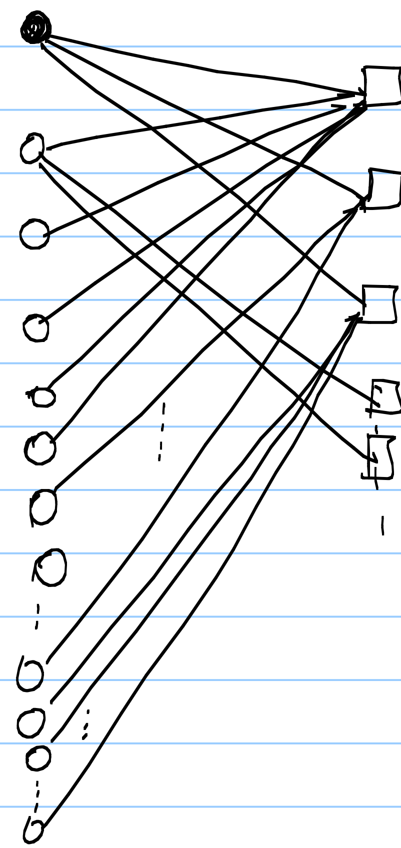
Lecture 22

Note Title 3/5/2008

Neighbourhood:

(3,6)-regular

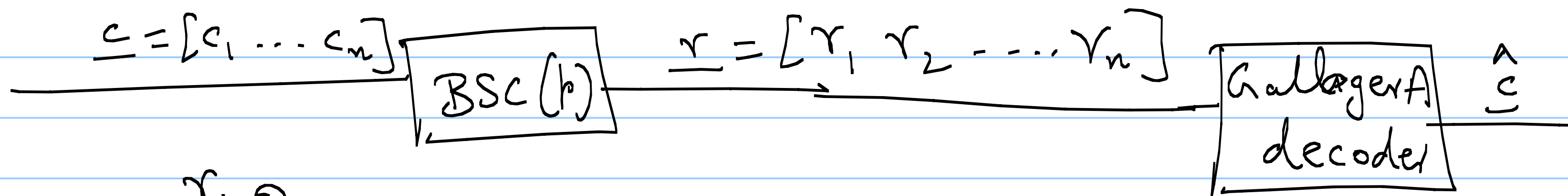


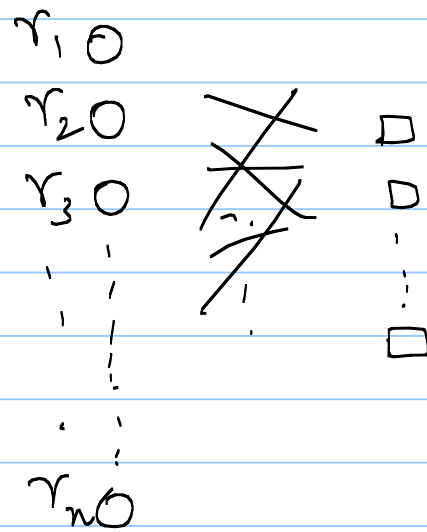
Gallager A decoder: hard-decision decoder

→ iterative

→ described on the Tanner graph.

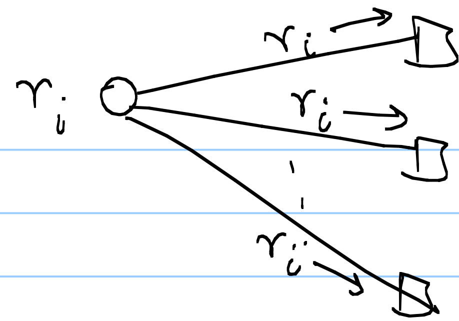
→ (w_c, w_r)-regular LDPC code





Iteration 1:

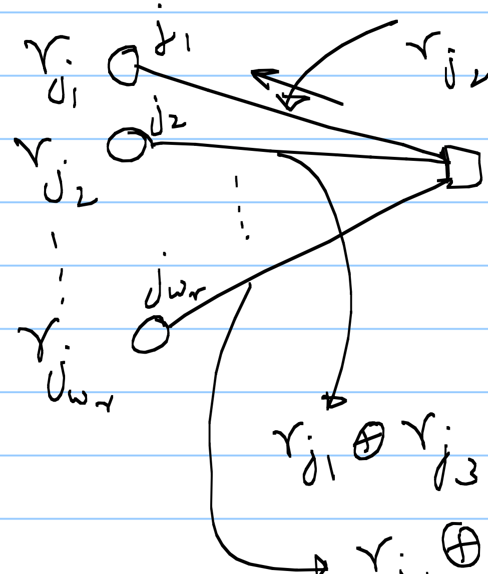
(a) Bit node i sends r_i to its neighbouring check nodes



Itr. 1

Step (a)

(b)



$r_{j_2} \oplus r_{j_3} \oplus \cdots \oplus r_{j_{w_r}}$

check node j

Step (a): received $r_{j_1}, \ldots, r_{j_{w_r}}$

$r_{j_1} \oplus r_{j_3} \oplus \cdots \oplus r_{j_{w_r}}$

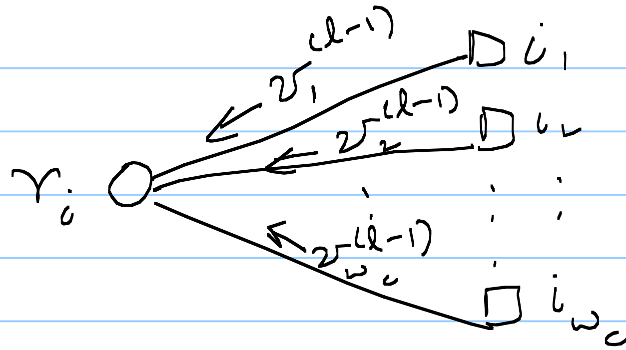
$r_{j_1} \oplus r_{j_2} \oplus \cdots \oplus r_{j_{w_r-1}}$

