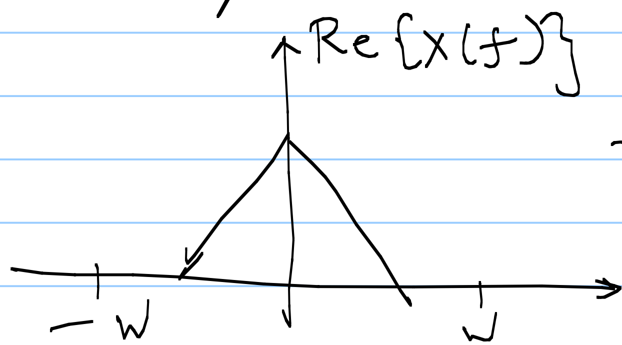
Lecture 3

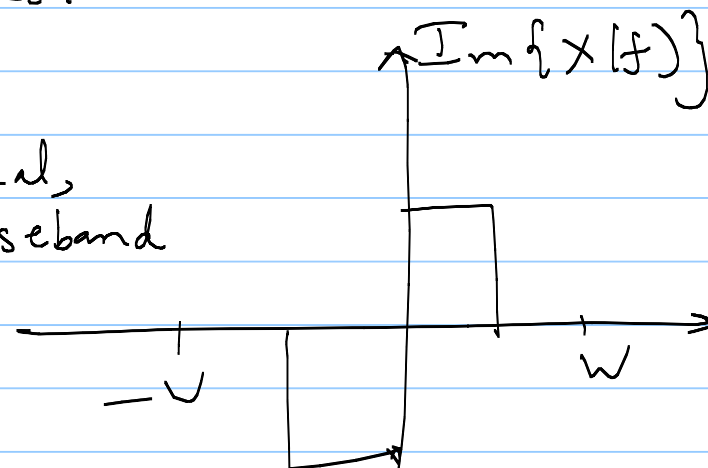
Note Title 7/31/2008

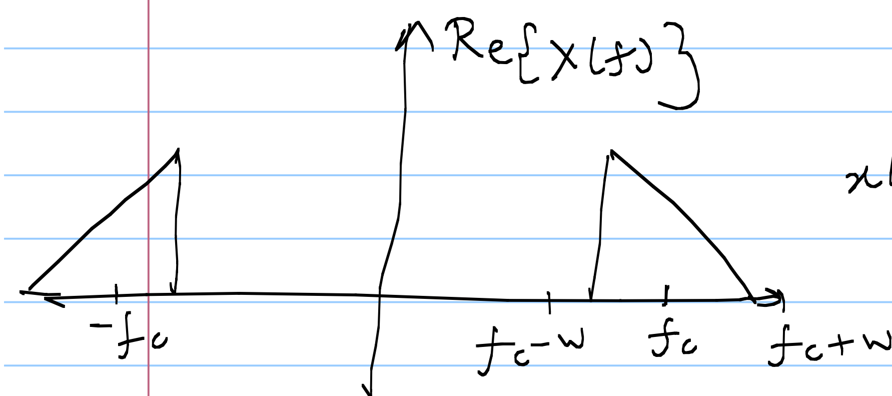
$$\|v\|_2 = +\sqrt{\langle v, v\rangle}$$

Baseband/Passband signals:

