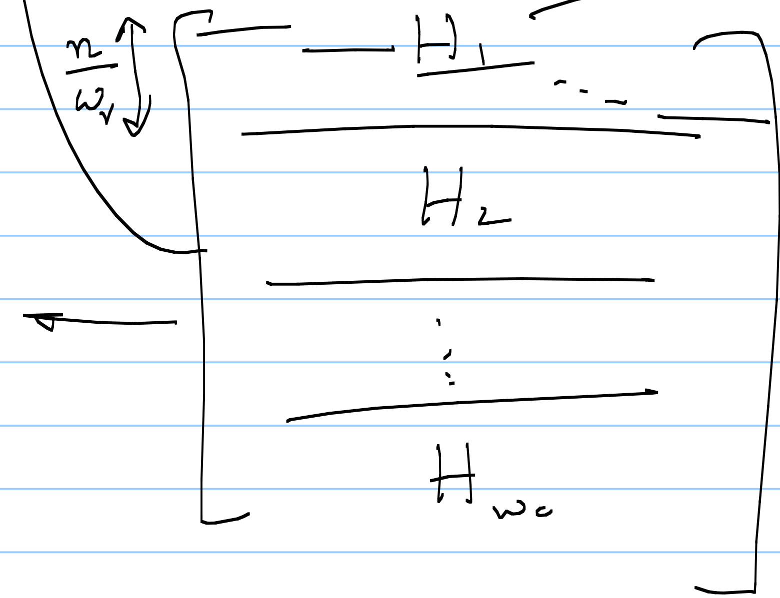
Lecture 21

Note Title 2/29/2008

(w_c, w_r)-regular LDPC matrix

$w_r \mid n$

$(1, w_r)$-regular



Gallager's Construction

Ex: $n = 8$ $(3,4)$-regular $n = 12$

9×12 matrix

$$
\begin{array}{c c}
 & 8 \\
6 & \left[\begin{array}{cccccccc}
1 & 1 & 1 & 1 & & & & \\
 & & & & 1 & 1 & 1 & 1 \\
\hline
1 & 1 & & & 1 & 1 & & \\
 & & 1 & 1 & & & 1 & 1 \\
\hline
1 & & 1 & & 1 & & 1 & \\
 & 1 & & 1 & & 1 & & 1
\end{array}\right]
\end{array}
$$

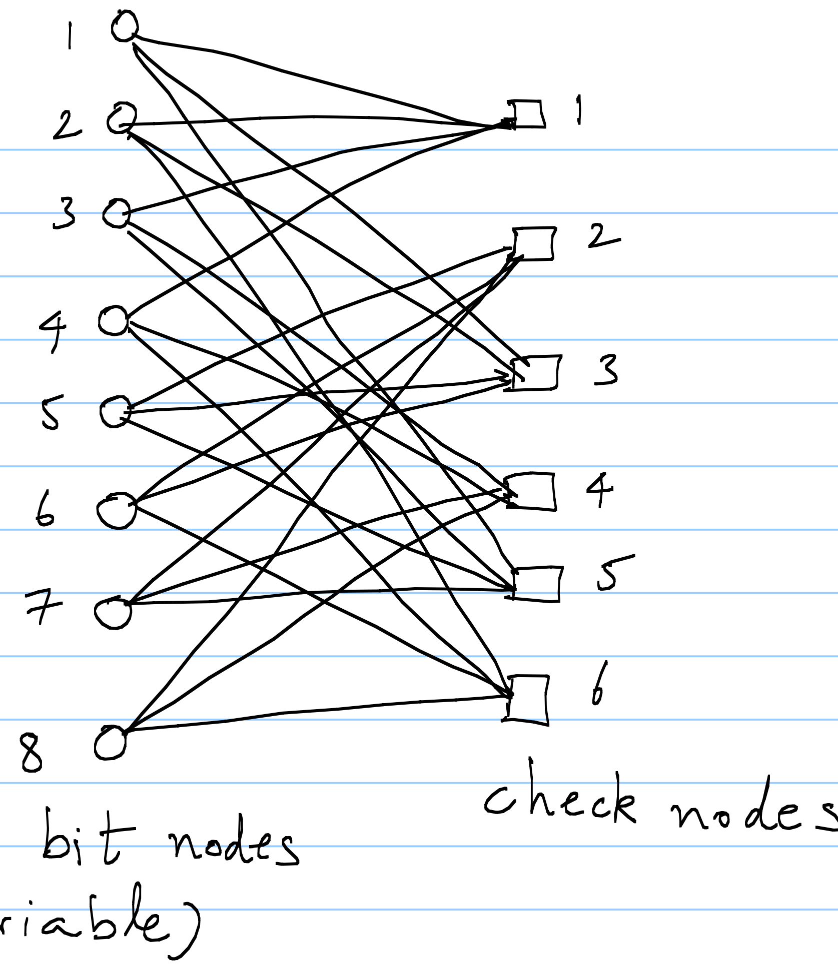
Tanner graph: graphical representation of a parity-check matrix. (H)