$$\frac{1-(1-2p)^{w_r-1}}{2}$$

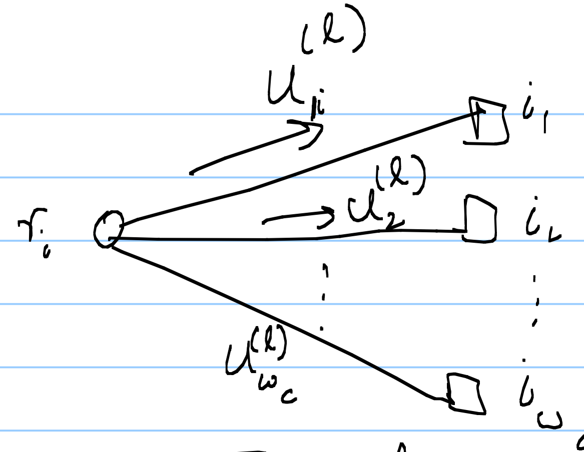
$\Pr(\text{Error})$

Iteration ℓ $\quad \ell \geq 2$

Step (a)



Itr $\ell-1$
Step (b)



Itr ℓ
step (a)

If $v_2^{(\ell-1)} = v_3^{(\ell-1)} = \dots = v_{w_c}^{(\ell-1)} = b$ (0 (or) 1),

$$u_1^{(\ell)} = b$$

Else $\quad u_1^{(\ell)} = r_i$

If $v_1^{(\ell-1)} = v_3^{(\ell-1)} = \dots = v_{w_c}^{(\ell-1)} = b$,

$u_2^{(\ell)} = b$, Else $u_2^{(\ell)} = r_i$

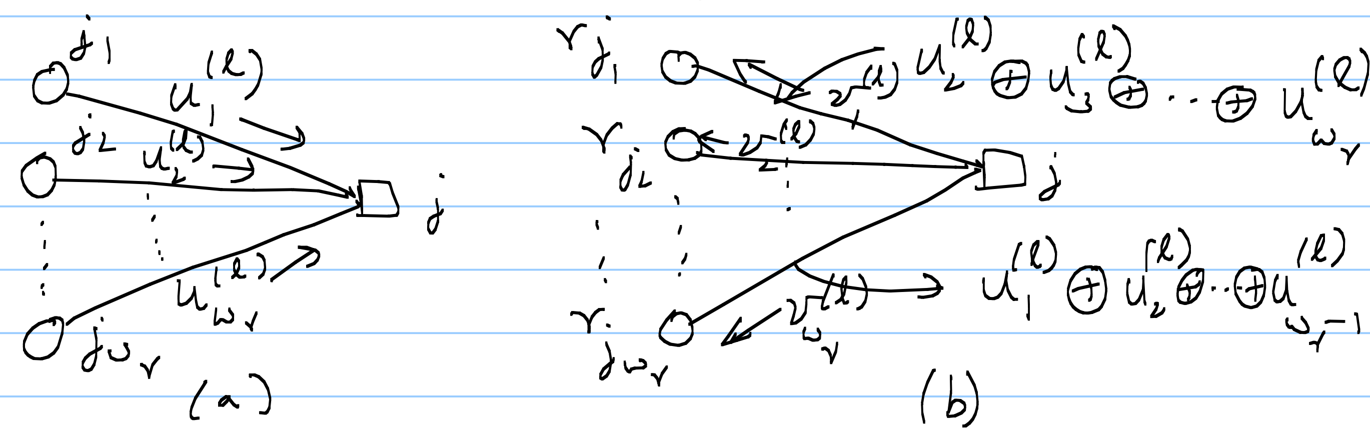
$q^{(\ell-1)} = $ (Prob that $v^{(\ell-1)}$ is in error)

<u>all-zero codeword:</u> error in $v^{(\ell-1)}$, if $v^{(\ell-1)} = 1$.

$$p^{(\ell)} = \text{Prob}\{u^{(\ell)} = 1\}$$

$$= (1-p)\left(q^{(\ell-1)}\right)^{w_c-1} + p\left(1-\left(1-q^{(\ell-1)}\right)^{w_c-1}\right)$$

<u>Step (b):</u>



$$q^{(\ell)} = \frac{1 - (1-2p^{(\ell)})^{w_r - 1}}{2}$$

$$p^{(\ell)} = f(p^{(\ell-1)}), \quad \ell = 1, 2, \ldots$$

$$p^{(0)} = p$$

Decision after iteration ℓ:

Bit i : $r_i, v_1^{(\ell)}, v_2^{(\ell)}, \ldots, v_{w_c}^{(\ell)}$

$$\hat{c}_i^{(\ell)} = \mathrm{maj}\{r_i, v_1^{(\ell)}, \ldots, v_{w_c}^{(\ell)}\}$$

$$\underline{\hat{c}}^{(\ell)} = [\hat{c}_1^{(\ell)} \ \hat{c}_2^{(\ell)} \ \ldots \ \hat{c}_n^{(\ell)}]$$

→ If there are no cycles of length $2l$ (or) lower in the Tanner graph, iid assumption holds up to iteration l.

→ iid assumption holds with high probability for any finite l as $n \to \infty$

Threshold : p^* : transition probability

$p^{(0)} = p < p^*$ $\quad\quad$ $p^{(l)} \longrightarrow 0$

$p^{(0)} = p > p^*$ $\quad\quad$ $p^{(l)} \longrightarrow$ finite value

$p^* =$ function of w_r & w_c.