Lecture 17

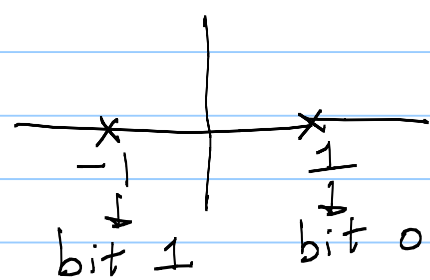
Note Title 2/21/2008

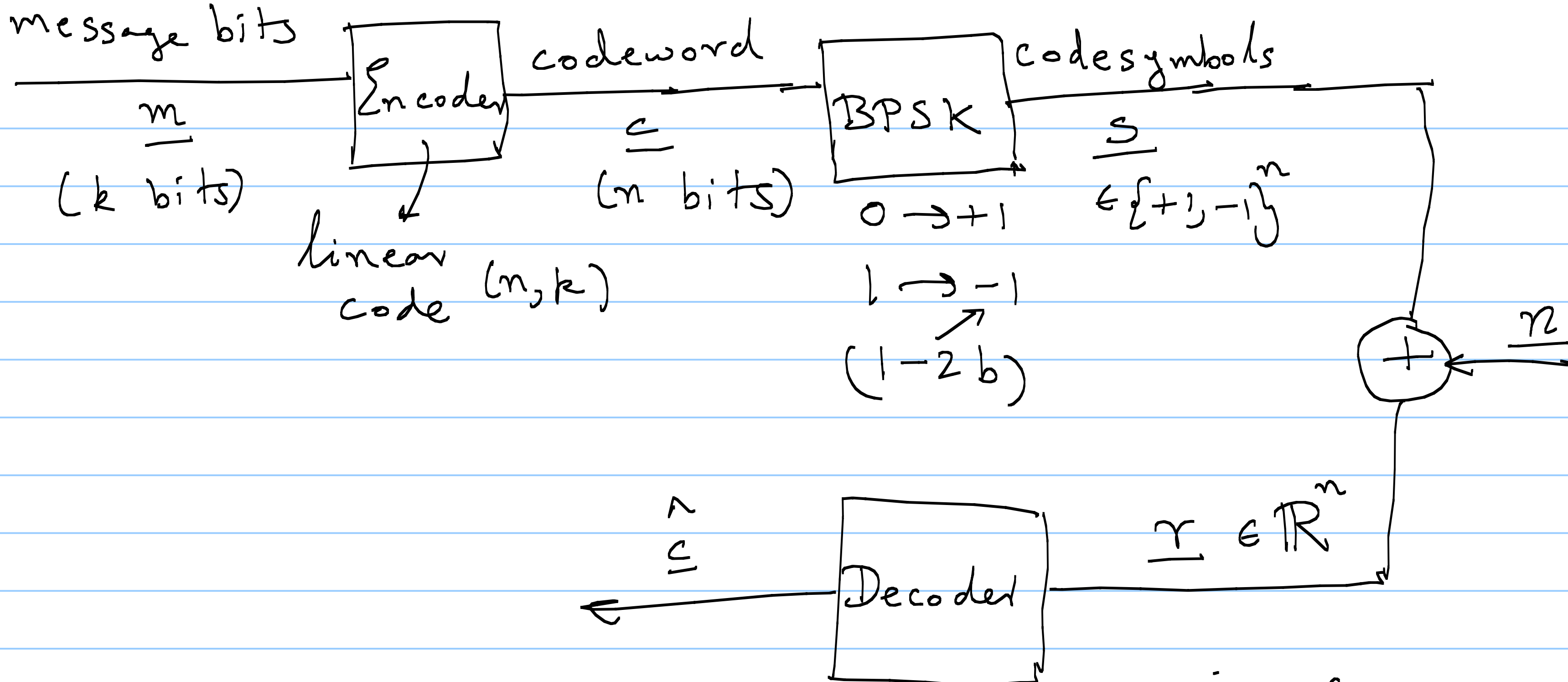
LDPC codes:
Low-density parity-check codes

→ Coding gain?

