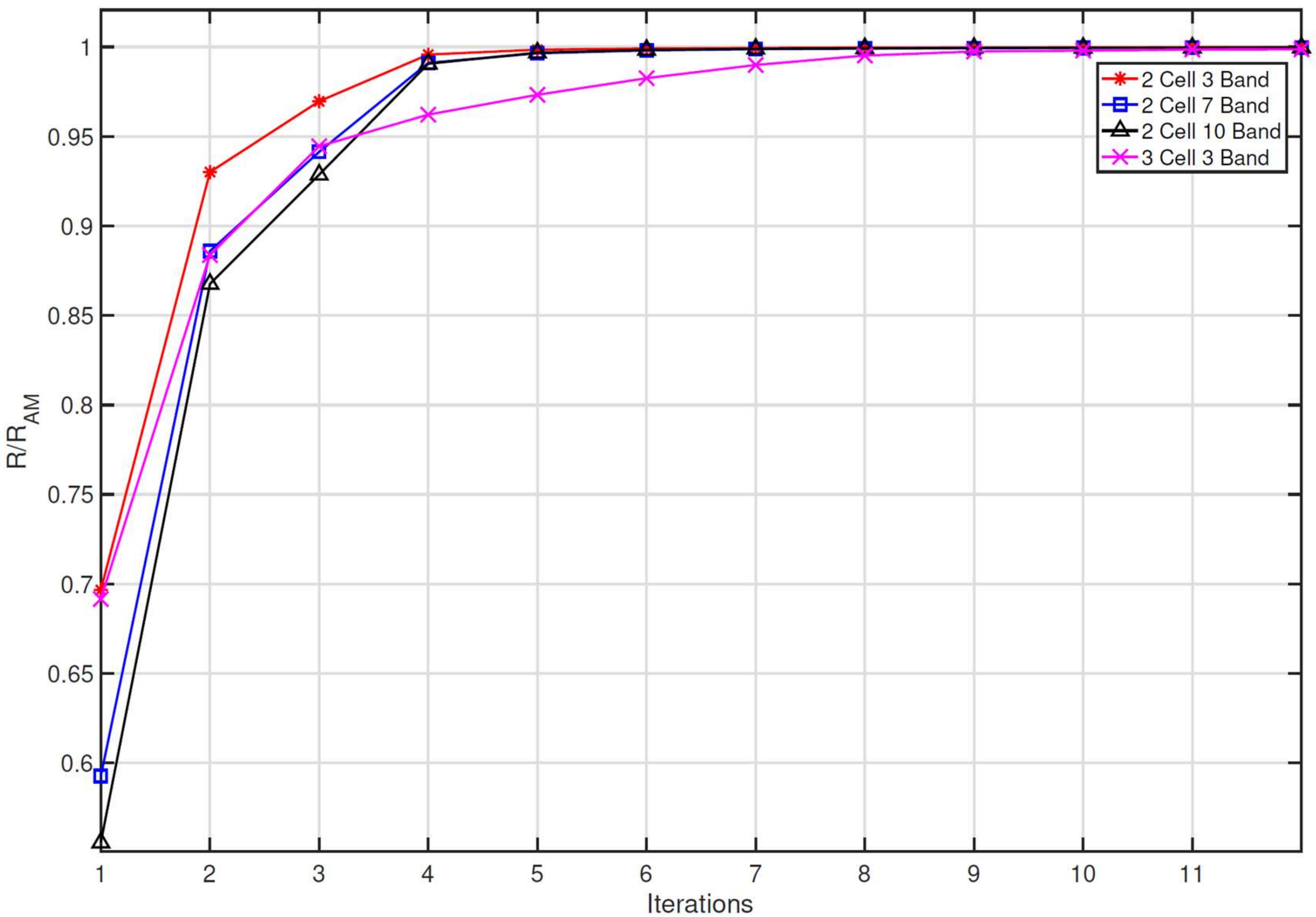


Gradient ascent convergence

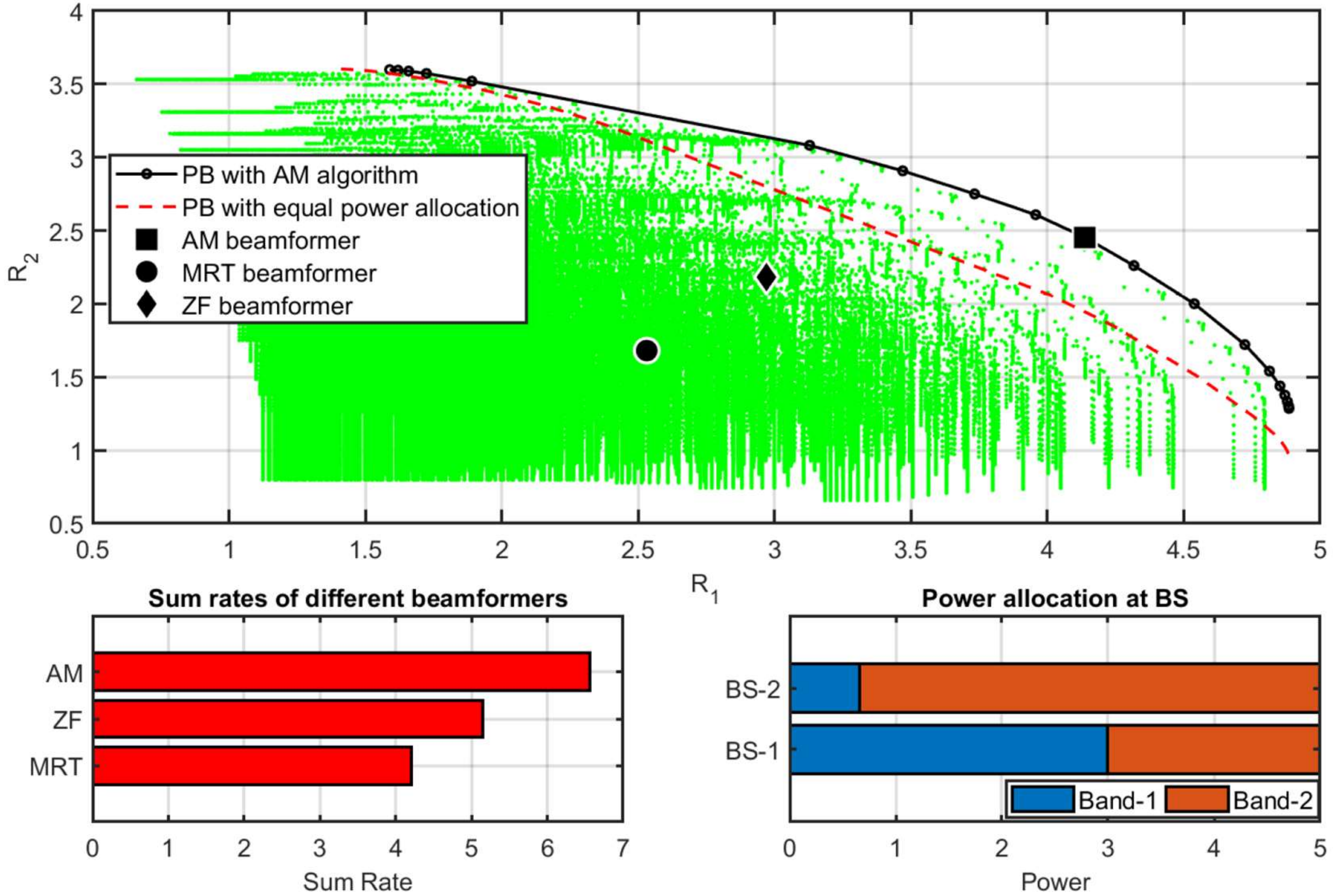


- Convergence to local maxima possible
- Try multiple initializations and choose the best

