Note Title

Summary:

- Reed-Solomon, BCH codes] can achieve
- Reed-Muller Codes

n: block length
t: required error-correcting capability

- Concatenations
- Weight distributions

bring two different codes together.

Product code: $C_1 : (n_1, k_1, d_1)$ G_1

$C_2 : (n_2, k_2, d_2)$ G_2

— Will result in $(\underbrace{n_1 n_2, k_1 k_2}, ?)$

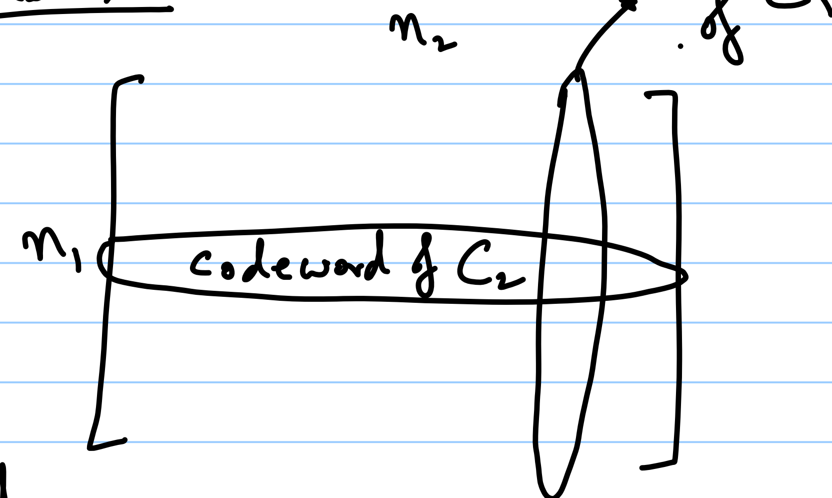
message is a $k_1 \times k_2$ matrix

$n_1 \times n_2$ matrix → Codeword $= G_1^T m G_2$ ← $k_1 \times k_2$ matrix

"product of C_1 & C_2"

- is this linear? ✓

Codeword matrix:



$d_{min} \geq \underline{\underline{d_1 d_2}}$

G_1, G_2 : in systematic form

$G_1^T m G_2$

Examples: $C = (7,4,3)$ Hamming Code
(1)

product of C with C: $(49, 16, \geq 9)$

$n = 63$, BCH, $t = 4$ $\quad (49, 25, \geq 9)$
shorten
$(63, 39, \geq 9)$

(2) $C_1 = (7,4,3)$ Hamming
$C_2 = (9,8,2)$ even-weight
Product of C_1 and C_2: $(63, 32, \geq 6)$

③ $C : (8,4,4)$

C with C : $(64, 16, \geq 16)$

BCH : $(63, 21, \geq 15)$
$\downarrow$
$t = 7$

Decoding product codes

Ex: $(7,4,3)$ Hamming with itself

$(49, 16, \geq 9)$

t errors

$$\begin{array}{cc} & 7 \\ 7 & \begin{bmatrix} & & & \\ & x & x & \\ & x & x & \\ & & & \end{bmatrix} \end{array}$$

- Try and decode one row at a time
(or)
1 col at a time
- iterate between row & column decoders

$$\Big[\underbrace{\overset{\times}{c_{11}}\, c_{12} \cdots c_{17}}\;\; \underbrace{\overset{\times}{c_{21}}\, c_{22} \cdots c_{27}}\;\; \cdots\;\; \underbrace{\overset{\times}{c_{71}}\, c_{72} \cdots c_{77}}\Big]$$

write codeword as a vector

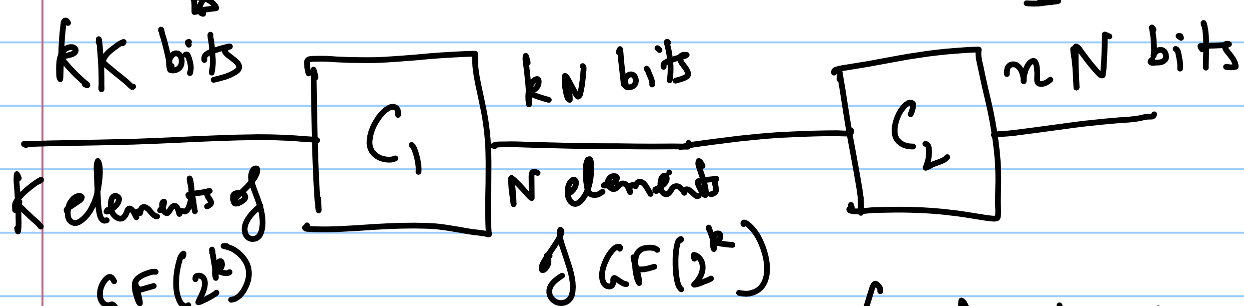
— has some "local" structure

Concatenated codes : (Forney)

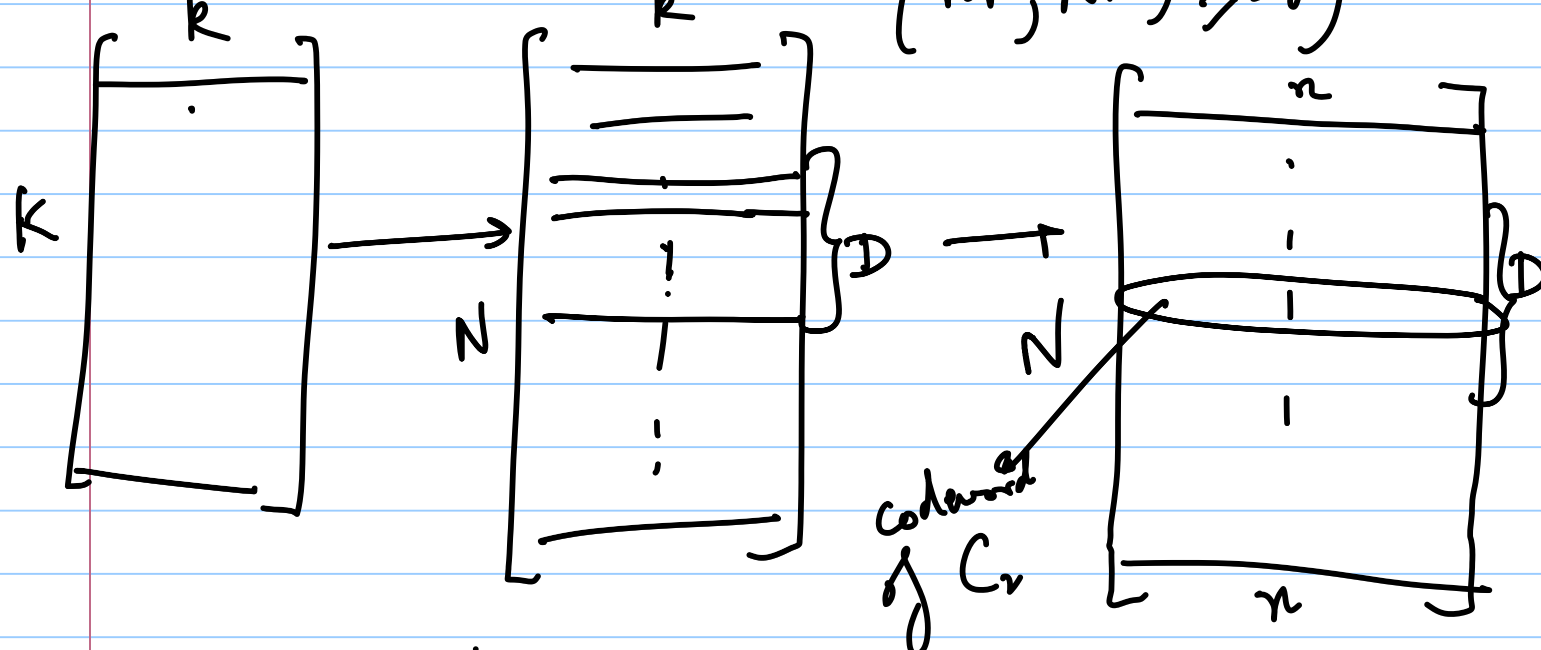
(N, K, D) over $GF(2^k)$: C_1

(n, k, d) over $GF(2)$: C_2

Concatenated code



$(nN, kK, \geq Dd)$



Ex: $(255, 245, 11)$ RS code over $GF(256)$

$\downarrow$ concatenat with $(9, 8, 2)$

$(2295, 1960, \geq 22)$

Decoder: - in a concatenation

- mark erasures, if inner decoder fails.

- error-correcting capability?