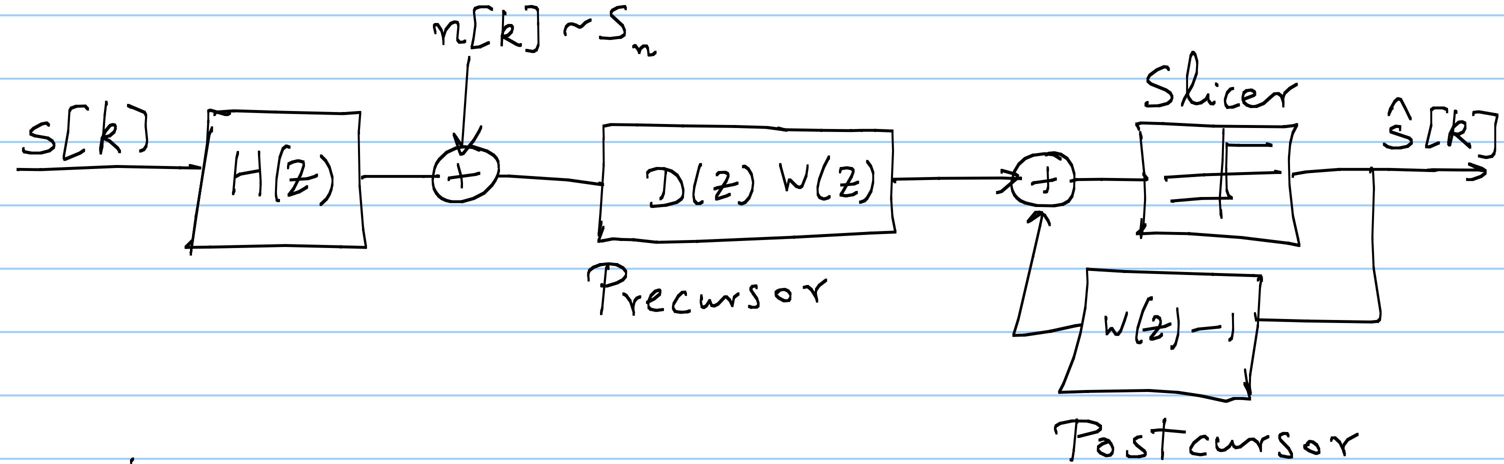
Lecture 29

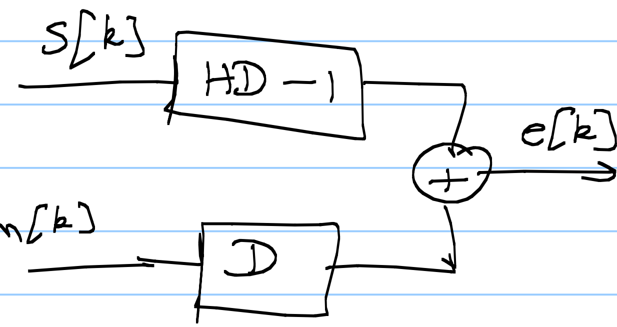
Note Title 9/24/2008



→ How to find $D(z)$?

ZF-DFE:

$$D = \frac{1}{H}$$



$$S_e = S_n\,|D|^2 = \frac{S_n}{|H|^2}$$

$$S_e = \frac{S_n}{|H|^2} = \varepsilon^2 M_e M_e^* \quad \text{(Spectral factorization)}$$

$$\varepsilon^2 = \left\langle \frac{S_n}{|H|^2} \right\rangle_G = \underline{\underline{MSE}}$$

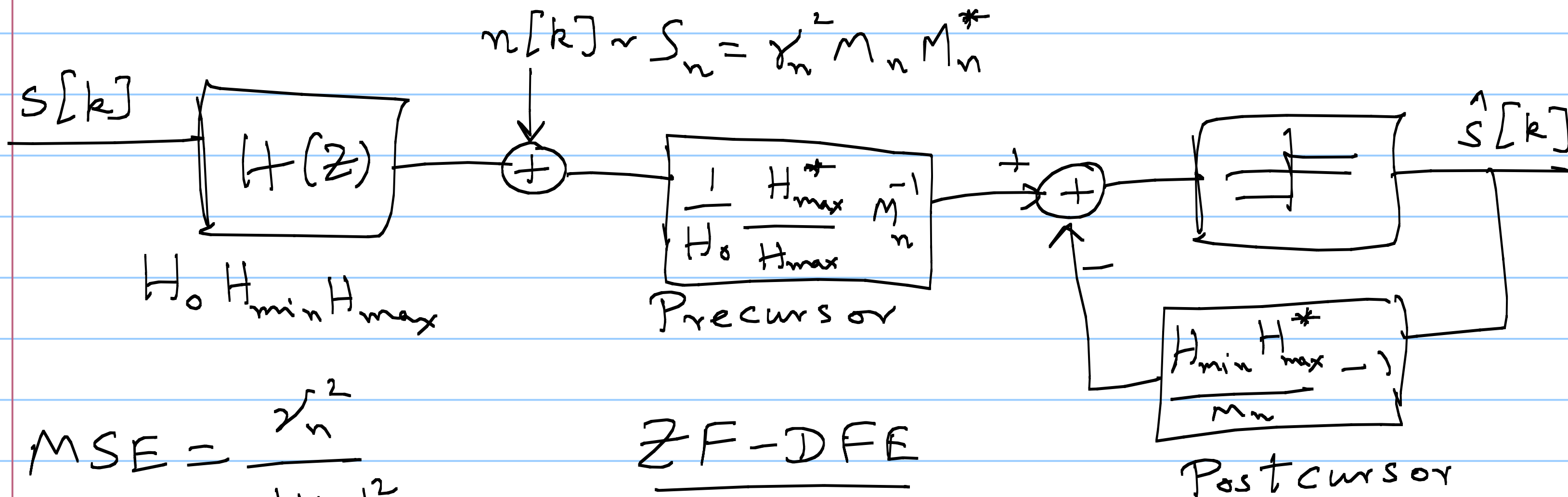
Whitening filter $\quad W = \frac{1}{M_e}$