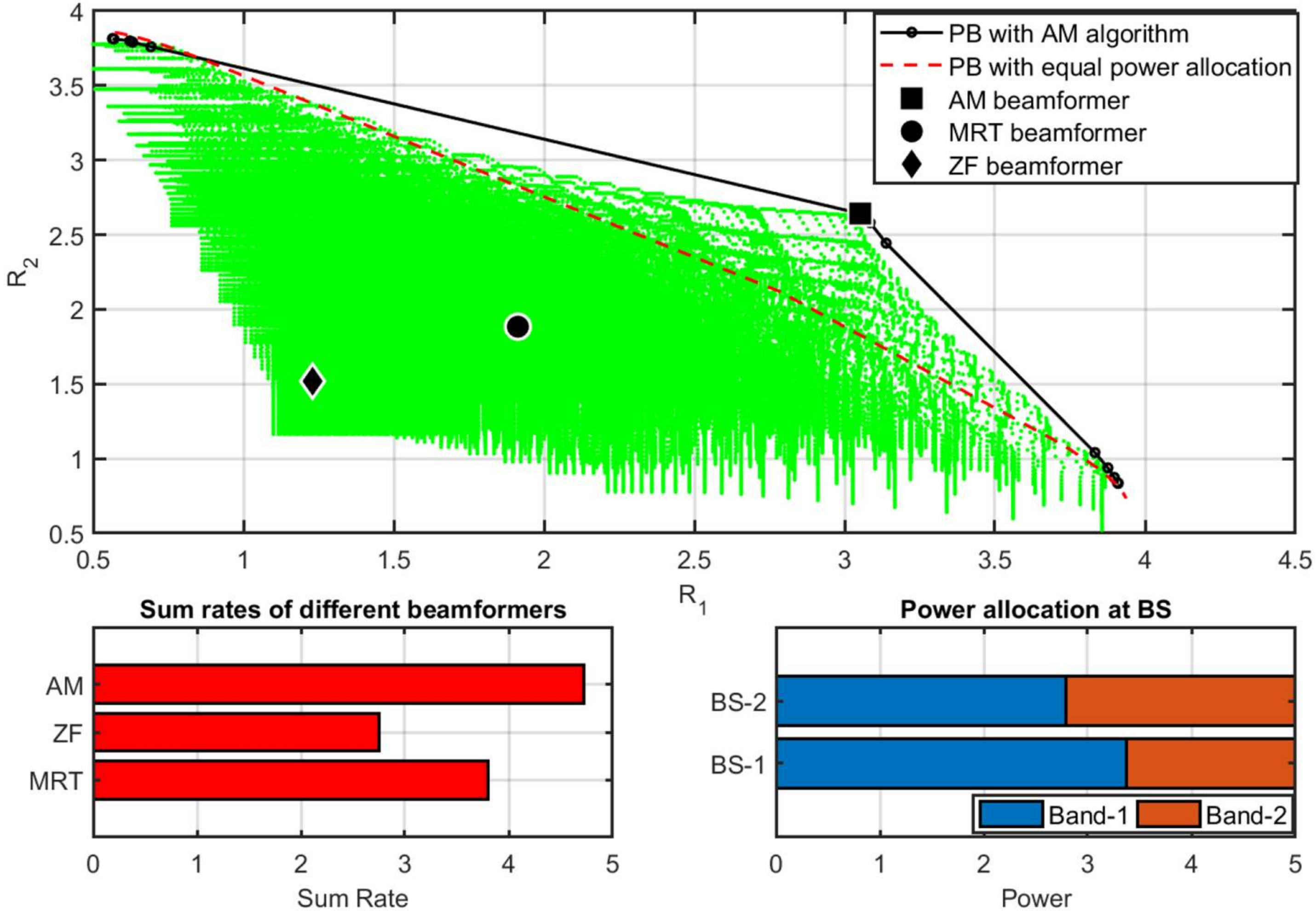
AM convergence



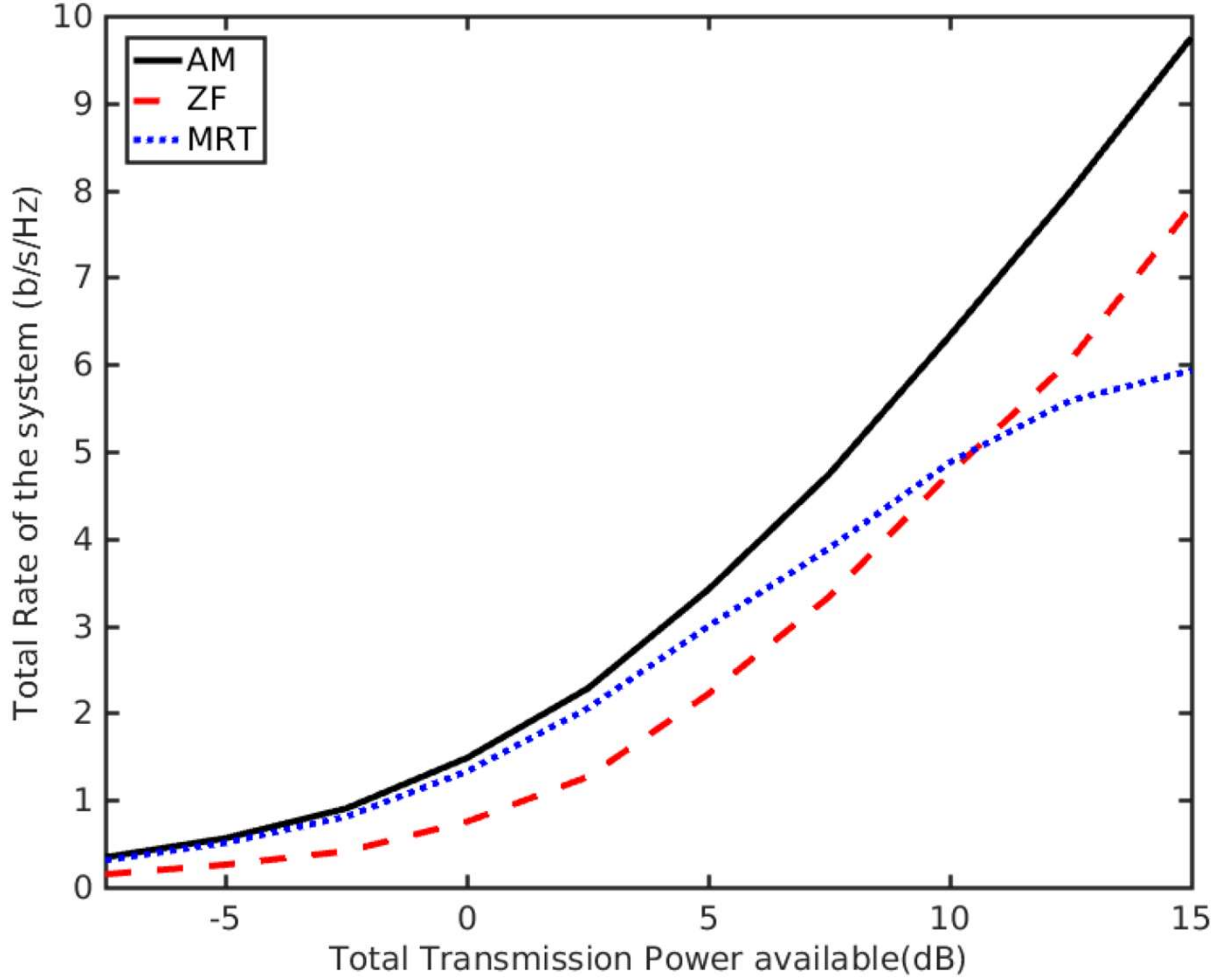
Simulation Results: 2-band



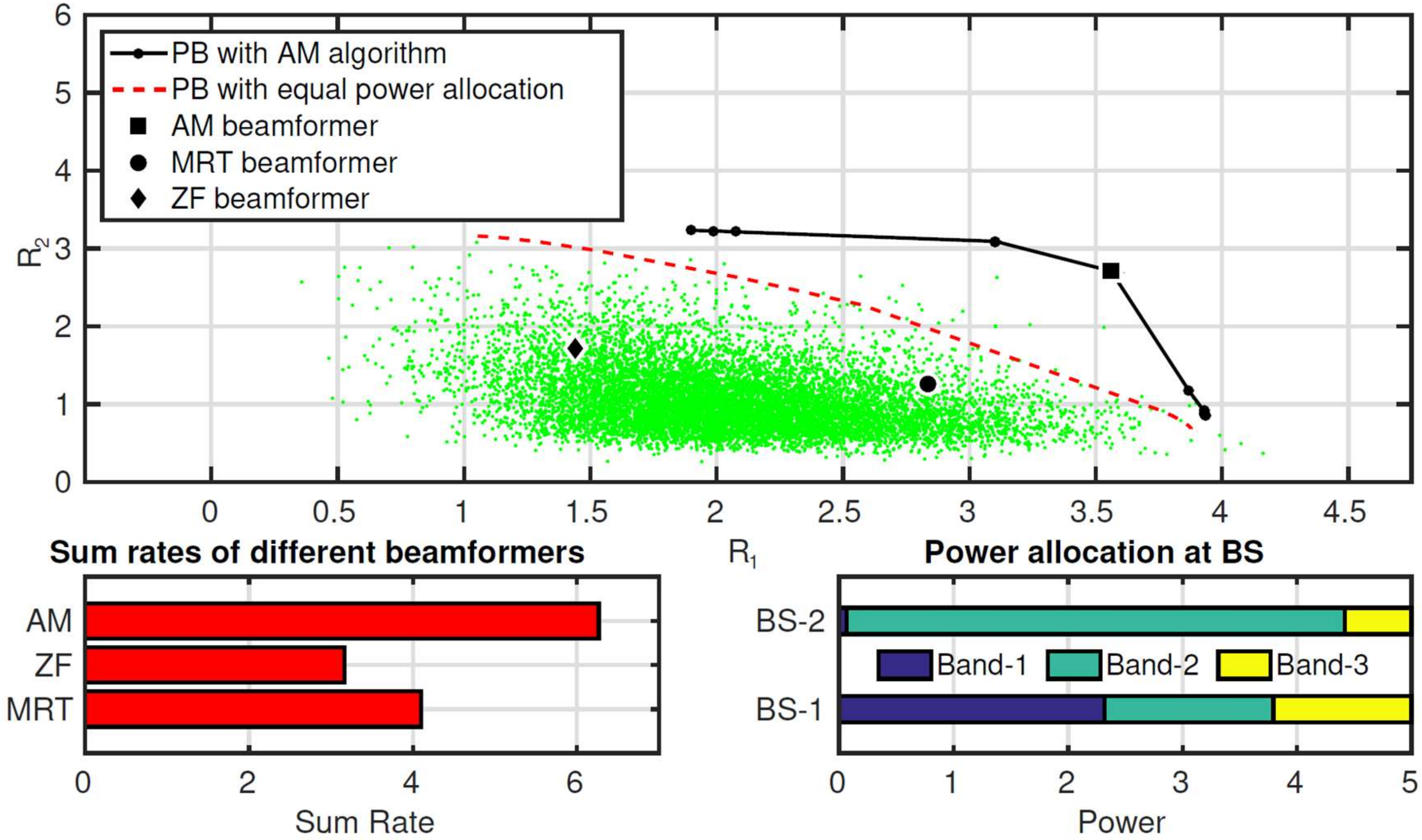
