

Quiz 2

Note Title

$$\textcircled{1} \quad RML(r, b) \quad n = 2^m = 64 \quad d = 2^{m-r}$$
$$m = 6 \quad k = \sum_{i=0}^r \binom{m}{i}$$

$\textcircled{2}$ C : binary BCH code, $n = 15$, $t = 2$

$$\text{(a)} \quad g(x) = (x^4 + x + 1)(x^4 + x^3 + x^2 + x + 1)$$
$$= 1 + x^4 + x^6 + x^7 + x^8$$

$$h(x) = \frac{x^{15} + 1}{g(x)} = (x + 1)(x^2 + x + 1)(x^4 + x^3 + 1)$$

$$g^\perp(x) = x^7 h(x^{-1}) = (x + 1)(x^2 + x + 1)(x^4 + x + 1)$$
$$= 1 + x + x^3 + x^7$$

$$\text{(b)} \quad G = \begin{bmatrix} \text{---} g(x) \text{---} \\ \text{---} xg(x) \text{---} \\ \vdots \end{bmatrix} \quad H = \begin{bmatrix} \text{---} g^\perp(x) \text{---} \\ \text{---} xg^\perp(x) \text{---} \\ \vdots \end{bmatrix}$$

$$\text{(c)} \quad k = n - \deg(g(x)) = 15 - 8 = 7$$

$$d \geq 5, \quad wt(\text{cyc}(g(x))) = 5 \Rightarrow d = 5$$

$n=7, t=2, \text{RS over GF}(8)$

3 (a) $g(x) = (x+\alpha)(x+\alpha^2)(x+\alpha^3)(x+\alpha^4)$ primitive $\alpha \in \text{GF}(8)$

(b) $r(x) = \alpha + x + x^2 + x^3 + x^4 + x^5 + \alpha x^6$

$$x^n + 1 = \underbrace{(x+\alpha)(x+\alpha^2)\dots(x+\alpha^{n-1})}_{\text{primitive polynomial}}$$

Quick
Answers $d_H(\underline{r}, [1 \ 1 \ 1 \ 1 \ 1 \ 1]) = 2 < \frac{d}{2}$

$$S_1 = \alpha^5, S_2 = 1, S_3 = \alpha, S_4 = \alpha^4$$

two-errors: $\sigma(x) = 1 + \sigma_1 x + \sigma_2 x^2$

$$(d^5 x + x^2 + \alpha x^3 + \alpha^4 x^4)(1 + \sigma_1 x + \sigma_2 x^2)$$

$$x^3: \alpha + \sigma_1 + \alpha^5 \sigma_2 = 0 \quad \sigma_1 = \alpha^2$$

$$x^4: \alpha^4 + \alpha \sigma_1 + \sigma_2 = 0 \quad \sigma_2 = \alpha^6$$

$$\sigma(x) = 1 + \alpha^2 x + \alpha^6 x^2$$

Zeros: $1, \alpha$. $X_1 = 1, X_2 = \alpha^6$

$$S_1: \alpha^5 = e_0 + e_6 \alpha^6 \quad c(x) = e_0 + e_6 x^6$$

$$S_2: 1 = e_0 + e_6 \alpha^5 \quad e_0 = \alpha^3, e_6 = \alpha^3$$