# Lecture 4

Note Title 8/4/2008

Discrete-time signals: $x[n]$ $\quad -\infty < n < -\infty$

$\downarrow$

sequence of complex numbers

Finite-energy sequences: $\displaystyle\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$

Convolution:

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-jn\omega}$$

ZT

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{jn\omega}\,d\omega$$

LTI system (linear filter) : $x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$

$$\updownarrow$$

$H(e^{j\omega})$, $H(z)$

$H(e^{j\omega}) \rightarrow$ freq. resp.

$H(z) \rightarrow$ transfer function

→ <u>ROC</u>

$h[n]$ : right-sided sequence

$h[n] = 0$ for $n \leq M$

→ right-sided and stable filters

$$\sum |h[n]|^2 < \infty$$

$\boxed{\text{monic \& causal}}$

$h[0] = 1$

→ minimum-phase : poles & zeros are inside unit-circle.

Folded spectrum:

$$q(t) \overset{FT}{\longleftrightarrow} Q(f)$$

$$q[n] = q(nT) \qquad f_s = 1/T$$

$$q[n] \overset{DTFT}{\longleftrightarrow} \tilde{Q}(e^{j\omega}) = \frac{1}{T} \sum_{m=-\infty}^{\infty} Q\left(f - \frac{m}{T}\right)$$

$$\omega = 2\pi f T = 2\pi \, f/f_s$$

No-aliasing condition:

$$f_s = \frac{1}{T} \geq 2\, BW(q(t))$$

# Random Signals

## Probability & Random Variables:

- → Random variable, CDF, PDF (PMF)
- → Mean, Expectation, Variance
- → Joint PDF, Vectors of RVs
- → Conditional probability:
  - → discrete & continuous
- → Conditional pdf  $X$ & $Y$: RVs

$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

Gaussian RV
(Normal)

$$X \sim N(\mu, \sigma^2)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$X \sim N(0,1) \qquad f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$$E[X] = \mu \qquad E[X^2] = \mu^2 + \sigma^2$$

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\,dt = \Pr(X \geq x)$$

$$Q(x) \leq \begin{matrix} \frac{1}{2} e^{-x^2/2} \\ \frac{1}{\sqrt{2\pi}\,x} e^{-x^2/2} \end{matrix}$$

$X, Y \sim$ Jointly Gaussian

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} e^{\frac{-(x^2 - 2\rho xy + y^2)}{2\sigma^2(1-\rho^2)}}$$

$$X \sim N(0, \sigma^2) \qquad Y \sim N(0, \sigma^2)$$

$$\rho = \frac{E[XY]}{\sigma^2}$$

$$\underline{X} = [X_1 \;\; X_2 \; \dots \; X_N]^T \qquad X_i \sim N(0, 1), \; (\underline{iid})$$

$$f_{\underline{X}}(\underline{x}) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{\frac{-\|\underline{x}\|^2}{2\sigma^2}}$$

General case: $\underline{m} = [E[X_1] \ldots E[X_N]]^T$

$$\underline{\underline{C}} = E[(\underline{X} - \underline{m})(\underline{X} - \underline{m})^T]$$

$$[\underline{\underline{C}}]_{ij} = E[(X_i - m_i)(X_j - m_j)]$$

$$f_{\underline{X}}(\underline{x}) = \frac{1}{(2\pi)^{N/2} |\underline{\underline{C}}|^{1/2}} e^{-\frac{1}{2}(\underline{x} - \underline{m})^T \underline{\underline{C}}^{-1} (\underline{x} - \underline{m})}$$

$\longrightarrow$ Linear combinations of jointly Gaussian RVs are jointly Gaussian

# Random Processes:

**DT**

$\{X_k\}$ : sequence of RVs

**CT**

$\{X(t)\}$ :

$X(t)$ : RV for each $t$

## Finite distributions:

joint pdf/pmf of a finite collection of RVs

$X_{k_1}, X_{k_2}, \ldots, X_{k_N}$

$\forall$ $N$ and $k_i$

$X(t_1), X(t_2), \ldots, X(t_N)$

$\forall$ $N$, $t_i$

Ex: $\sum A_k \cos(2\pi f_k t + \theta)$