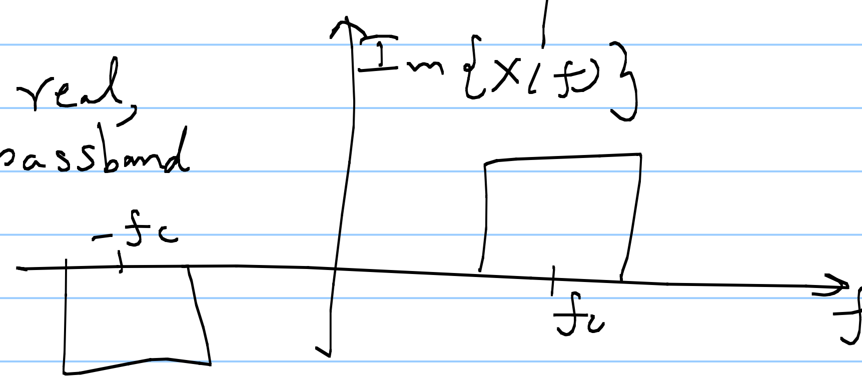


$x(t)$: real, baseband





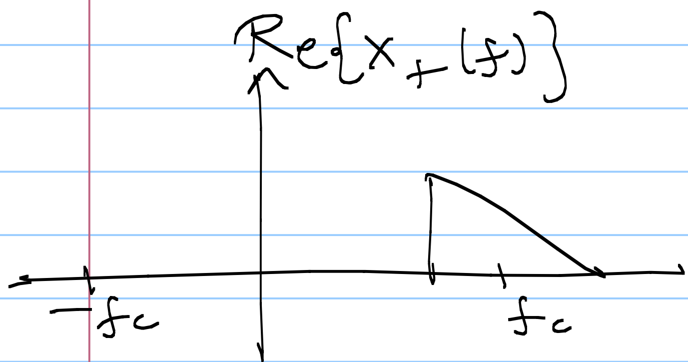
$x(t)$: real, passband

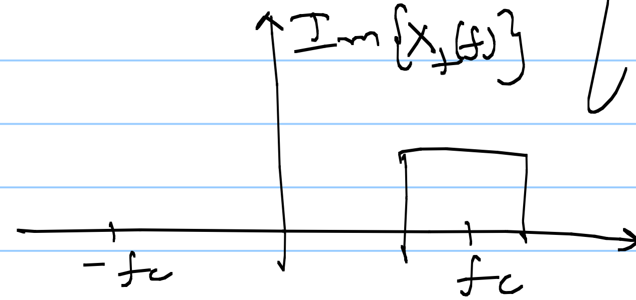


$x(t)$: real, passband, centre frequency f_c

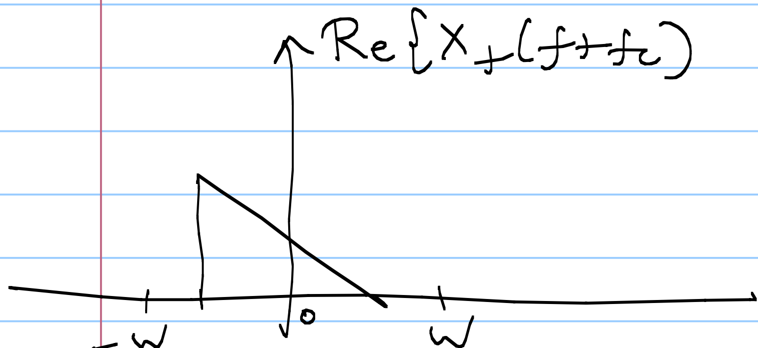
$$X(f) = X^*(-f)$$

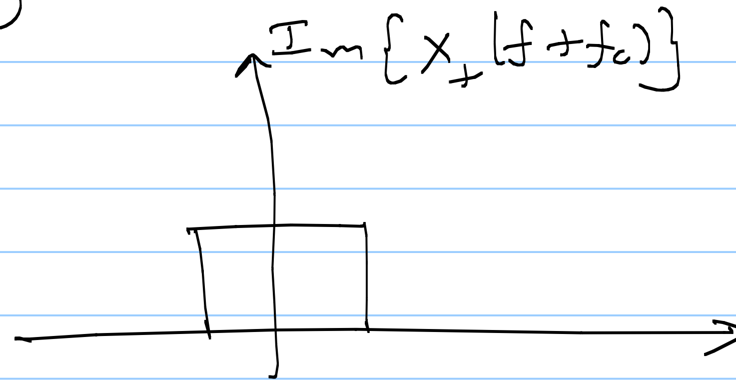
$$X_+(f) = \begin{cases} X(f), & f > 0 \\ 0, & f \le 0 \end{cases}$$





$$X_+(f) = X(f)\,u(f)$$





$$\|X_+(f)\|_2 = \frac{1}{\sqrt{2}}\|X(f)\|_2$$

Complex envelope of $x(t)$

$$\tilde{x}(t) \xleftrightarrow{FT} \sqrt{2}\, X_+(f+f_c)$$

$$= \sqrt{2}\, X(f+f_c)\, u(f+f_c)$$

Going back:

$$x(t) = \sqrt{2}\, \mathrm{Re}\left\{ \tilde{x}(t)\, e^{j2\pi f_c t} \right\}$$

Use: $x(t) \longleftrightarrow X(f)$

$$\mathrm{Re}\{x(t)\} \longleftrightarrow \frac{X(f)+X^*(-f)}{2}$$

$$= \frac{x(t)+x^*(t)}{2}$$

Use:

