Lecture 35

Note Title 4/10/2008

7. (F) Consider a convolutional code with the generator matrix

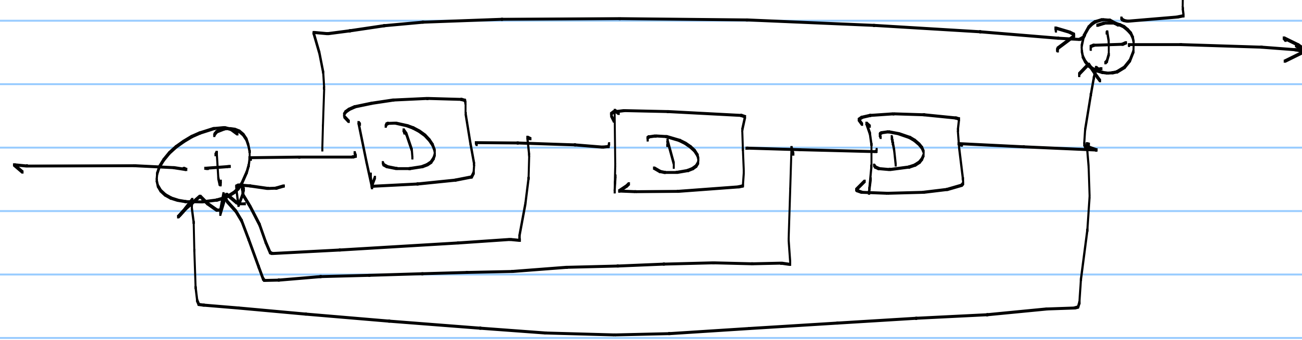
$$G(D) = \begin{bmatrix} 1 + D^3 & 1 + D + D^2 + D^3 \end{bmatrix}.$$

(a) Draw the circuit for a non-systematic encoder for the code.

(b) Draw the circuit for a systematic encoder for the code.

(b)

$$G(D) = \begin{bmatrix} \frac{1+D^3}{1+D+D^2+D^3} & 1 \end{bmatrix}$$



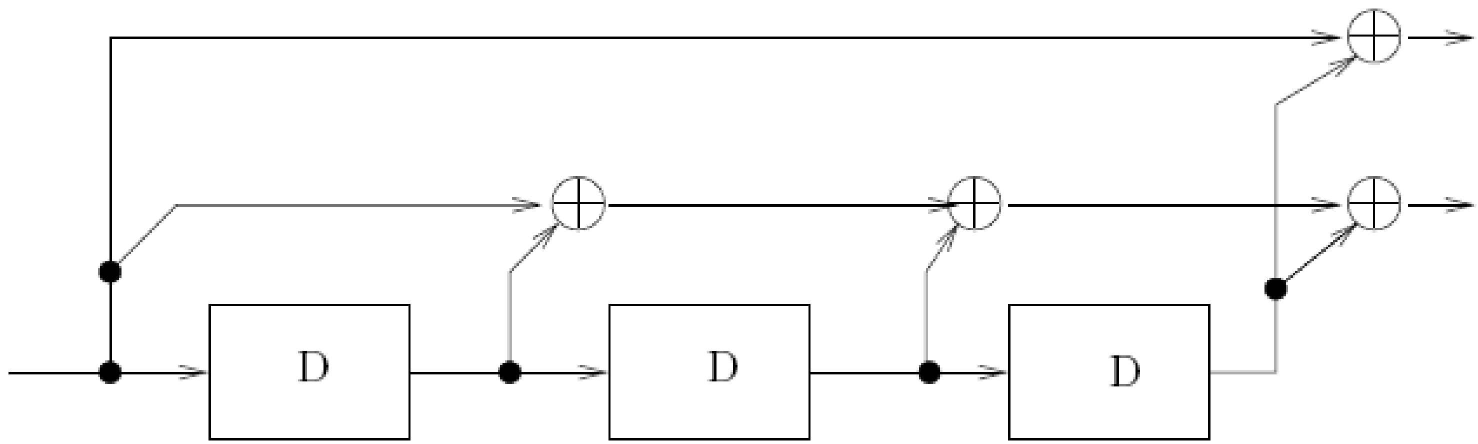


Figure 1: Encoder for Problem 8

(c) Draw one stage of the trellis for the systematic encoder.

8. (F) Consider the convolutional encoder shown in Fig. 1.

(a) Draw one stage of the trellis for the encoder.

(b) Write down the transfer function matrix for the convolutional code.

(c) Encode the infinite message sequence (11111111.......) (all 1s).

(b) $G(D) = [1+D^3 \quad 1+D+D^2+D^3]$

(c)

