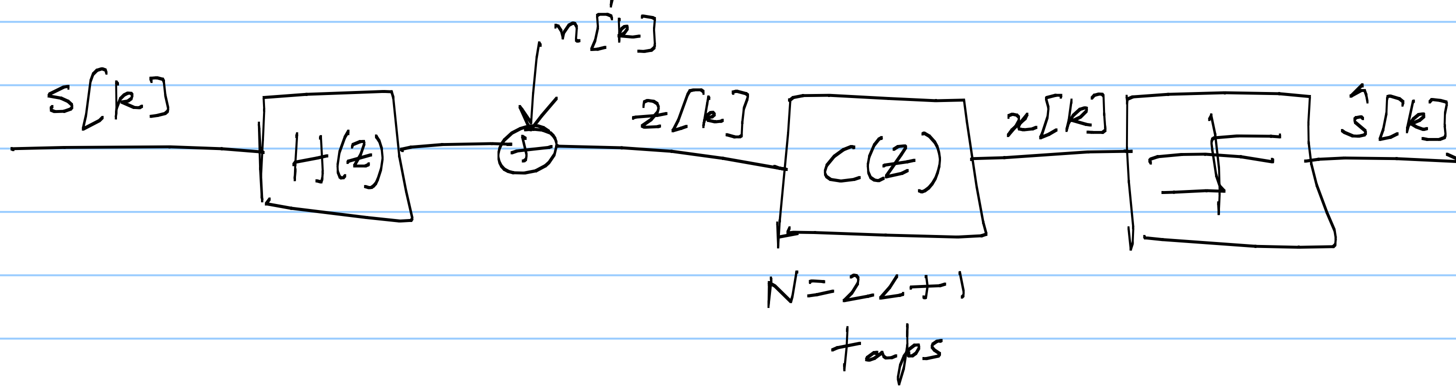
Lecture 33

→ Constrained Complexity equalizers



$\underline{c}$: filter

$\underline{z}_k$: ?

$$x[k] = \underline{c}^T \underline{z}_k$$

$$e[k] = s[k] - \underline{c}^T \underline{z}_k$$

$$MSE = E\left[|e[k]|^2\right] = E\left[|s[k]|^2\right] - 2\,\mathrm{Re}\left\{\underline{c}^{*T}\underline{\alpha}\right\} + \underline{c}^{*T}\Phi\,\underline{c}$$

$$\boxed{\Phi\, \underline{c}_{opt} = \underline{\alpha}}$$

Update:
$$\underline{c}_{j+1} = \underline{c}_j - \frac{\beta}{2} \nabla_{\underline{c}_j} \underbrace{E\left[|e[k]|^2\right]}_{MSE_j}$$

j-th iteration

$$= \underline{c}_j + \beta(\underline{\alpha} - \phi \underline{c}_j)$$

$$\underline{q}_j = \underline{c}_j - \underline{c}_{opt}$$

$$\underline{q}_{j+1} = \sum_{i=1}^{n} (1-\beta\lambda_i)^j \underbrace{\underline{v}_i}_{\text{orthonormal eigenvectors of } \phi.} \underline{v}_i^{*T} \underline{q}_0$$

$$\boxed{0 \le \beta < \frac{2}{\lambda_{max}}}$$

$$(\lambda_{min} = \lambda_1 < \cdots < \lambda_n = \lambda_{max})$$

$$\left.\begin{array}{l}\text{Speed of}\\ \text{Convergence}\end{array}\right\} \Longleftrightarrow \left\{\begin{array}{l}\text{spread of}\\ \text{eigenvalues}\end{array}\right.$$

$$\min_{\theta} S_z(e^{j\theta}) \le \lambda_i \le \max_{\theta} S_z(e^{j\theta})$$

$$n \to \infty \qquad \lambda_{min} \to \min_{\theta} S_z(e^{j\theta})$$

$$\lambda_{max} \to \max_{\theta} S_z(e^{j\theta})$$

Stochastic Gradient or LMS:

