Lecture 26

Note Title 3/13/2008

→ (w_c, w_r)-regular LDPC codes
→ $(\lambda(x), \rho(x))$- LDPC code

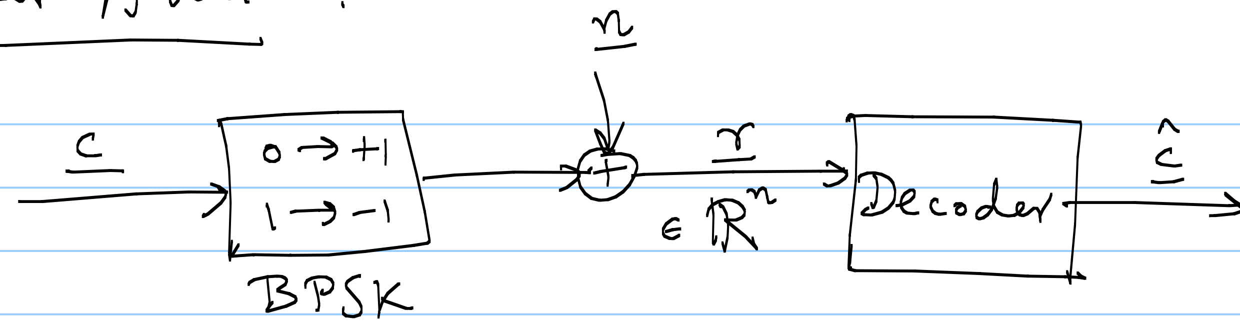
BSC

1) Construction
2) Message-passing decoder (Gallager A)

3) Density Evolution
↳ Threshold : function of $\lambda(x), \rho(x)$.

→ Optimization: find "good" $\rho(x), \lambda(x)$

BPSK over AWGN:



$$\underline{r} = [r_1 \ r_2 \ \ldots \ r_n]$$

→ Soft-decision message-passing decoders are "approximations" of bitwise-MAP decoders.

↳ approx. compute $Pr(c_i = 0 \mid \underline{r})$ (or

"likelihood-ratio" $L_i = \dfrac{Pr(c_i = 0 \mid \underline{r})}{Pr(c_i = 1 \mid \underline{r})}$

(or)

"LLR" $\log L_i$

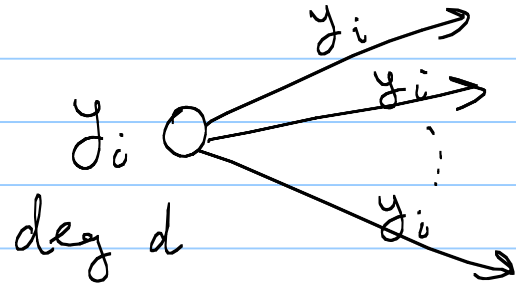
Channel-LLR

$$l_i = \frac{Pr(c_i = 0 \mid r_i)}{Pr(c_i = 1 \mid r_i)} = e^{\frac{2 r_i}{\sigma^2}}$$

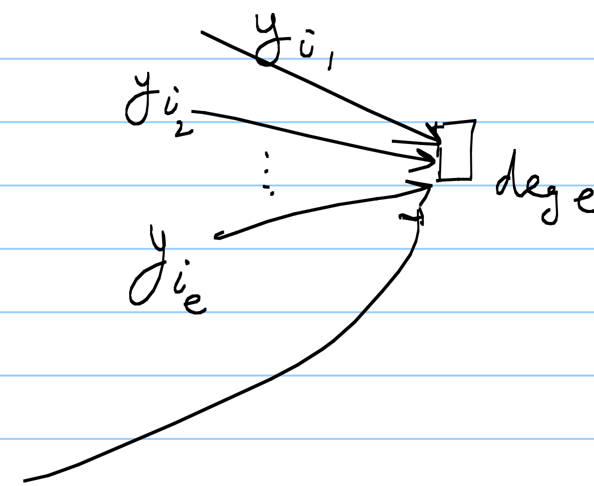
$$y_i = \log l_i = \frac{2}{\sigma^2} r_i$$

- → channel-LLR y_i "assigned" to i-th bit node.
- → Each node will send its best estimate of LLR to its neighbours.
- → avoid resending information

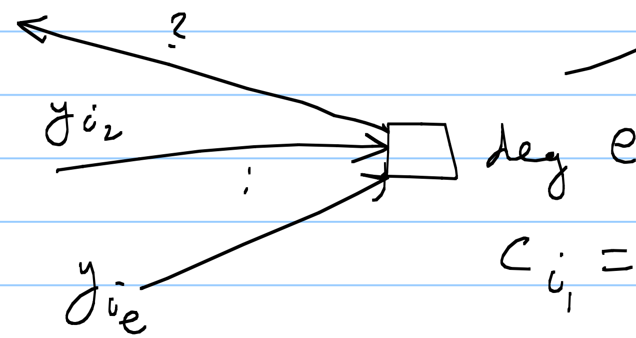
Iteration 1: Step (a)



Step (a)



Step (b)



$$c_{i_1} = c_{i_2} + c_{i_3} + \cdots + c_{i_e}$$

$LLR = y_{i_2}$ - - - . . $LLR = y_{i_e}$

→ Calculation at a check node:

given LLRs of bits $i_2, \ldots, i_e$, compute LLR for bit i_1

Toy example: 1) $X = Y + Z$

$$\Pr(Y=0) = p_{y0}$$

$$\Pr(Z=0) = p_{Z0}$$

$$p_{X0} = p_{y0}\, p_{Z0} + (1 - p_{y0})(1 - p_{Z0})$$

2)

$$X = Y_1 + Y_2 + Y_3 + \cdots + Y_{100}$$

$$\Pr(Y_i = 0) = p_{i0}$$

$$p_{X0} = \sum_{2^{99} \text{ cases}} (\text{product of } 100 \text{ terms})$$

Revisit (1) $X = Y + Z$

$$(p_{X0} - p_{X1}) = (p_{Y0} - p_{Y1})(p_{Z0} - p_{Z1})$$

Revisit (2)

$$p_{X0} - p_{X1} = \prod_{i=1}^{100} (p_{i0} - p_{i1})$$

$$\frac{p_{X0} - p_{X1}}{p_{X0} + p_{X1}} = \prod_{i=1}^{100} \left(\frac{p_{i0} - p_{i1}}{p_{i0} + p_{i1}} \right)$$

$$\frac{l_x - 1}{l_x + 1} = \prod_{i=1}^{100} \left(\frac{l_i - 1}{l_i + 1} \right)$$

$$\frac{e^{y_x}-1}{e^{y_x}+1} = \prod_{i=1}^{100} \frac{e^{y_i}-1}{e^{y_i}+1}$$

$$\tanh\left(\frac{y_x}{2}\right) = \prod_{i=1}^{100} \tanh\left(\frac{y_i}{2}\right)$$

$s_i = \mathrm{sgn}(y_i)$

$s_x = \mathrm{sgn}(y_x)$ sign

magnitude

$$s_x = \prod_{i=1}^{100} s_i$$

$$\log \tanh \frac{|y_x|}{2} = \sum_{i=1}^{100} \log \tanh \frac{|y_i|}{2}$$

$$\left(\begin{aligned} f(x) &= \log \tanh \frac{|x|}{2} \\ |x| &= \log \tanh \frac{f(x)}{2} \end{aligned} \right)$$

$$\underline{\underline{|y_x| = f\left(\sum_{i=1}^{100} f(|y_i|)\right)}}$$