Lecture 18

Note Title 2/22/2008

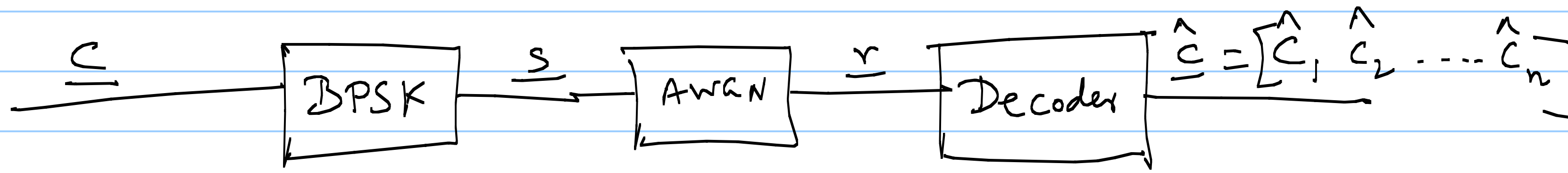
→ BPSK over AWGN

$$\hat{\underline{c}} = \arg \min_{\underline{u} \in C} \| \underline{r} - (1 - 2\underline{u}) \|^2$$

$$\left(= \arg \max_{\underline{u} \in C} \underline{r} \cdot (1 - 2\underline{u}) \right)$$

Bitwise - MAP decoder:

(Maximum a posteriori probability)



"Bitwise": decode one bit at a time

How does one find $\hat{c}_i$?

if $\Pr(c_i = 0 \mid \underline{r}) > \Pr(c_i = 1 \mid \underline{r})$,

then $\hat{c}_i = 0$

else $\hat{c}_i = 1$.

Other ways:

1) $\Pr(c_i = 0 \mid \underline{r}) > 1/2$

APP

2) $$\frac{\Pr(c_i = 0 \mid \underline{r})}{\Pr(c_i = 1 \mid \underline{r})} > 1$$

APP ratio

log APP ratio

3) $$\log \frac{\Pr(c_i = 0 \mid \underline{r})}{\Pr(c_i = 1 \mid \underline{r})} > 0$$

$$\xrightarrow{\underline{c}} \boxed{\text{BPSK}} \xrightarrow{\underline{s}} \boxed{\text{AWGN}} \xrightarrow{\underline{r}} \boxed{\text{Decoder}} \longrightarrow \underline{\hat{c}} = [\hat{c}_1 \; \cdots \; \hat{c}_n]$$

APP $\quad \Pr(c_i = 0 \mid \underline{r})$

if $\quad \Pr(c_i = 0 \mid \underline{r}) > \Pr(c_i = 1 \mid \underline{r}), \quad \hat{c}_i = 0$

$$\Pr(c_i = 0 \mid \underline{r}) = \frac{f(\underline{r} \mid c_i = 0) \cdot \Pr(c_i = 0)}{f(\underline{r})}$$

$$\text{APP ratio} = \frac{\Pr(c_i = 0 \mid \underline{r})}{\Pr(c_i = 1 \mid \underline{r})} = \frac{f(\underline{r} \mid c_i = 0)}{f(\underline{r} \mid c_i = 1)} \, \frac{\cancel{\Pr(c_i = 0)}}{\cancel{\Pr(c_i = 1)}}$$

$= \text{likelihood ratio}$

How to compute $f(\underline{r} \mid c_i = 0)$ & $f(\underline{r} \mid c_i = 1)$?

log-APP ratio $\log \dfrac{\Pr(c_i = 0 \mid \underline{r})}{\Pr(c_i = 1 \mid \underline{r})}$

log-likelihood ratio (LLR) $\log \dfrac{f(\underline{r} \mid c_i = 0)}{f(\underline{r} \mid c_i = 1)}$