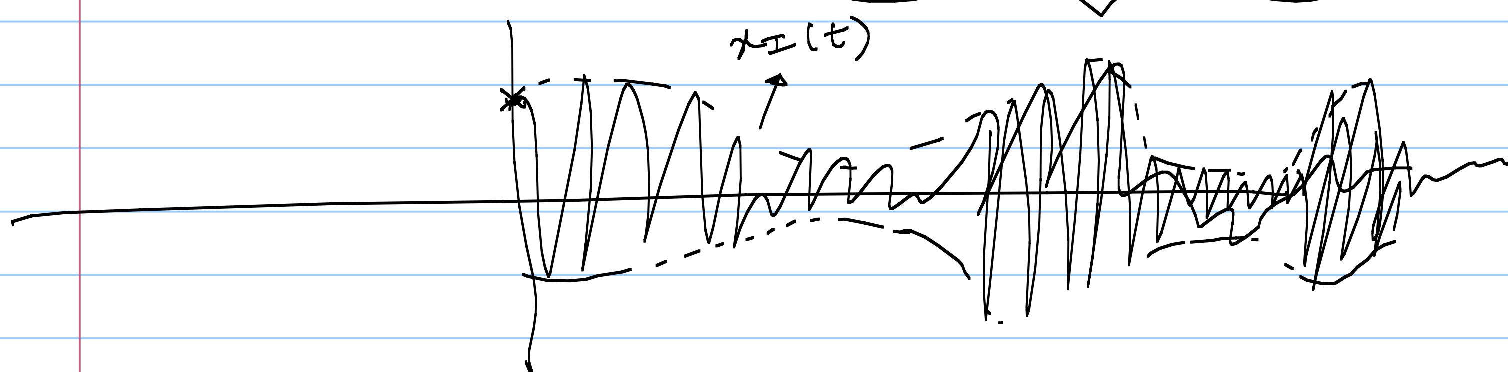
$$X(f) = X_+(f) + X_+^*(-f)$$

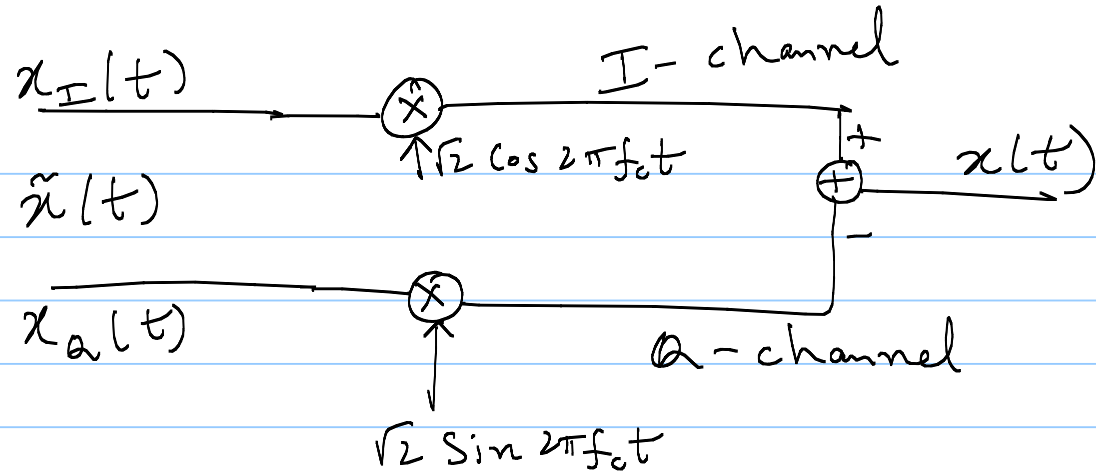
$\longrightarrow$ Equivalence between $x(t)$ and $\tilde{x}(t)$

$$x(t) = \sqrt{2}\,\mathrm{Re}\left\{ \tilde{x}(t)\, e^{j2\pi f_c t} \right\}$$

$$\tilde{x}(t) = \underset{\text{in-phase}}{x_I(t)} + j\,\underset{\text{Quadrature}}{x_Q(t)}$$