nodes connected by edges

Nodes: columns & rows of H

Edges: 1s in H.

$$
\begin{array}{c|cccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\hline
1 & 1 & 1 & 1 & 1 & & & & \\
2 & & & & & 1 & 1 & 1 & 1 \\
3 & 1 & 1 & & & 1 & 1 & & \\
4 & & & 1 & 1 & & & 1 & 1 \\
5 & 1 & & 1 & & 1 & & 1 & \\
6 & & 1 & & 1 & & 1 & & 1
\end{array}
$$



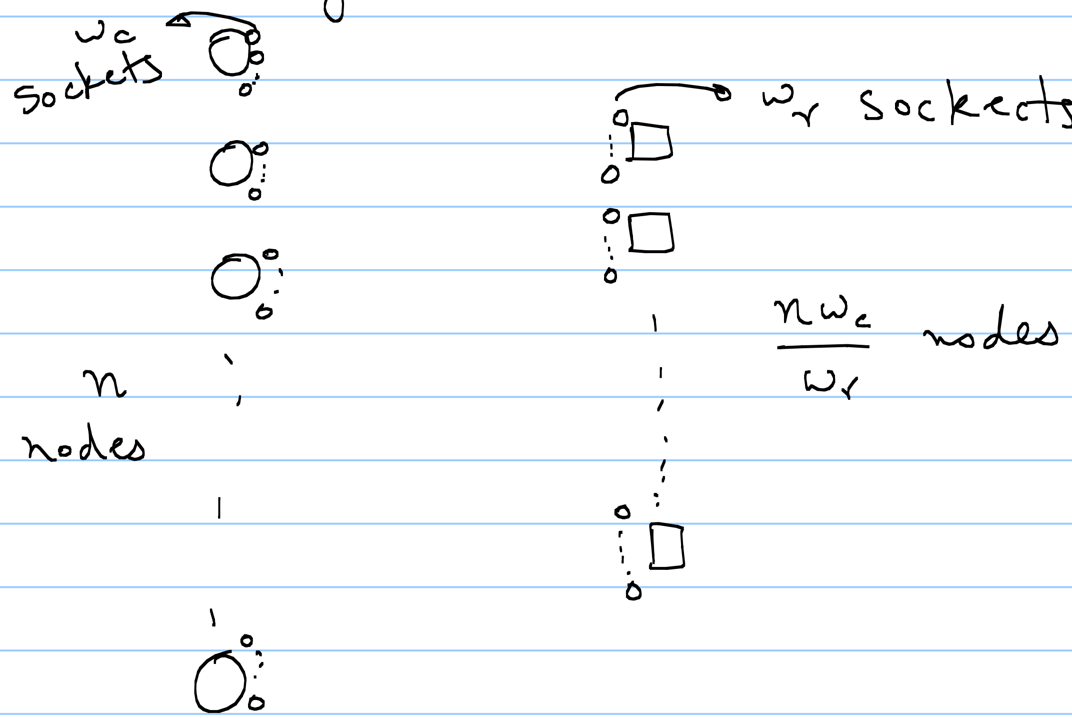
bit node degree $= w_c$

check node degree $= w_r$

# of edges $= n w_c$

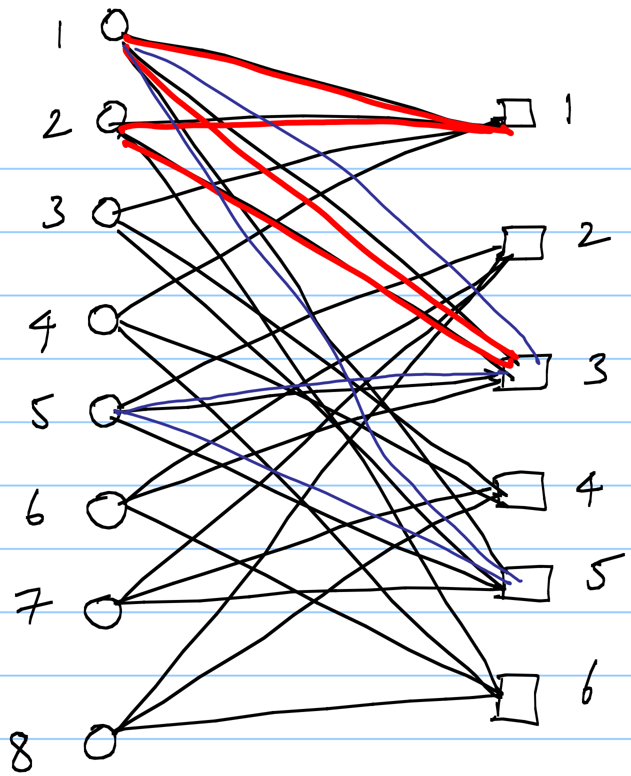
Socket Construction (Rudiger Urbanke, Thomas Richardson)

(n, w_c, w_r)-regular LDPC codes



$\pi : \{1, 2, \ldots, n w_c\} \longrightarrow \{1, 2, \ldots, n w_c\}$ bijection

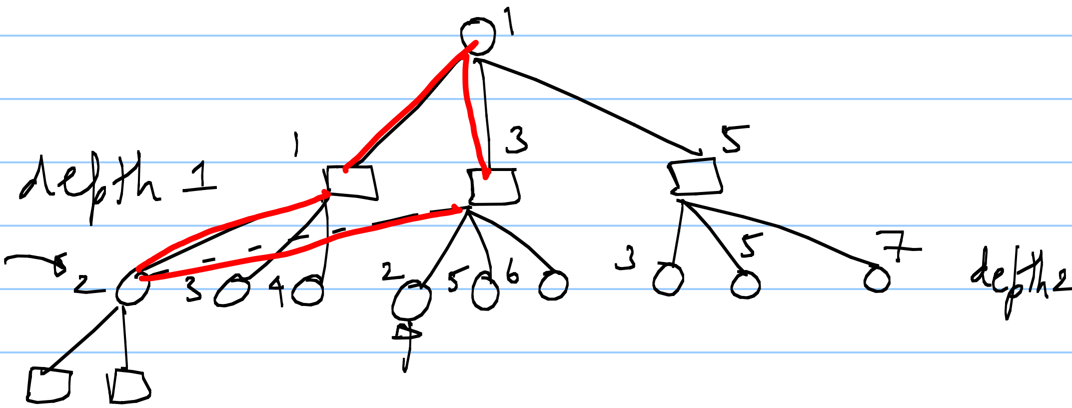
$\rightarrow$ Connect Socket $i_{(LHS)}$ to Socket $\pi(i)_{(RHS)}$



(8, 3, 4) - regular
LDPC code

Neighbourhood:

bit node 1



(n, w_c, w_r) - code.

# of neighbours at depth 1 $= w_c$ checks

" depth 2 $= w_c(w_r - 1)$ bits

" depth 3 $= w_c(w_r - 1)(w_c - 1)$ checks

⋮

- Repetition of nodes in neighbourhood

$\Updownarrow$

cycles in Tanner graph

- length $2l$ cycle in Tanner graph

$\Updownarrow$

repetition at depth l for nodes in that cycle.

- Desirable: avoid short cycles.
- Given l. $\exists$ n large enough s.t.

$\Pr(\text{length} \leq 2l \text{ cycle}) \to 0$