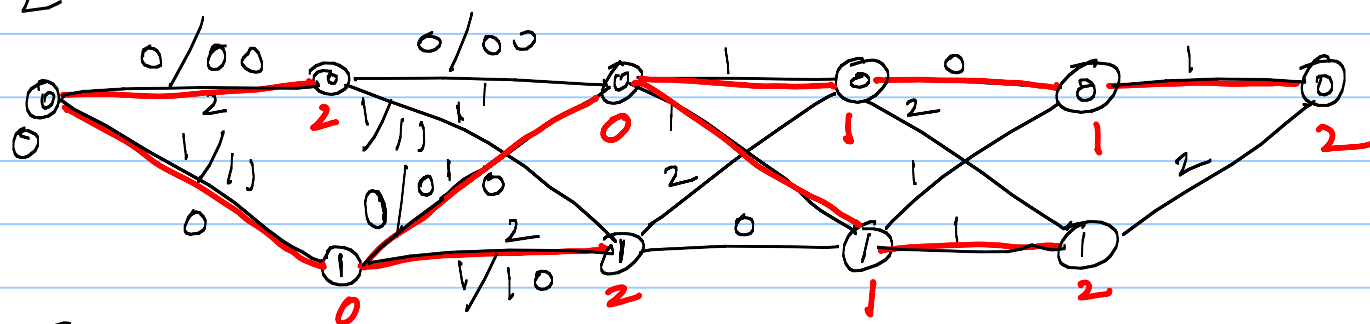
$$u(D) = \frac{1}{1+D}$$

$$v(D) = \left[\frac{1+D^3}{1+D} \quad \frac{1+D+D^2+D^3}{1+D} \right]$$

$$= \left[1+D+D^2 \quad 1+D^2 \right]$$

(9) $G(D) = [\,1 \quad 1+D\,]$

$\underline{r} = [\,11 \quad 01 \quad 10 \quad 00 \quad 10$



$\hat{\underline{u}} = [\,1 \quad 0 \quad 0 \quad 0\,]$

(2) $(n,1)$ repetition code

$$C = \{ 0 0 \ldots 0, \; 1 1 \ldots 1 \}$$

$$+1 \, +1 \ldots +1, \; -1 \, -1 \ldots -1$$

$$\underline{r} = [r_1 \; r_2 \; \ldots \; r_n]$$

ML

$$\hat{\underline{c}} = \arg\max_{\underline{s}} \; \underline{r} \cdot \underline{s}$$

$$\hat{\underline{c}} = \begin{cases} 0 0 \ldots 0 & \text{if } \sum r_i > 0 \\ 1 1 \ldots 1 & \text{else} \end{cases}$$

bitwise-MAP:

$$l_i = e^{\frac{2r_i}{\sigma^2}}$$

$$L_1 = l_1 l_2 l_3$$

$$= e^{\frac{2}{\sigma^2}\sum r_i}$$

$\hat{c}$ = same as ML

$$\Pr(\text{error}) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$ → same as uncoded.

3. Consider a code with parity-check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Design soft ML and bitwise-MAP decoders over an AWGN channel under BPSK modulation.

(b) Design a hard-decision syndrome decoder after converting the above channel to a BSC.

(c) Is there a received word for which the above three decoders provide three different outputs?

$$C = \{00000, 11001, 10110, 01111\}$$

$$\underline{r} = [r_1 \ r_2 \ r_3 \ r_4 \ r_5]$$

Correlations: $a_1 = r_1 + r_2 + r_3 + r_4 + r_5$

$$a_2 = -r_1 - r_2 + r_3 + r_4 - r_5$$

$$a_3 = -r_1 + r_2 - r_3 - r_4 + r_5$$

$$a_4 = r_1 - r_2 - r_3 - r_4 - r_5$$

$$C = \{00000, 11001, 10110, 01111\}$$

$$L_4 = \frac{l_1 l_2 l_3 l_4 l_5 + l_3 l_4}{l_2 l_5 + l_1}$$

$$L_5 = \frac{l_1 l_2 l_3 l_4 l_5 + l_2 l_5}{l_3 l_4 + l_1}.$$

(11) RS code $(2^m - 1, 2^m - 1 - 2t, 2t+1)$

$(n) \quad (k) \quad (d)$

(b) $p_s = 1 - (1-p)^m \longrightarrow p_s \approx mp$ as $p \to 0$

$\Pr(\text{block error}) = \Pr(> t \text{ symbol errors in } n \text{ symbols})$

$$= \sum_{j=t+1}^{n} \binom{n}{j} p_s^j (1-p_s)^{n-j}$$

BCH code $(\underset{(n)}{2^m - 1}, 2^m - 1 - mt, 2t+1)$

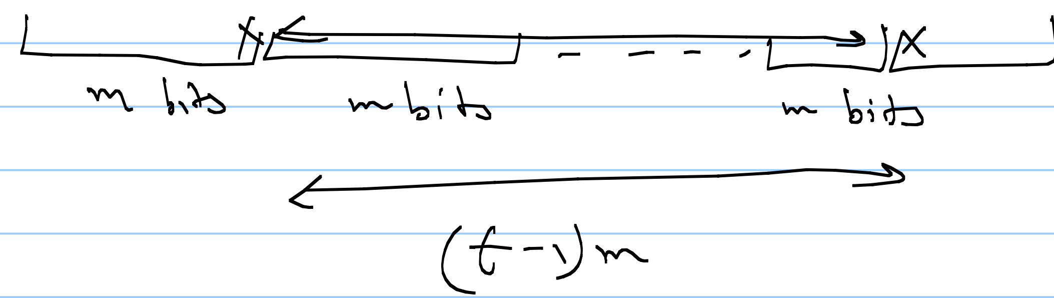
$$\Pr(\text{block error}) = \sum_{j=t+1}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

13. Let C be a t-error-correcting RS code of length $n = 2^m - 1$ over GF(2^m).

(a) Determine the exact burst-error-correcting capability of C in bits.

(b) Let M codewords of C be symbol-interleaved by a row-column interleaver. Determine the burst-error-correcting capability after interleaving.

(a) $(t-1)m+1$



(b) $(t-1)Mm + M$

$\boxed{tMm - m + 1}$