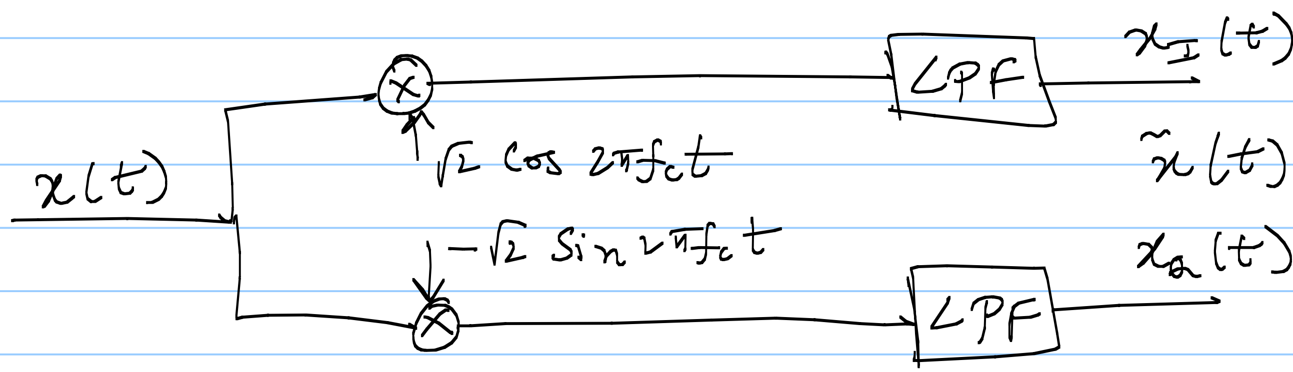
$$x(t) = \underbrace{\sqrt{2}\, x_I(t) \cos 2\pi f_c t}\; - \sqrt{2}\, x_Q(t) \sin 2\pi f_c t$$





Upconversion

"Quadrature Amplitude Modulation"

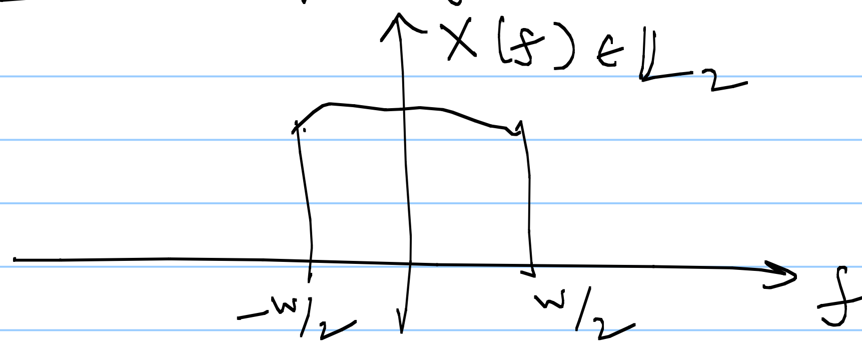


"coherent"

Downconversion

Nyquist Sampling Theorem

$x(t) \in \mathbb{L}_2$



$$X(f) = \sum_{n=-\infty}^{\infty} X_n \, e^{-j\frac{2\pi f n}{W}}, \qquad |f| \le \frac{W}{2}$$

where $$X_n = \frac{1}{W} \int_{-W/2}^{W/2} X(f) \, e^{j\frac{2\pi f n}{W}} \, df = \frac{1}{W} x\left(\frac{n}{W}\right)$$

$$X(f) = \sum_{n=-\infty}^{\infty} \frac{1}{W} x\left(\frac{n}{W}\right) e^{-j\frac{2\pi f n}{W}} \, \mathrm{rect}_{[-\frac{W}{2}, \frac{W}{2}]}(f)$$

$$x(t) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{W}\right) \underbrace{\operatorname{sinc}\left(W\left(t - \frac{n}{W}\right)\right)}$$

orthogonal basis expansion

$$\left\langle \operatorname{sinc}\left(W\left(t-\frac{n}{W}\right)\right), \operatorname{sinc}\left(W\left(t-\frac{m}{W}\right)\right) \right\rangle = 0,$$

if $m \neq n$

DSP: