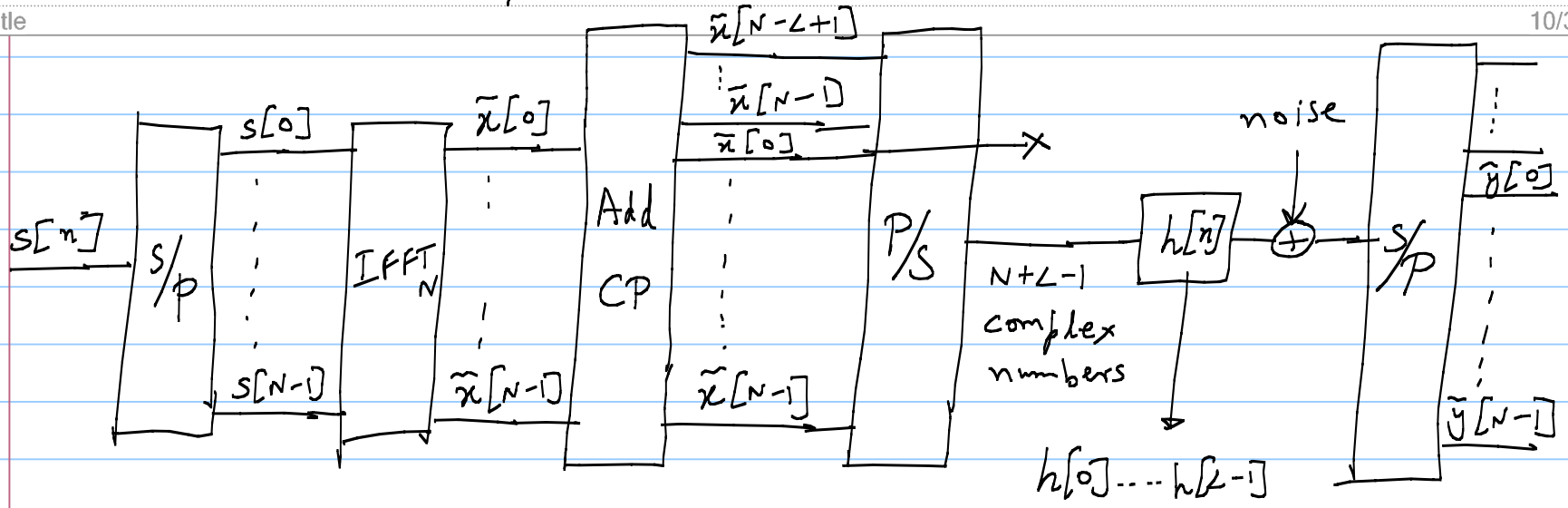
Lecture 41

Note Title 10/30/2008



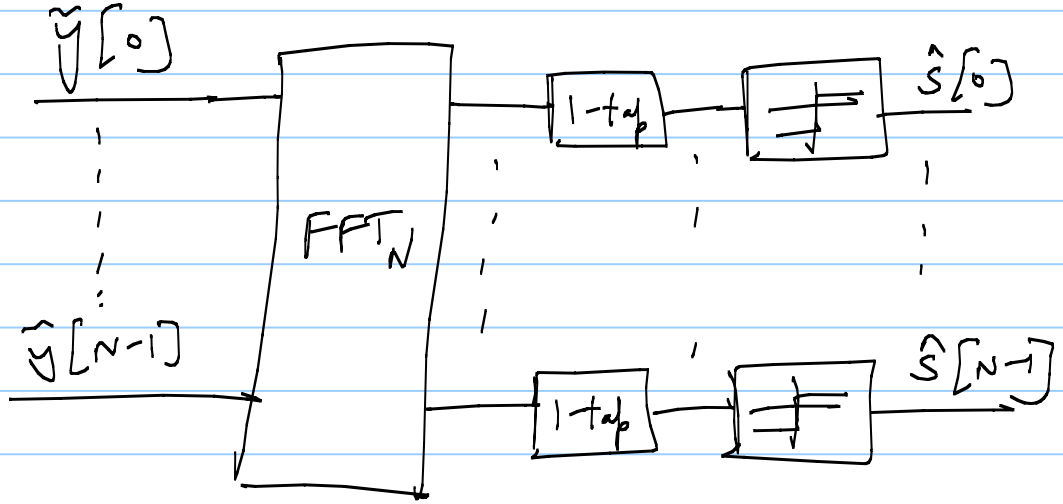
$$\begin{bmatrix} \tilde{y}[0] \\ \tilde{y}[1] \\ \vdots \\ \tilde{y}[N-1] \end{bmatrix} = \begin{bmatrix} h[0] & 0 & \cdots & 0 & h[L-1] & \cdots & h[1] \\ h[1] & h[0] & 0 & \cdots & 0 & h[L-1] & \cdots & h[2] \\ \vdots & & \ddots & & & & \ddots & \vdots \\ h[L-1] & & & & & & & h[L-1] \\ 0 & & & & & & & 0 \\ \vdots & & & & & & & \vdots \\ 0 & \cdots & \cdots & h[L-1] & \cdots & \cdots & h[0] \end{bmatrix} \begin{bmatrix} \tilde{x}[0] \\ \tilde{x}[1] \\ \vdots \\ \tilde{x}[N-1] \end{bmatrix}$$

$$\text{Circulant matrix} = FFT_N^{-1} \begin{bmatrix} H[0] & & & \\ & H[1] & & 0 \\ & & \ddots & \\ & 0 & & H[N-1] \end{bmatrix} FFT_N$$

$$H[k] = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} h[n]\, e^{-j\frac{2\pi nk}{N}}$$

$$FFT_N \begin{bmatrix} \tilde{y}[0] \\ \vdots \\ \tilde{y}[N-1] \end{bmatrix} = \begin{bmatrix} H[0] & & & \\ & H[1] & & 0 \\ & & \ddots & \\ & 0 & & H[N-1] \end{bmatrix} \begin{bmatrix} S[0] \\ \vdots \\ S[N-1] \end{bmatrix}$$