Simulation Results: 2-band



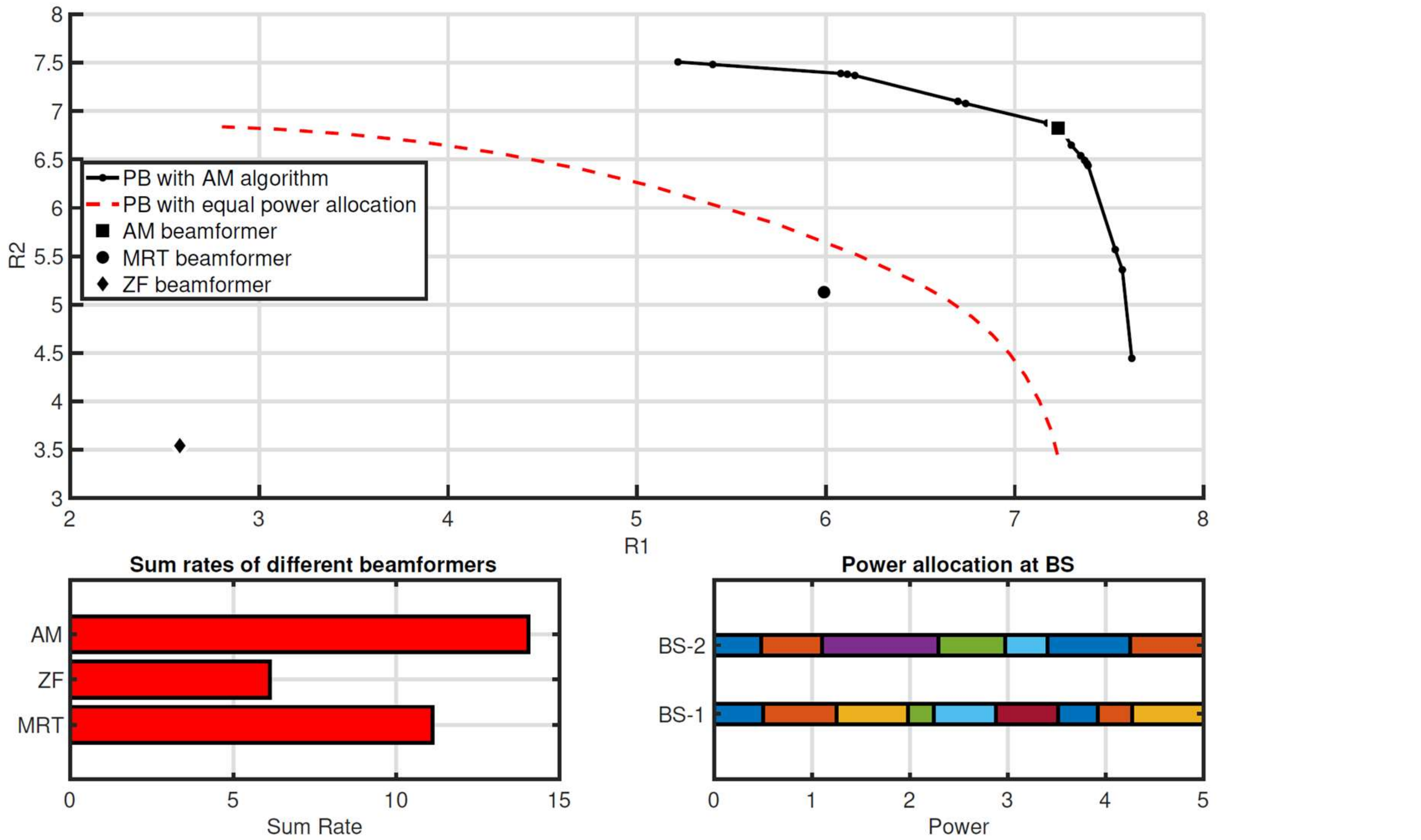
Simulation Results: 2-band



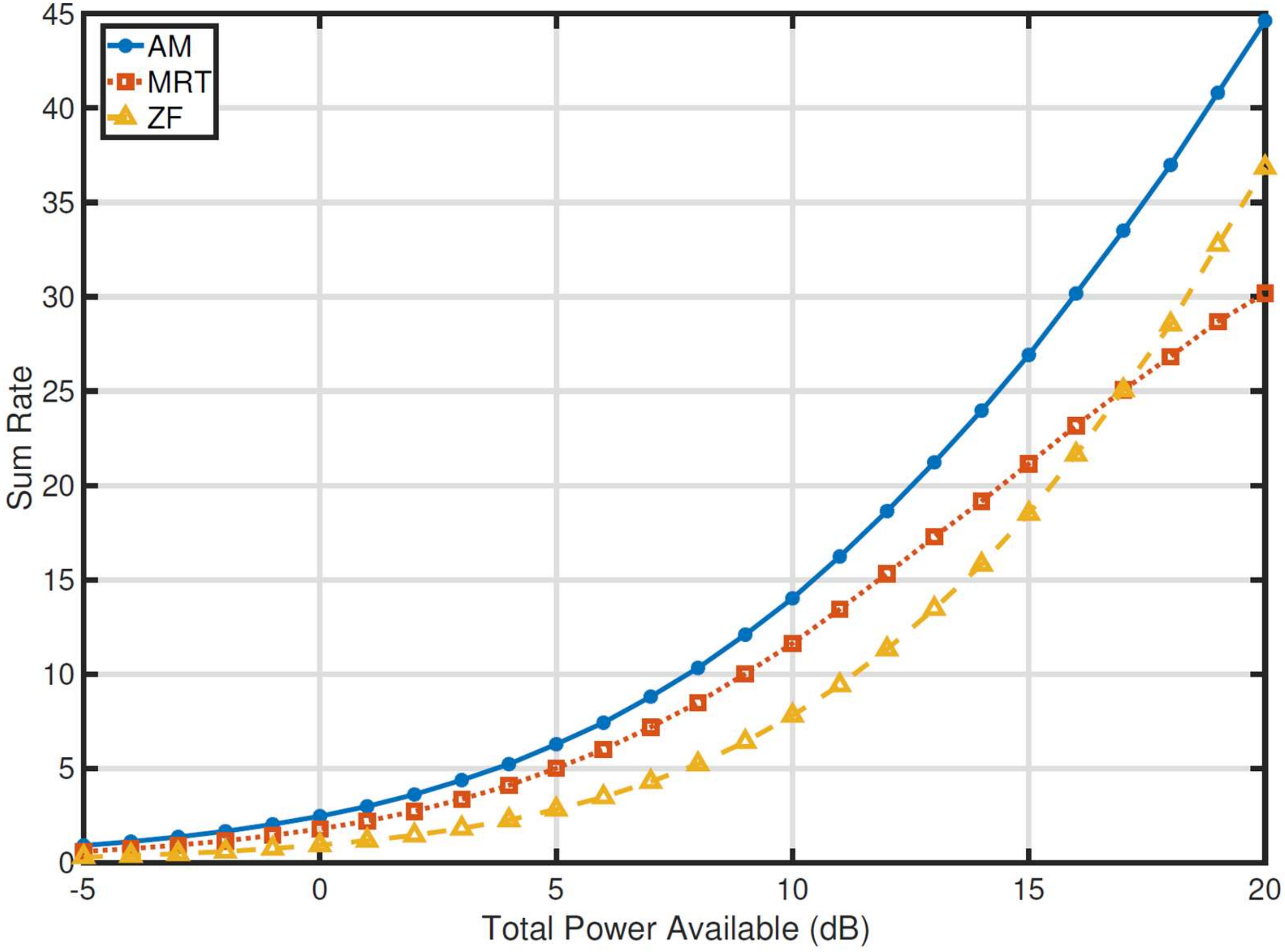
