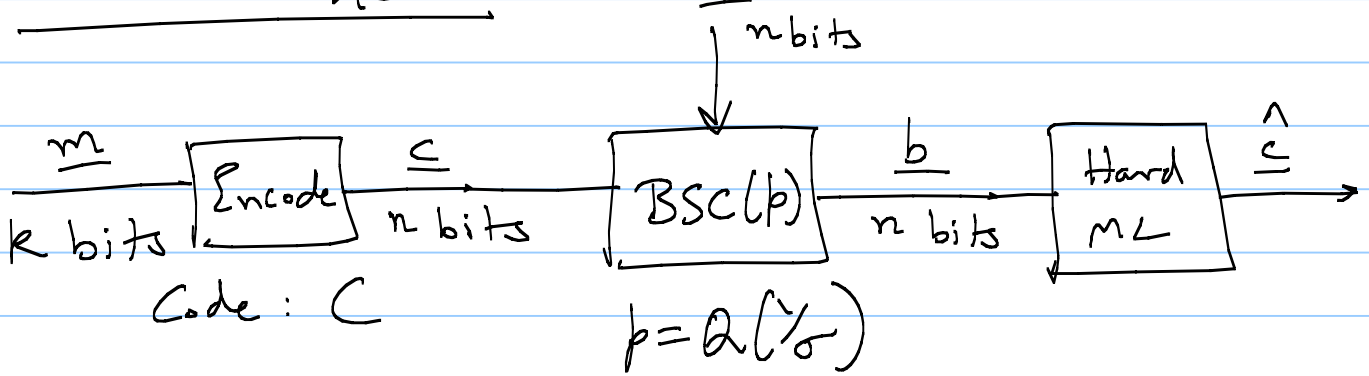
Lecture 45

Note Title 11/6/2008

Hard ML decoder



$$\hat{c} = \arg\min_{c \in C} d_H(b, c)$$

$\hat{c} = b + \hat{e}$ → Try to find the most likely error

$$\hat{e} \in \{v \in \{0,1\}^n : b + v \in C\}$$

$$\hat{e} = \arg\min_{v} w_H(v)$$

Syndrome decoder for linear codes

Ex:

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

PC matrix of a (6, 3, 3)

$$\hat{\underline{c}} = \underline{b} + \hat{\underline{e}}$$

$$H\hat{\underline{c}}^T = \underline{0} = H\underline{b}^T + H\hat{\underline{e}}^T \Rightarrow$$

- Computed at the Rx: $H\underline{b}^T$
- Called syndrome $\underline{s} = H\hat{\underline{e}}^T$: $H\underline{b}^T = H\hat{\underline{e}}^T$

$\underline{s}$ is 3×1

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

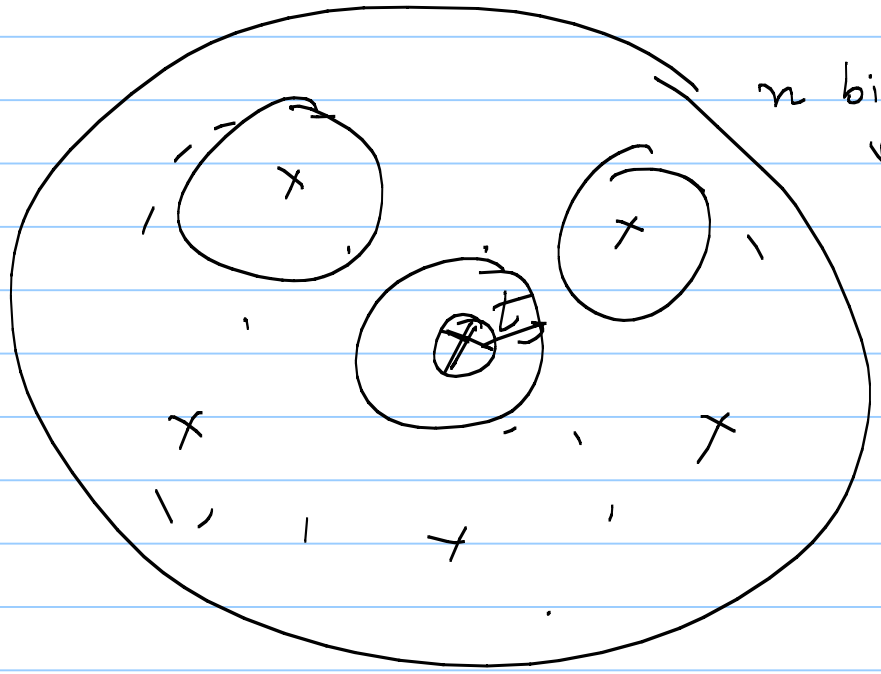
$$\underline{b} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$\underline{s} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \\ \hat{e}_5 \\ \hat{e}_6 \end{bmatrix}$$

$$\underline{\hat{e}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{\hat{c}} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

How many errors can be corrected?

