Note Title

Tutorial on Linear Block Codes

(1) C_1: pc matrix

$$H_1 = \begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \\ \begin{matrix} m_1 & m_2 & p_1 & p_2 \end{matrix} \end{array}$$

C_2: pc matrix

$$H_2 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

row1 = row1 $\oplus$ row2

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(a) $C_1 = \{0000, 0101, 1011, 1110\}$

$p_1 = m_1$

$p_2 = m_1 \oplus m_2$

$C_2 = \{0000, 0101, 1011, 1110\}$

(b) $G = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ for both codes.

(2)

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(a) Find n & k

$n \to 8$.

$k \to \text{rank}(G) = 4$ (Gaussian elimination)

(b) Minimum distance:

list all codewords (16 of them)

Find minimum weight

Ans: $d=4$

$(8,4,4)$ code.

(3)
$$\left.\begin{aligned} u &= [u_1 \; u_2 \; \ldots \; u_n] \\ v &= [v_1 \; v_2 \; \ldots \; v_n] \\ w &= [w_1 \; w_2 \; \ldots \; w_n] \end{aligned}\right\} \text{binary vectors}$$

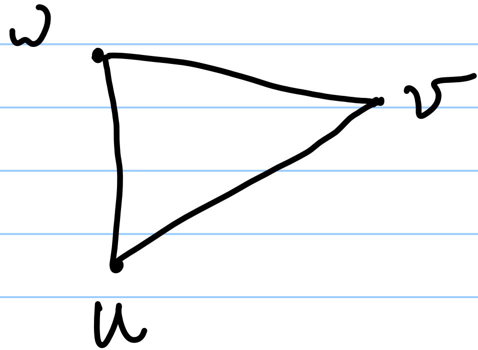
(a) $$d_H(u,v) = wt(u \oplus v)$$

(b) $$u * v = [u_1v_1 \;\; u_2v_2 \; \ldots \; u_nv_n]$$

$$wt(u+v) = d_H(u,v) = \underline{wt(u)} + \underline{wt(v)} - 2\,wt(u*v)$$

(c) $$\left.\begin{aligned} &wt(u), wt(v) : \text{even} \\ &\Downarrow \\ &wt(u+v) : \text{even} \end{aligned}\right]$$

(d) $$d_H(u,v) \le \underline{d_H(u,w)} + \underline{d_H(w,v)}$$



Same ?

(e) $$wt(u+v) \ge wt(u) - wt(v)$$

$$d_H(u,w) \qquad d_H(u,v) \qquad d_H(w,v)$$

$$v = [v_0 \; v_1 \; \dots \; v_{n-1}], \quad wt(v): \text{even}$$

$$v(x) = v_0 + v_1 x + \dots + v_{n-1} x^{n-1}$$

$$wt(v): \text{even} \iff v(1) = 0$$

$$x+1 \mid v(x)$$

4) Construct H for a $(n, 8, 4)$ code with minimum n.

① Construct H for $\underline{(n-1, 8, 3)}$

② Extend to get $(n, 8, 4)$

$$n-1+1=13$$

$$r = 4 \quad \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & & 1 \end{bmatrix} \quad 5$$

$n-1$ distinct nonzero columns

$$n-1-r=k=8$$

$\underline{\underline{n-1}} = 8 + \underline{\underline{r}}$, possible for $r=4,5,\ldots$

$(n,16,4)$ + $(n,32,4)$ with minimal n

5 (a) Find dual of $(n, 1, n)$ repetition code

$$\updownarrow$$

(b) $(n, n-1, 2)$ even-weight code

6 C : has an invertible generator matrix G

G : $n \times n$ matrix

rank $= n$

$\Rightarrow C = \{0, 1\}^n$

⑦ $(n_1, k, d_1) : G_1$

$(n_2, k, d_2) : G_2$

(a) $$G = \begin{array}{c} \qquad n_1+n_2 \\ k\begin{bmatrix} G_1 & G_2 \end{bmatrix} \end{array}$$

$n = n_1 + n_2$

$k = \text{rank}(G)$

$d \geq d_1 + d_2$

?

$mG = m[G_1 \quad G_2]$

$\underbrace{mG}_{\substack{\text{nonzero} \\ wt \geq d_1+d_2}} = [\underbrace{mG_1}_{\substack{\text{nonzero} \\ wt \geq d_1}} \quad \underbrace{mG_2}_{\substack{\text{nonzero} \\ wt \geq d_2}}]$

(b)

$$G = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix}$$

$mG = [m_1 G_1 \ m_2 G_2]$

$m = [m_1 \ m_2]$

$n = n_1 + n_2$

$\text{rank}(G) = 2k$

$$d = \min(d_1, d_2)$$

(20)

$H_1 = [I_{n-k} \ P_1]$ $(n, k, 3)$ code

$H_2 = [I_{n-k} \ P_2]$ $(n, k, 5)$ code

(a) $H = \begin{bmatrix} H_1 & H_2 \end{bmatrix}$ length $= 2n$

dim $= k$

$d = 2$

$H_1 = \begin{bmatrix} I_{n-k} & P_1 \end{bmatrix}$ $(n, k, 3)$ code

$H_2 = \begin{bmatrix} I_{n-k} & P_2 \end{bmatrix}$ $(n, k, 5)$ code

(b) $H = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}$ length $= 2n$

dim $= 2k$

$d = 3$