→ Channel model: BSC(p)
AWGN (Additive White Gaussian Noise)

→ BPSK modulation





$$\underline{n} \sim N(\underline{0}, \sigma^2 I_n)$$

Ex: $n = 3$

$$C = \{000, 111\}$$

$$\underline{s} = \{[+1 \ \ +1 \ \ +1], [-1 \ \ -1 \ \ -1]\}$$

PDF $f(\underline{r}) = \frac{1}{2} f(\underline{r} \mid \underline{s} = [1\ 1\ 1]) + \frac{1}{2} f(\underline{r} \mid \underline{s} = [-1\ -1\ -1])$

$\underline{r} = [r_1\ \ r_2\ \ r_3]$ $\quad N([1\ 1\ 1], \sigma^2 \underline{I})$ $\quad N([-1\ -1\ -1], \sigma^2 \underline{I})$

$$= \frac{1}{2} \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^3 e^{-\frac{[(r_1-1)^2 + (r_2-1)^2 + (r_3-1)^2]}{2\sigma^2}}$$

$$+ \frac{1}{2} \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^3 e^{-\frac{[(r_1+1)^2 + (r_2+1)^2 + (r_3+1)^2]}{2\sigma^2}}$$

Conditional pdf: (in general)

$$\underline{r} = [r_1 \; r_2 \; \cdots \; r_n] \quad \text{"received vector"}$$

$$f(\underline{r} \mid \underline{s} = [s_1 \; s_2 \; \cdots \; s_n])$$

$$= \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^n e^{-\frac{\|\underline{r} - \underline{s}\|^2}{2\sigma^2}}$$

ML decoder: (optimal if all codewords are equally likely)

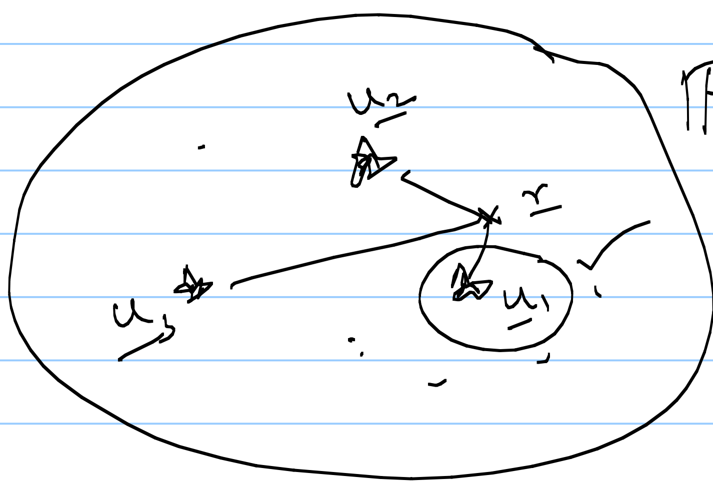
$$\hat{\underline{c}} = \arg\max_{\underline{u} \in C} f(\underline{r} \mid \underline{s} = 1 - 2\underline{u})$$

$$S = \{ 1 - 2\underline{u} : \underline{u} \in C \} \quad \text{"set of all possible code symbols"}$$

$$\underline{\hat{s}} = \arg\max_{\underline{u} \in S} f(\underline{r} \mid \underline{s} = \underline{u})$$

$$= \arg\max_{\underline{u} \in S} \underbrace{\left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n}_{\text{drop this}} e^{-\frac{\|\underline{r} - \underline{u}\|^2}{2\sigma^2}}$$

$$= \arg\min_{\underline{u} \in S} \|\underline{r} - \underline{u}\|^2$$



$\mathbb{R}^n$

Geometric view

Ex: $n = 3$ $\quad S = \{[1\ 1\ 1],\ [-1\ -1\ -1]\}$

$$\underline{r} = [0.2 \quad 1.1 \quad 0.3] \qquad \hat{\underline{s}} = [1\ 1\ 1]$$

$$\underline{r} = [0.2 \quad -1.1 \quad 0.3] \qquad \hat{\underline{s}} = [-1\ -1\ -1]$$

5.54 $\qquad\qquad$ 3.14

$[1\ 1\ 1]$ $\qquad\qquad$ $[-1\ -1\ -1]$

Hard versus Soft decoders

Hard: 1-bit suboptimal comparator is used first

Soft: Uses "real" valued $\underline{r}$

Ex: $\rightarrow$ ML decoder is difficult to implement for large n & k.

$$\underline{C} = \{0000, 0101, 1010, 1111\}$$

$$\|\underline{r} - \underline{u}\|^2 = \sum_{i=1}^{n} (r_i - u_i)^2$$

$$= \underbrace{\sum r_i^2} + \underbrace{\sum u_i^2}_{= n} - 2 \sum r_i u_i$$

$$\hat{\underline{c}} = \arg \max_{\underline{u} \in C} \underline{r} \cdot (1 - 2\underline{u})$$

(BPSK)

$$\underline{r} = [0.5 \quad 0.5 \quad -0.7 \quad -0.9] \quad \hat{\underline{c}} = [1 \ 1 \ 1 \ 1]$$