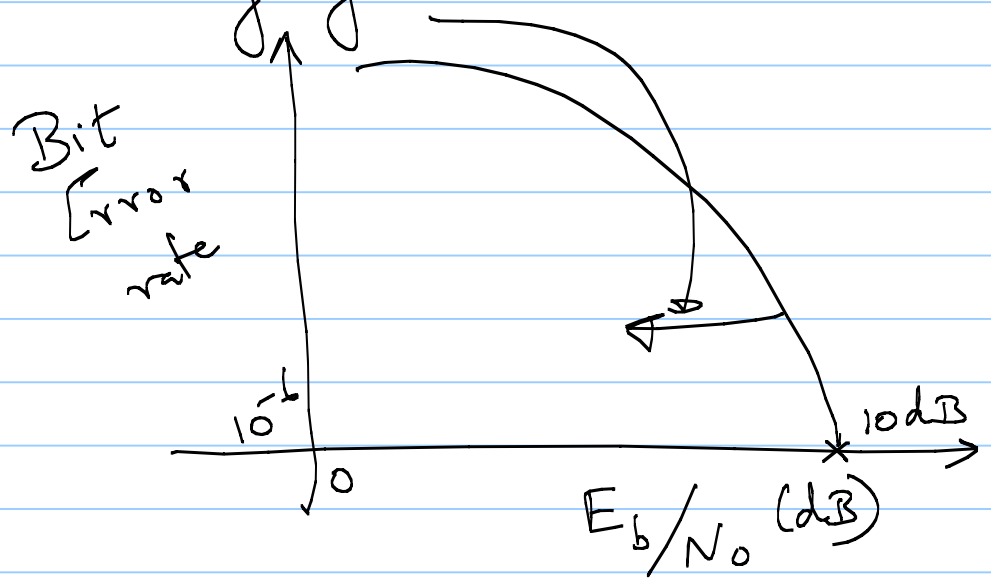
At the receiver,



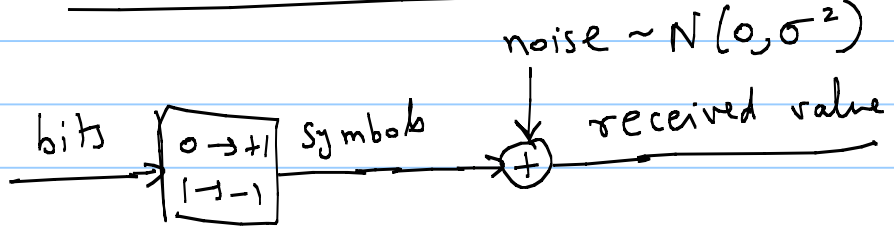
→ Frequency domain equalization

Error Control Codes

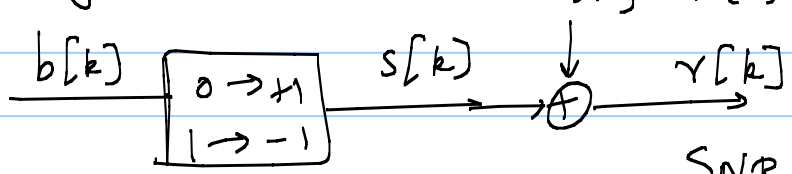
→ Coding gain.



BPSK over AWGN : (no ISI)



Coding: add controlled redundancy



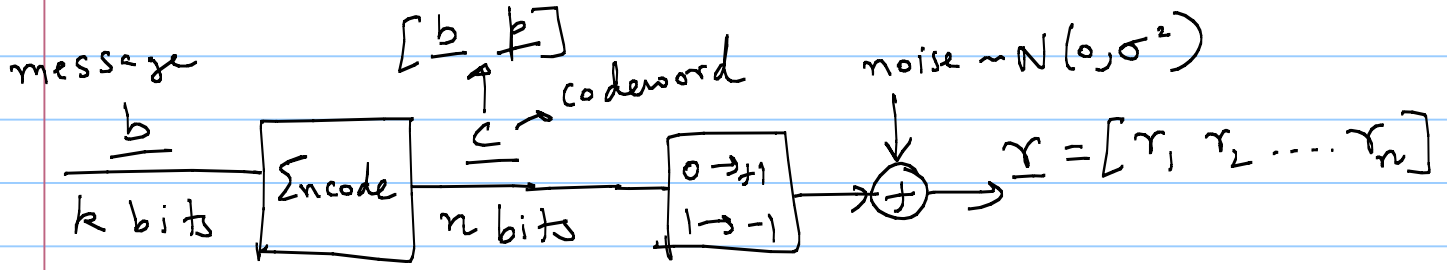
$E_s = E_b = 1$

$SNR = \frac{1}{\sigma^2}$

(No coding)

$\frac{E_b}{N_0} = \frac{1}{2\sigma^2}$

$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$



$R = \text{Rate} = k/n \leq 1$

$E_b = \frac{n}{k}$

$\underline{b} \in \{0,1\}^k$

$\underline{c} \in \{0,1\}^n$

Code = Set of all codewords $\subseteq \{0,1\}^n$

$\frac{E_b}{N_0} = \frac{n}{k \cdot 2\sigma^2} = \frac{1}{2R\sigma^2}$