$$\underline{c}_{j+1} = \underline{c}_j - \frac{\beta}{2} \nabla_{\underline{c}_j} E\left[|e[k]|^2 \right]$$

$$\updownarrow$$

$$\underline{c}_{k+1} = \underline{c}_k - \frac{\beta}{2} \nabla_{\underline{c}_k} |e[k]|^2$$

$$|e[k]|^2 = |s[k] - \underline{c}^T \underline{z}_k|^2$$

$$= |s[k]|^2 - 2\,\mathrm{Re}\left\{\underline{c}^{*T} \underline{z}_k^{*} s[k]\right\} + \underline{c}^{*T} \underline{z}_k^{*} \underline{z}_k^{T} \underline{c}$$

$$\nabla_{\underline{c}} |e[k]|^2 = -2\, e[k]\, \underline{z}_k^{*}$$

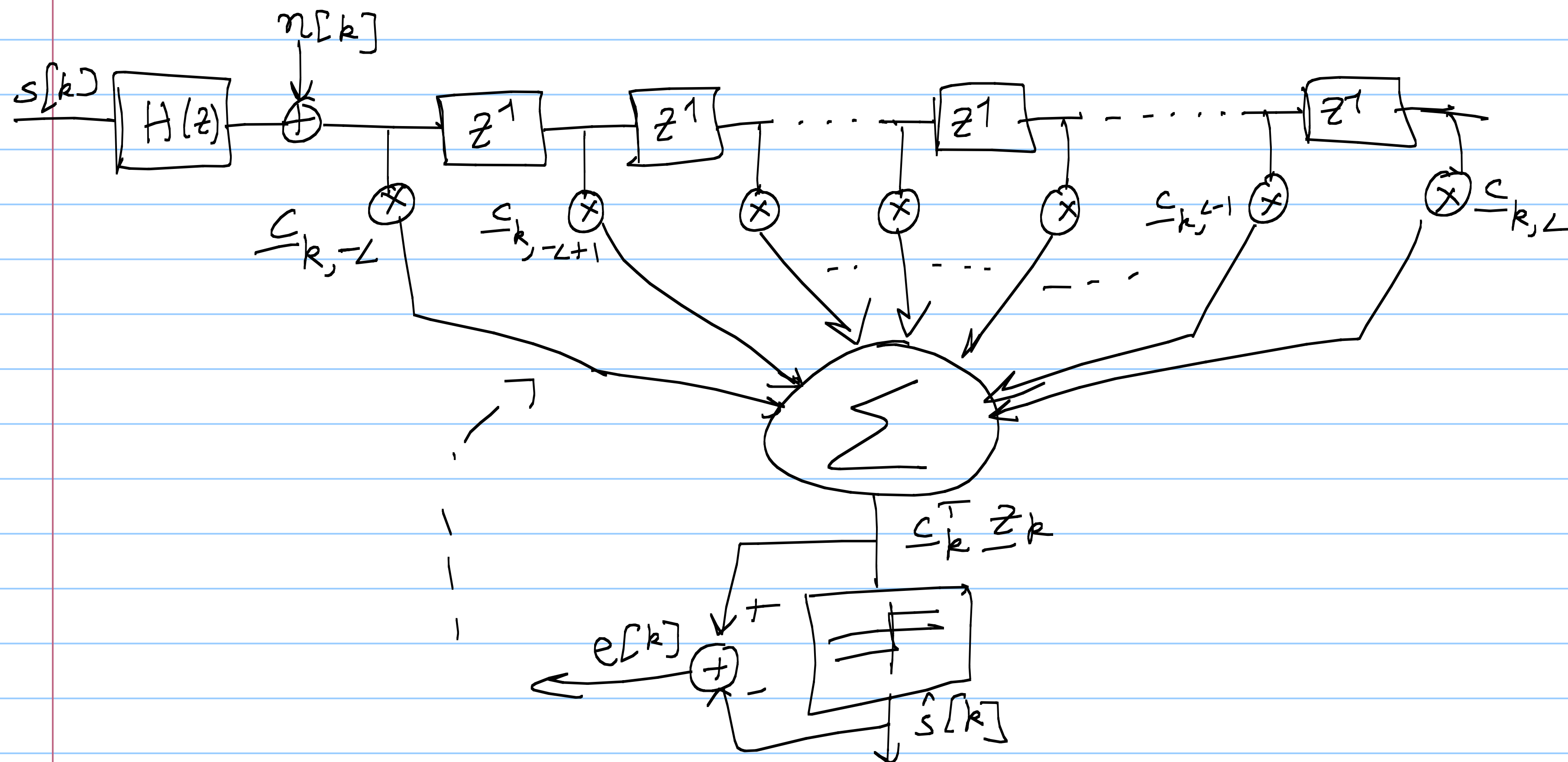
$$\boxed{\underline{c}_{k+1} = \underline{c}_k + \beta\, e[k]\, \underline{z}_k^{*}}$$

