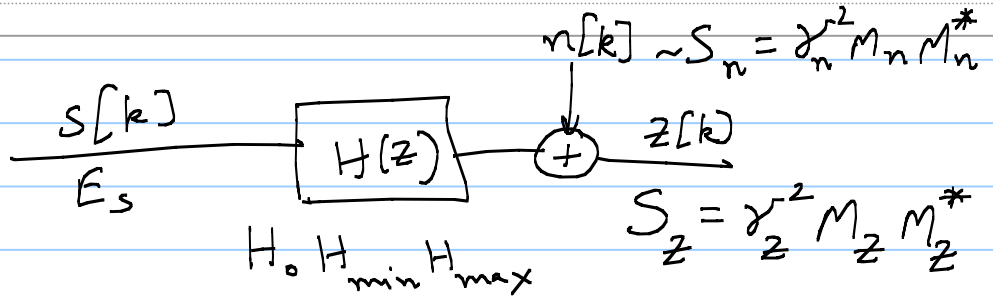
Lecture 30

Note Title 9/25/2008

Summary:



	LE		DFE		
	D	MSE	DW	W−1	MSE
ZF	$\frac{1}{H}$	$\left\langle \frac{S_n}{\lvert H\rvert^2} \right\rangle_A$	$\frac{1}{H_0}\frac{H_{max}^*}{H_{max}} M_n^{-1}$	$\frac{H_{min} H_{max}^*}{M_n} - 1$	$\left\langle \frac{S_n}{\lvert H\rvert^2} \right\rangle_G$
MMSE	$\frac{E_s H^*}{S_z}$	$\left\langle \frac{S_n}{\lvert H\rvert^2 + \frac{S_n}{E_s}} \right\rangle_A$	$\frac{E_s}{\gamma_z^2}\frac{H^*}{M_z^*} M_n^{-1}$	$M_z M_n^{-1} - 1$	$\left\langle \frac{S_n}{\lvert H\rvert^2 + \frac{S_n}{E_s}} \right\rangle_G$

Example: $H(z) = 1 + c z^{-1}$, $S_n = N_0$, $\mathcal{X} = \{\pm 1\}$

(c: complex)

ZF-LE: $D = \dfrac{1}{1 + c z^{-1}}$ $\quad MSE = \left\langle \dfrac{N_0}{(1 + c z^{-1})(1 + c^* z)} \right\rangle_A$

$|c| < 1$: ✓ $\quad = \dfrac{N_0}{|1 - |c|^2|}$ (?)

$|c| > 1$: non-realizable

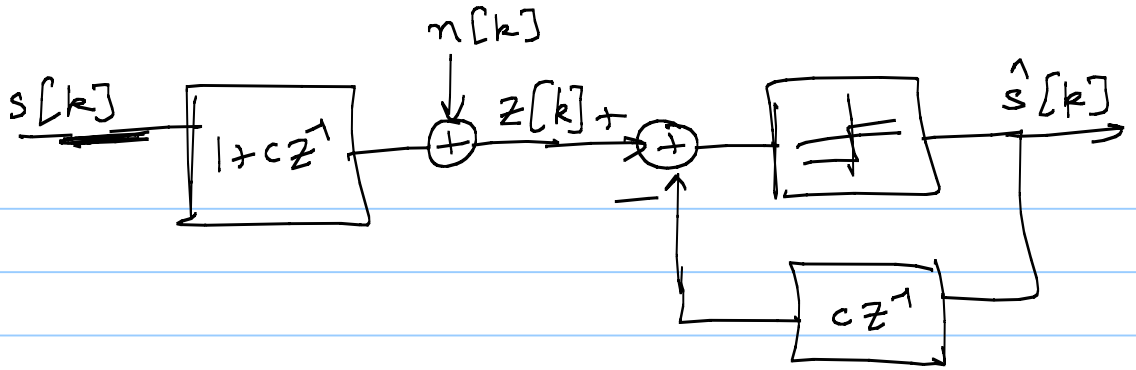
ZF-DFE:

(i) $|c| < 1$ $\quad DW = 1$

$W - 1 = c z^{-1}$

$MSE = \left\langle N_0 \cdot \dfrac{1}{1 + c z^{-1}} \cdot \dfrac{1}{1 + c^* z} \right\rangle_G$

$= N_0$



$MSE = N_0$

(ii) $|c| > 1$

$$H(z) = 1 + c z^{-1} = \underbrace{(z^{-1})}_{H_0} \times \underbrace{\left(1 + \frac{1}{c} z\right)}_{H_{max}}$$

$$DW = \frac{1}{c} \cdot \frac{1 + \frac{1}{c^*} z^{-1}}{1 + \frac{1}{c} z}$$

"IIR anti-causal"

$$w - 1 = \frac{1}{c} z^{-1}$$

$$MSE = \left\langle \frac{N_0}{|c|^2 \; H_{max} \cdot H_{max}^*} \right\rangle_a = \frac{N_0}{|c|^2}$$

Remarks: - $|c|>1$ $\Rightarrow$ more complicated Precursor for lower MSE.

- $z[k] = s[k] + c\,s[k-1] + n[k]$

- C : Crossing U.C.

MMSE-LE:

$$D = \frac{1 + c^* z}{(1+c^* z)(1+c z^{-1}) + N_0}$$

$$S_z = (1+c^* z)(1+c z^{-1}) + N_0 = \frac{1+|c|^2+N_0}{1+|\alpha|^2}(1-\alpha z^{-1})(1-\alpha^* z)$$

$|\alpha| < 1$

$$MSE = \left\langle \frac{N_0}{S_z} \right\rangle_A = \frac{N_0}{1+|c|^2+N_0}\left(\frac{1+|\alpha|^2}{1-|\alpha|^2}\right) ?$$

MMSE-DFE:

$$DW = \frac{1}{\gamma_z^2} \cdot \frac{H^*}{M_z^*}$$

$$= \frac{1+|\alpha|^2}{1+|c|^2+N_0} \cdot \frac{1+c^* z}{1-\alpha^* z}$$ ← 2

$$W-1 = -\alpha z^{-1}$$

$$MSE = \frac{N_0 (1+|\alpha|^2)}{1+|c|^2+N_0}$$

→ Constrained complexity equalizers.

→ Training receiver + adaptive equalizers → Fractionally-spaced equalizers.

→ More practical receiver structures.