C : (n, k) code

$$f(\underline{r} \mid c_i = 0) = \sum_{\left(\substack{\underline{u} \in C: \\ u_i = 0}\right)} \underbrace{f(\underline{r} \mid \underline{u})}_{\checkmark} \cdot \underbrace{\Pr(\underline{u} \mid c_i = 0)}_{1/2^{k-1}}$$

$$f(\underline{r} \mid c_i = 1) = \sum_{\substack{\underline{u} \in C: \\ u_i = 1}} \overbrace{f(\underline{r} \mid \underline{u})} \cdot \overbrace{\Pr(\underline{u} \mid c_i = 1)}$$

Likelihood ratio

$$\frac{f(\underline{r} \mid c_i = 0)}{f(\underline{r} \mid c_i = 1)} = \frac{\sum_{\substack{\underline{u} \in C: \\ u_i = 0}} f(\underline{r} \mid \underline{u})}{\sum_{\substack{\underline{u} \in C: \\ u_i = 1}} f(\underline{r} \mid \underline{u})}$$

$$f(\underline{r} \mid \underline{u}) = \prod_{j=1}^{n} \underbrace{f(r_j \mid u_j)}_{=\ \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(r_j - (1-2u_j))^2}{2\sigma^2}}}$$

$$\frac{f(\underline{r} \mid c_i = 0)}{f(\underline{r} \mid c_i = 1)} = \frac{\displaystyle\sum_{\underline{u} \in C : u_i = 0} \prod_{j=1}^{n} f(r_j \mid c_j = u_j)}{\displaystyle\sum_{\underline{u} \in C : u_i = 1} \prod_{j=1}^{n} f(r_j \mid c_j = u_j)}$$

$$= \underbrace{\frac{f(r_i \mid c_i = 0)}{f(r_i \mid c_i = 1)}} \underbrace{\frac{\displaystyle\sum_{\underline{u} \in C : u_i = 0} \prod_{\substack{j=1 \\ j \neq i}}^{n} f(r_j \mid c_j = u_j)}{\displaystyle\sum_{\substack{\underline{u} \in C : \\ u_i = 1}} \prod_{\substack{j=1 \\ j \neq i}}^{n} f(r_j \mid c_j = u_j)}}_{\text{No } r_i \text{ here}}$$

Ex: $C = \{0000, 1010, 0101, 1111\}$

$$\underline{r} = [r_1 \quad r_2 \quad r_3 \quad r_4]$$

$$L_i = \frac{f(\underline{r} \mid c_i = 0)}{f(\underline{r} \mid c_i = 1)}$$

$$L_1 = \frac{f(r_1|0)}{f(r_1|1)} \cdot \left(\frac{\overbrace{f(r_2|0)f(r_3|0)f(r_4|0)}^{0000} + \overbrace{f(r_2|1)f(r_3|0)f(r_4|1)}^{0101}}{\underbrace{f(r_2|0)f(r_3|1)f(r_4|0)}_{1010} + \underbrace{f(r_2|1)f(r_3|1)f(r_4|1)}_{1111}} \right)$$

$$l_1 = \frac{f(r_1|0)}{f(r_1|1)} = e^{\frac{2r_1}{\sigma^2}} \qquad \log l_1 = \frac{2r_1}{\sigma^2}$$

$$\underset{\text{output likelihoods}}{L_1} = l_1 \cdot \frac{l_2 l_3 l_4 + l_3}{l_2 l_4 + 1} , \quad \underset{\text{Channel likelihoods}}{l_i} = e^{\frac{2 r_i}{\sigma^2}}$$

$$\text{channel LLR} = \frac{2 r_i}{\sigma^2}$$

In general,

$$L_i = \underset{\text{intrinsic}}{l_i} \cdot \underbrace{\frac{\displaystyle\sum_{\substack{\underline{u} \in C: \\ u_i = 0}} \prod_{\substack{j : u_j = 0 \\ (j \neq i)}} l_j}{\displaystyle\sum_{\substack{\underline{u} \in C: \\ u_i = 1}} \prod_{j : u_j = 0} l_j}}_{\text{extrinsic.}}$$