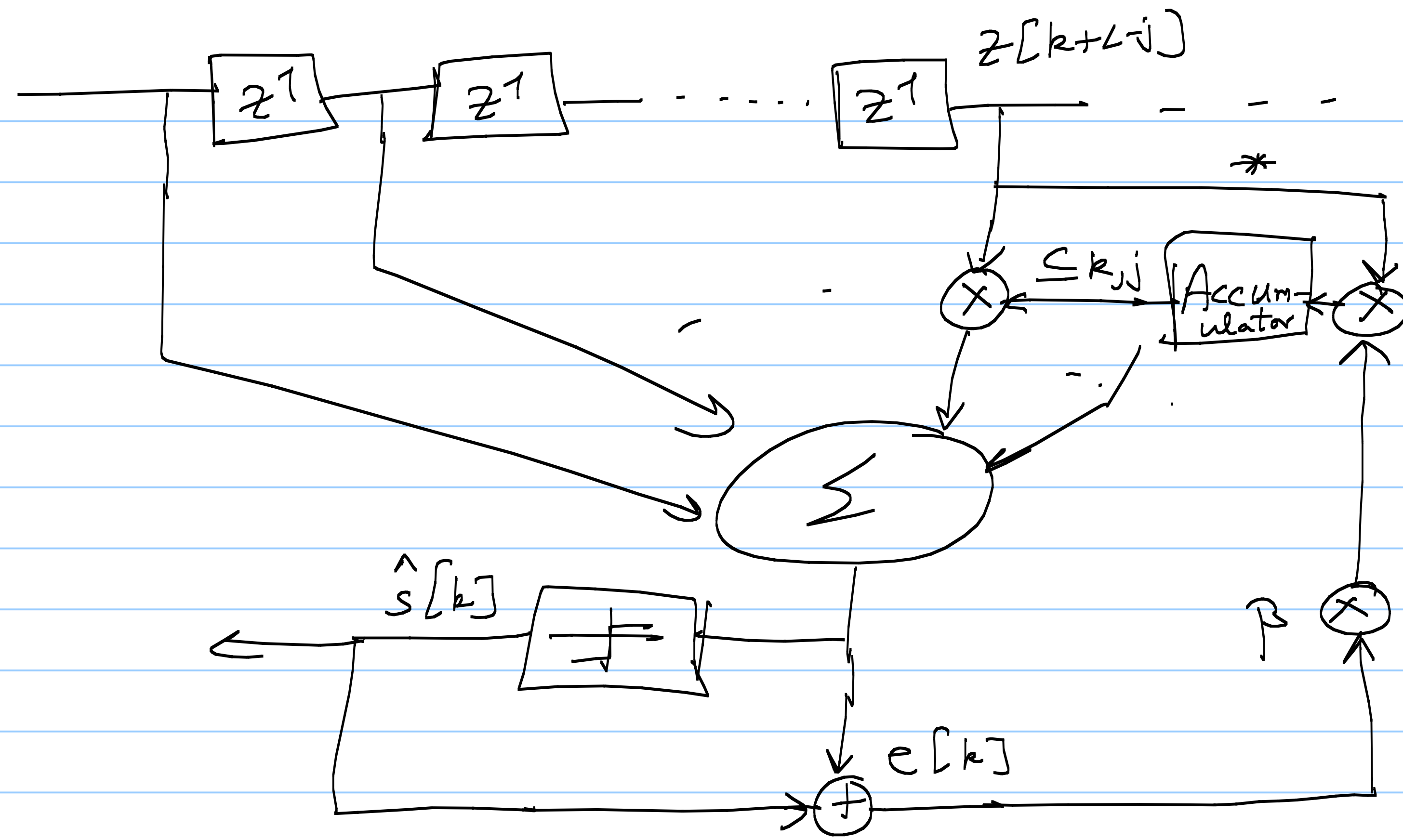
$$\begin{array}{c} 0 \rightarrow \\ \vdots \\ \\ \vdots \\ 2L \rightarrow \end{array} \begin{bmatrix} \underline{c}_{k+1,-L} \\ \vdots \\ \underline{c}_{k+1,0} \\ \vdots \\ \underline{c}_{k+1,L} \end{bmatrix} = \begin{bmatrix} \underline{c}_{k,-L} \\ \vdots \\ \underline{c}_{k,0} \\ \vdots \\ \underline{c}_{k,L} \end{bmatrix} + \beta e[k] \begin{bmatrix} z^{*}[k+L] \\ \vdots \\ z^{*}[k] \\ \vdots \\ z^{*}[k-L] \end{bmatrix}$$

