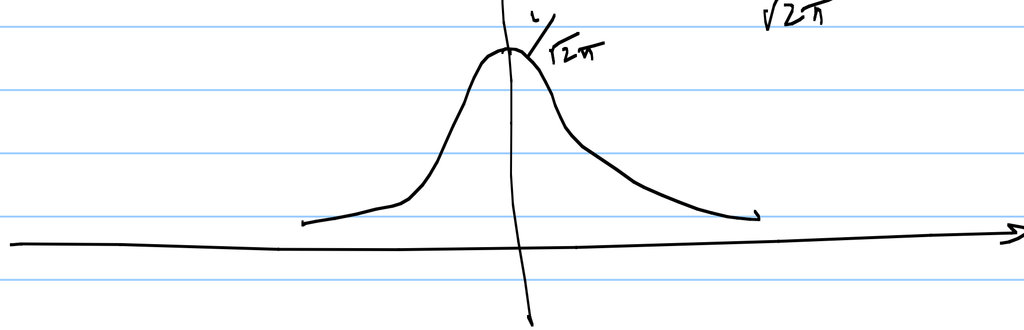
Lecture 5

Note Title 8/5/2008

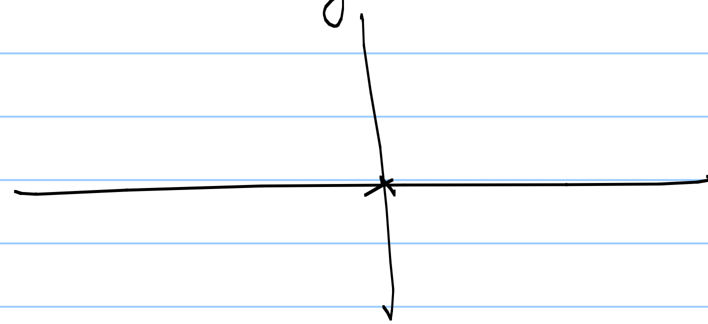
Gaussian RV: $X \sim N(0,1)$

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$



$X, Y \sim$ Jointly Gaussian iid $N(0,1)$

$f_{XY}(x,y)$



$$Y = X + N$$

$X \downarrow$ Uniform $\{-1, 1\}$

$N \downarrow N(0, \sigma^2)$

$X \sim \text{Unif}\,[a, b]$

Uniform $[-1, 1]$

Random Processes

DT

$\{X_k\}$

CT

$\{X(t)\}$

→ Finite distributions

$$X(t) = A\cos(\omega t + \phi)$$

Strict sense stationarity:

$$f_{X_{k_1}, X_{k_2}, \ldots, X_{k_N}} = f_{X_{k_1+\delta} X_{k_2+\delta} \cdots X_{k_N+\delta}}$$

$\forall \delta, k_i, N$

CT
Similar

Mean

$m_x[k] = E[X_k]$ — deterministic

$m_x(t) = E[X(t)]$

Autocorrelation

$R_{xx}(k_1, k_2) = E[X_{k_1} X_{k_2}]$

$R_{xx}(t_1, t_2) = E[x(t_1) x(t_2)]$

Wide Sense Stationarity (WSS)

$m_x[k] = m_x$ — - ..

$R_{xx}(k_1, k_2) = R_{xx}(k_1+\delta, k_2+\delta)$ — - ..

WSS: $R_{xx}(m) = E[X_{k+m} X_k]$ $\qquad$ $R_{xx}(\tau) = E[X(t+\tau)X(t)]$

deterministic

Spectrum:

$R_{xx}(m) \overset{DTFT}{\longleftrightarrow} S_X(e^{jw})$ $\qquad$ $R_{xx}(\tau) \overset{FT}{\longleftrightarrow} S_X(f)$

$\updownarrow$ ZT $\qquad$ PSD

$S_X(z)$

Cross-correlation: X, Y: two RPs

$R_{xy}(k_1, k_2) = E[X(k_1) Y(k_2)]$ $\qquad$ ----

Jointly WSS: $R_{xy}(k_1+\delta, k_2+\delta) = R_{xy}(k_1, k_2)$ $\qquad$ ---

$R_{xy}(m) = E[X(k+m) Y(k)]$ $\qquad$ ---

Filtering a WSS RP

$$X_k \longrightarrow \boxed{h[k]} \longrightarrow Y_k \qquad\qquad X(t) \longrightarrow \boxed{h(t)} \longrightarrow Y(t)$$

$$h[k] \leftrightarrow H(z), \quad h[k] \leftrightarrow H(e^{j\omega}) \qquad\qquad h(t) \leftrightarrow H(f)$$

$$m_y = m_x H(e^{j0}) \qquad\qquad m_y = m_x H(0)$$

$$R_{yy}(m) = R_{xx}(m) * h[m] * h^*[-m] \qquad\qquad R_{yy}(\tau) = R_{xx}(\tau) * h(\tau) * h^*(-\tau)$$

$$S_y(e^{j\omega}) = S_x(e^{j\omega}) \left| H(e^{j\omega}) \right|^2 \qquad\qquad S_y(f) = S_x(f) \left| H(f) \right|^2$$

$$S_y(z) = S_x(z)\, H(z)\, H^*(1/z^*)$$

PSD: real & non-negative.

Gaussian RP:

→ finite distributions are jointly Normal

	DT	CT
WSS:	m_x, $R_{xx}(m)$	m_x, $R_{xx}(\tau)$
White:	$m_x = 0$	$m_x = 0$
	$R_{xx}(m) = \frac{N_0}{2}\delta[m]$	$R_{xx}(\tau) = \frac{N_0}{2}\delta(\tau)$
	$S_x(e^{j\omega}) = \frac{N_0}{2} \quad \forall \omega$	$S_x(f) = \frac{N_0}{2} \quad \forall f$

→ Noise usually modeled as a white Gaussian process.

Sampling a CT RP

$$\{X(t)\} \xrightarrow{T\text{ secs}} \{Y_k\} \qquad Y_k = X(kT)$$

CT

$m_X, R_{XX}(\tau)$

$\Updownarrow$

$S_X(f)$

DT

$m_Y = m_X$

$R_{YY}(m) = R_{XX}(mT)$

$\Updownarrow \quad \omega = 2\pi f T$

$$S_Y(e^{j\omega}) = \frac{1}{T} \sum_{m=-\infty}^{\infty} S_X\left(f - \frac{m}{T}\right)$$

PAM RP:

$$Y(t) = \sum_{k=-\infty}^{\infty} X_k\, h(t-kT)$$

$\{X_k\}$: DT RP $\quad m_X, R_{XX}(m) \Longleftrightarrow S_X(e^{j\omega})$

T: symbol time, $h(t)$: deterministic

$$Z(t) = Y(t-\theta), \quad \theta \sim \text{Unif}[0, T]$$

$$Z(t) = \sum_{k=-\infty}^{\infty} X_k \, h(t-kT-\theta)$$

$$\downarrow$$

WSS

$$S_Z(f) = \frac{1}{T} |H(f)|^2 \underbrace{S_X\left(e^{j2\pi f T}\right)}$$

Summary:

$$y(t) = x(t) * h(t) + n(t)$$

$$x(t) = Tx(\underline{b}) \qquad \hat{\underline{b}} = Rx(y(t))$$

$x(t)$: Power, BW $\qquad$ $n(t)$: Power

Bit rate & Prob. of error

$\longrightarrow$ Signals/systems, DSP, PRP

Barry, Lee, Messers... : 1 – 3 Ch.

Proakis (5^{th} ed): 1 – 2 Ch.