Simulation Results: 3-band



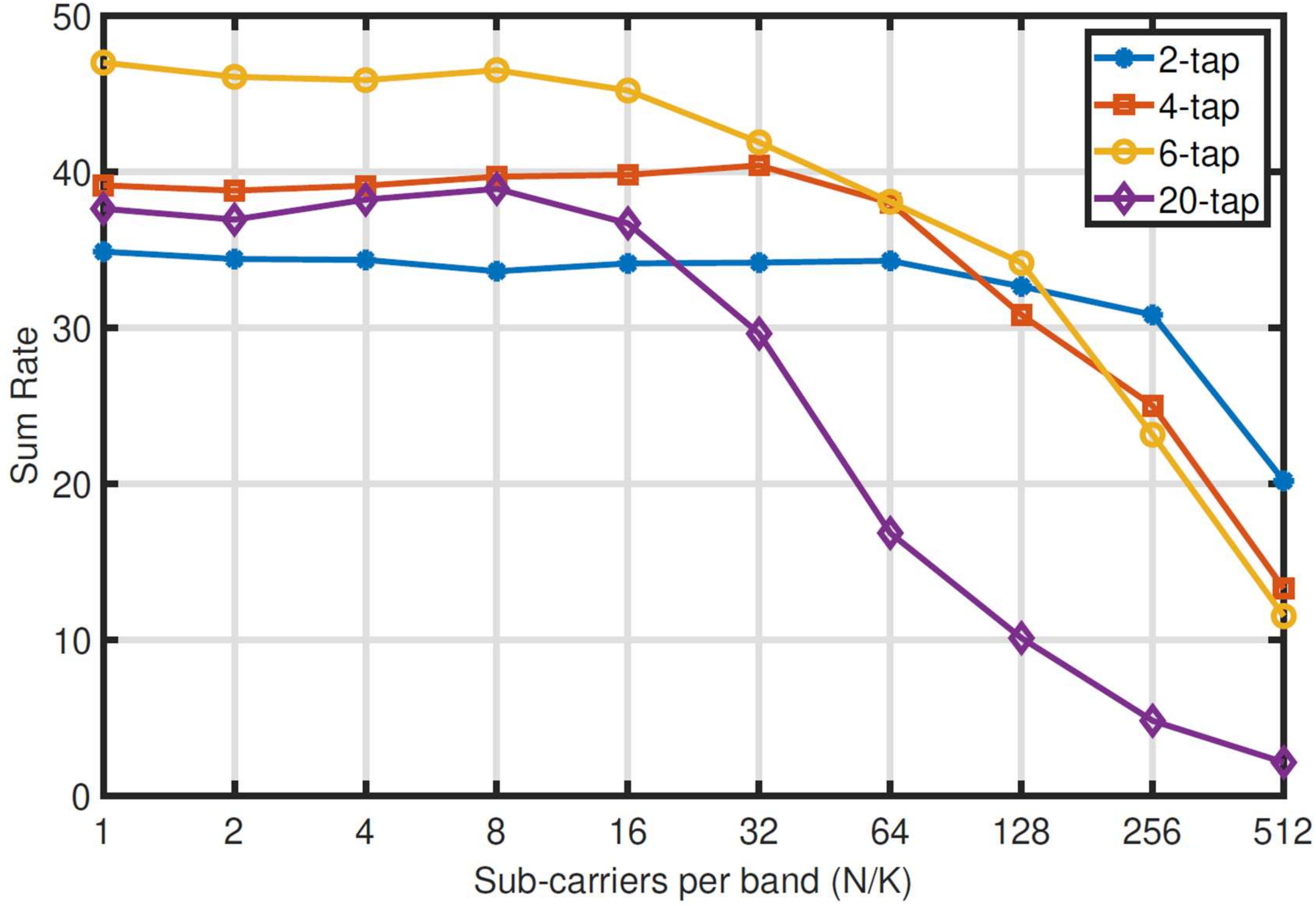
Simulation Results: 10-band



Simulation Results: 10-band



Simulation Results: Frequency selective channel



Summary

- Beamforming for the multicell downlink
  - Single-user detection and Gaussian codebooks

- Distributed solution with limited coordination
  - Single band case:
    - Closed form solution for given interference constraints
    - Gradient ascent for weighted sum rate maximization
  - Multiple band case:
    - Alternating maximization: Power allocation and beamforming
    - Significant gain over equal power allocation, MRT, ZF

https://www.ee.iitm.ac.in/~skrishna/