

Problems

Note Title

⑥ GF(16)

$$(a) x + y + z = 0$$

$$(b) x^2 + y^2 + z^2 = 0$$

$$(c) x + y + z = 0$$

$$\underbrace{x^3 + y^3 + z^3}_{=0} = 0$$

$$\underline{\underline{z=0, x=y}}$$

⑦ BCH code

⑧ BCH code

⑨ $n=63, t$ BCH code

(a) Smallest t s.t. $k \neq n - 6t$

$$t=5$$

(b) $n=255$. Smallest t s.t. $k \neq n - 8t$

$$t=9$$

10) $t=1$, RS code over $GF(8)$

(a) Find a nonzero codeword $C = [c_1, c_2, \dots, c_7]$
with $c_4 = c_5 = c_6 = c_7 = 0$

$$H = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \\ 1 & \alpha^4 & \alpha^8 & \alpha^{12} & \alpha^{16} & \alpha^{20} & \alpha^{24} \end{bmatrix}$$

$$H C^T = 0 \Rightarrow \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^4 & \alpha^8 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 0 & \alpha + \alpha^4 & \alpha^2 + \alpha^8 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = 0$$

(b) Nonzero codeword with $c_2 = c_4 = c_5 = c_6 = 0$

11) RS code

12) BCH code

13) $n=15, t=1$

(a) C_1 : Binary BCH, $k=11$

C_2 : RS code, $t=1$, over $GF(8)$
 $N=5$ over $GF(8)$

$$K = 5 - 2t = 3 \text{ over } GF(8)$$

↓ expanded

$$k=9$$

(b) C_1 : $P_r(\text{block error}) = 1 - (1-p)^{15}$

C_2 : $p_s = 1 - (1-p)^3 \rightarrow 105 p^2 \approx p \rightarrow 0$

$P_r(\text{block error}) = 1 - (1-p_s)^9$
 $- 5 p_s (1-p_s)^9$

$\rightarrow 90 p^2 \approx p \rightarrow 0$

(c) C_1 over C_2 : k is higher

C_2 over C_1 : field is smaller

(14) (2,1) code $C_1 = \{00, 11\}$

(6,4) code C_2 (systematic)

2 bits $\rightarrow C_1 \rightarrow$ 4 bits $\rightarrow C_2 \rightarrow$ 6 bits

(17) BCH, RS

(18) Done in class

(19) burst error-correcting capability

(20) bug

(21) bug.