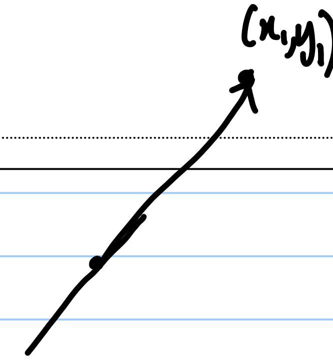
Decoding RM codes

Note Title



Ex: $EG(4,2)$

Points: $\{0,1\}^4$ 16 points

origin

0000 0001 0010 0011

0100 0101 0110 0111

1000 1001 1010 1011

1100 1101 1110 1111

lines:

$\{0000, 0110\}$,

$\{0000, 0111\}$

⋮

15 such lines

Points: 0000 0001 0010 - - 0110 - 1111 -

Subset $[\ 1 \quad 0 \quad 0 \ \ldots\ 0 \quad 1 \quad 0 \ \ldots\ 0\ ]$

$\{0000, 0110\}$

$\longleftarrow$ 16 $\longrightarrow$

2-flat $[\ 1 \quad 1 \quad 0 \ \ldots\ 0 \quad 1 \quad 1 \quad 0 \ \ldots\ 0\ ]$

$$
\begin{array}{cc}
 & \text{16 points} \\
\text{lines}\;\begin{array}{c} 1 \\ 2 \\ \vdots \\ L \end{array} & \begin{bmatrix} & 1 & 1 & \\ & & & \\ & & & \\ & & & \end{bmatrix} \; \text{2 1s}
\end{array}
$$

15 1s in each col.

$$16 \times 15 = L \times 2$$

$$L = \binom{16}{2}$$

$\#$ of 2-flats through 0000

$$= \frac{\binom{15}{2}}{3}\,? = \frac{15 \times 14}{2 \times 3} = 35$$

$$(\#\text{ of 2-flats})\,\cancel{4} = \overset{4}{\cancel{16}} \times 35$$

$$= 140$$

Result: Incidence vectors of $(m-r)$-flats in $EG(m,2)$ are codewords of $RM(r,m)$
(minimum-weight)

Pf: $(m-r)$-flat is defined by r equations of the form

$$(\deg = 1) \rightarrow \sum_{j=1}^{m} a_{ij} v_j + b_i = 0 \quad , \quad i = 1, 2, \dots, r$$

$$a_{ij} \in \{0,1\}$$
$$b_i \in \{0,1\}$$

incidence vector: $f(v_1, v_2, \dots, v_m)$

$$\begin{array}{ccc} 00\cdots0 & v_1\cdots v_m & 11\cdots1 \\ \Big[& 1 & \Big] \end{array}$$

evaluation

$$f(v_1, v_2, \dots, v_m) = \prod_{i=1}^{r} \Big(\underbrace{\sum_{j=1}^{m} a_{ij} v_j + b_i + 1} \Big)$$

$(\deg = r)$

RM(2, 4) : incidence vectors of
2-flats in EG(4, 2)
3-flats

RM(3, 4) : 1-flat,
2-flats + 3-flats

dual

RM(1, 4) : 3-flats

$J = 2^{m-r} - 1$

Main result: RM(r, m) can be decoded by (r+1)-step majority-logic decoding up to $\lfloor \frac{1}{2}(2^{m-r}-1) \rfloor$ errors.

Ex: $RM(1,4)$; dual: $RM(2,4)$ (contains 2-flats)

2-step majority-logic decodable
with $J=7$

1-flat through origin : collect all 2-flats that contain this 1-flat

form a set of orthogonal parity checks on the 1-flat

1st step

repeat for 7 such 1-flats (or more)

2nd step: Use 1-flats to find e_0.

origin

0000	0001	0010	0011
0100	0101	0110	0111
1000	1001	1010	1011
1100	1101	1110	1111

1-flat

0000, 0001

$$
\begin{aligned}
e_0 + e_1 + e_2 + e_3 &= S_1 = r_0 + r_1 + r_2 + r_3 \\
e_0 + e_1 + e_4 + e_5 &= S_2 = \vdots \\
e_0 + e_1 + e_6 + e_7 &= S_3 = \\
e_0 + e_1 + e_8 + e_9 &= S_4 = \\
e_0 + e_1 + e_{10} + e_{11} &= S_5 = \\
e_0 + e_1 + e_{12} + e_{13} &= S_6 = \\
e_0 + e_1 + e_{14} + e_{15} &= S_7 =
\end{aligned}
$$

repeat for $e_0 + e_i$, $i = 2, 3, \dots, 7$

2nd step:

$$
\hat{e}_0 = \text{maj} \left\{ \begin{array}{l} \hat{e}_0 + \hat{e}_1 \\ \hat{e}_0 + \hat{e}_2 \\ \vdots \\ \hat{e}_0 + \hat{e}_7 \end{array} \right\}
$$

In general, $RM(r, m) \xrightarrow{\text{dual}} RM(m-r-1, m)$

1st step: r-flat : $(r+1)$-flats $(r+1)$-flat that contain the r-flat

proceed .....