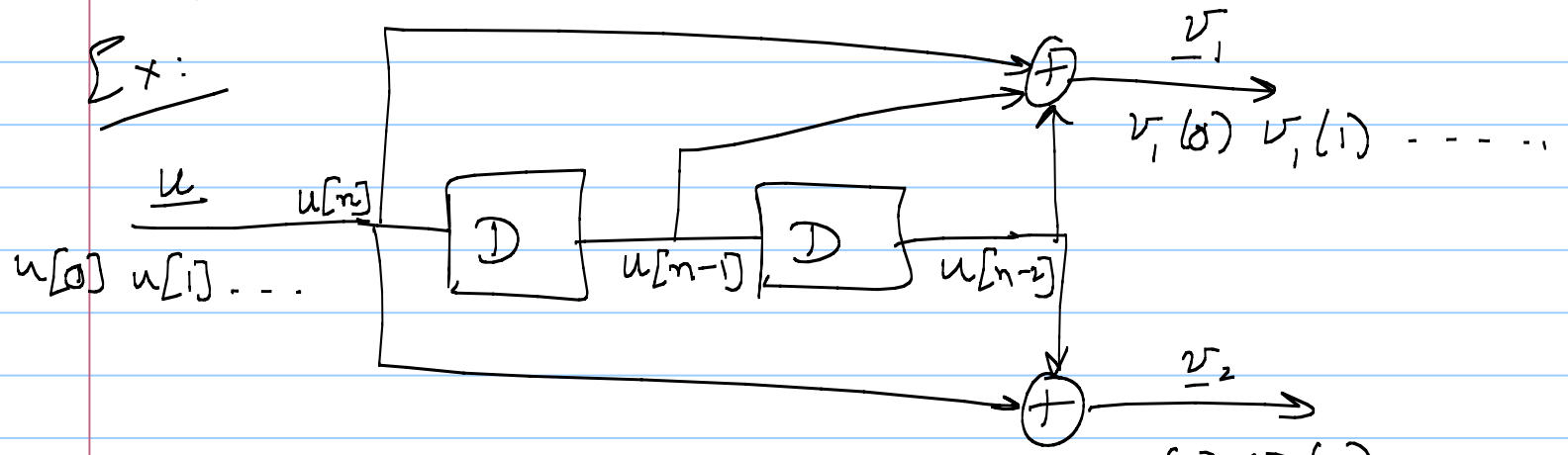


$t = \left\lfloor \frac{d-1}{2} \right\rfloor$ is correctable.

$$P_e \approx c p^{t+1} = c\, Q\left(\frac{1}{\sigma}\right)^{t+1}$$

Convolutional Codes:

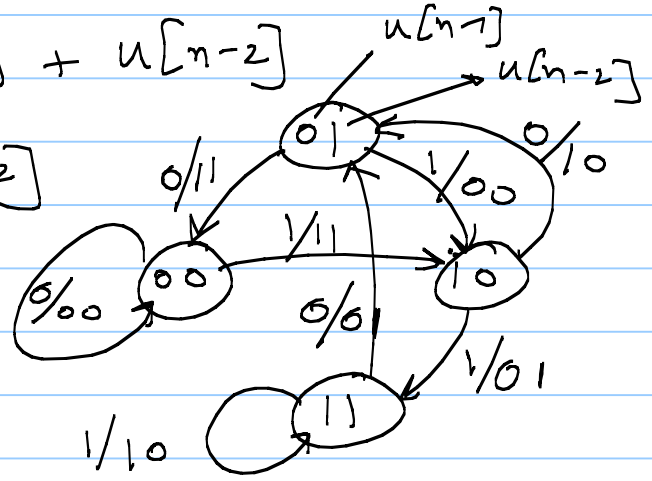
Ex:



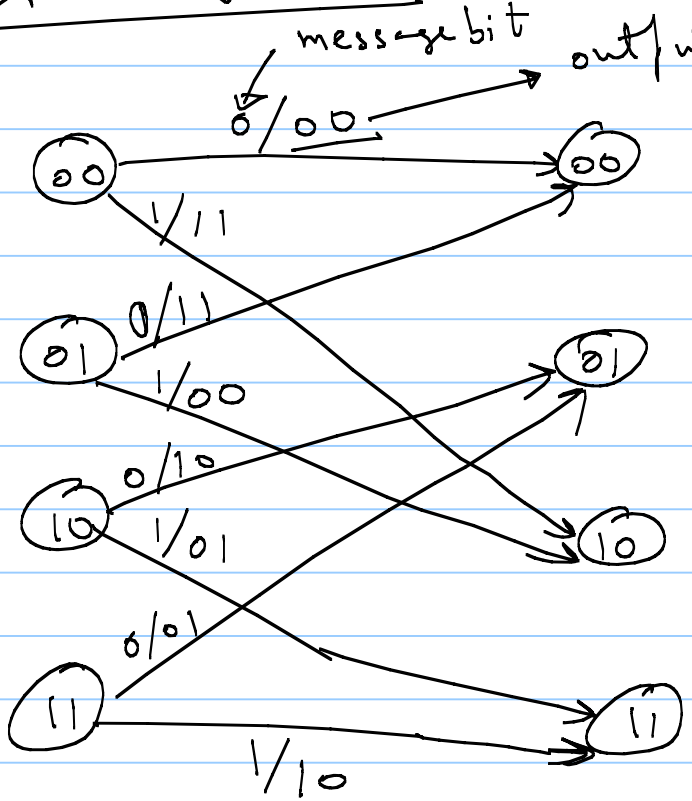
rate-1/2 encoder

$$v_1[n] = u[n] + u[n-1] + u[n-2]$$

$$v_2[n] = u[n] + u[n-2]$$



State diagram to Trellis



message

$\underline{u} = u[0] \cdots u[k-1]$

$\underline{v} = v_1[0]\, v_2[0] \cdots v_1[k-1]\, v_2[k-1]$ → $\underline{r}$

$\hat{\underline{v}} = \arg \min_{\text{paths}} |\underline{r} - \underline{v}|^2$

Soft ML decoding: implementable as a Viterbi algorithm.