j-th element of $\underline{c}_{k+1}$:

$$\underline{c}_{k+1,j} = \underline{c}_{k,j} + \beta\, e[k]\, \underline{z}^*[k+L-j]$$

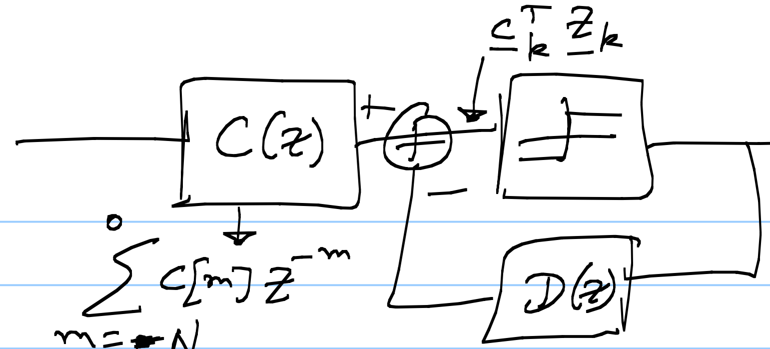
?





z^{-1}
z^{-1}
z^{-1}
$z[k+L-j]$
*
$c_{k,j}$
Accumulator
Σ
$\hat{s}[k]$
β
$e[k]$

Adaptive DFE:



$$C(z) = \sum_{m=-N}^{0} c[m] z^{-m} \qquad D(z) = \sum_{m=1}^{K} d[m] z^{-m}$$

→ Define $\underline{z}_k$ & $\underline{c}_k$ so that

$$e[k] = s[k] - \underline{c}_k^T \underline{z}_k$$

$$\underline{c}_{k+1} = \underline{c}_k + \beta\, e[k]\, \underline{z}^*_k$$

$$\underline{c}_k = \begin{bmatrix} c[-N] \\ \vdots \\ c[0] \\ d[1] \\ \vdots \\ d[K] \end{bmatrix} \qquad \underline{z}_k = \begin{bmatrix} z[k+N] \\ \vdots \\ z[k] \\ \hat{s}[k-1] \\ \vdots \\ \hat{s}[k-K] \end{bmatrix}$$