Suppose $H = H_0 H_{max} H_{min}$ and $S_n = \gamma_n^2 M_n M_n^*$

$$S_e = \frac{\gamma_n^2 \, M_n \, M_n^*}{|H_0|^2 \, H_{min} H_{max}^* \, H_{min}^* H_{max}}$$

$$M_e = \frac{M_n}{H_{min} H_{max}^*}, \quad \varepsilon^2 = MSE = \frac{\gamma_n^2}{|H_0|^2}$$

$$DW = \frac{1}{H_0 H_{min} H_{max}} \cdot \frac{H_{min} H_{max}^*}{M_n} = \frac{1}{H_0} \frac{H_{max}^*}{H_{max}} M_n^{-1}$$



ZF-DFE

$$MSE = \frac{\gamma_n^2}{|H_0|^2}$$

$$= \left\langle \frac{S_n}{|H|^2} \right\rangle_G$$

→ slicer error is white

→ $\left| \frac{H_{max}^*}{H_{max}} \right| = 1$ "all-pass"

MMSE-DFE:

$$D = \frac{E_s H^*}{E_s |H|^2 + S_n} \qquad S_z = E_s |H|^2 + S_n$$

$$= \gamma_z^2 M_z M_z^*$$

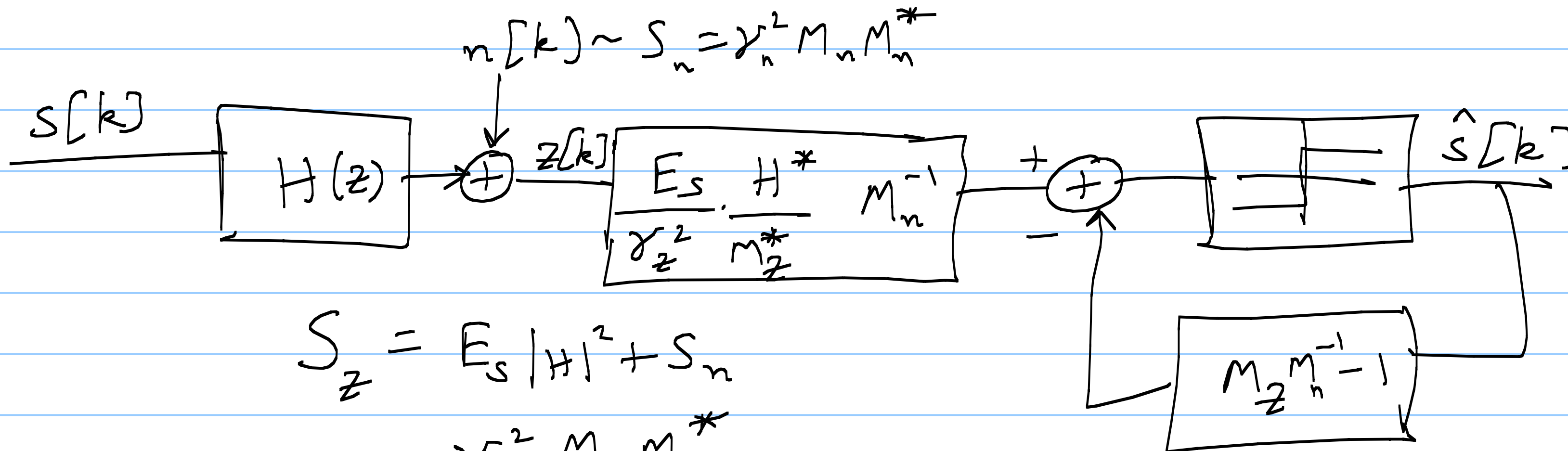
$$S_e = E_s S_z^{-1} S_n$$

$$= \left(\frac{E_s \gamma_n^2}{\gamma_z^2} \right) \frac{M_n}{M_z} \cdot \frac{M_n^*}{M_z^*}$$

$$M_e = \frac{M_n}{M_z} \text{ and } MSE = \frac{E_s \gamma_n^2}{\gamma_z^2}$$

$$W = \frac{M_z}{M_n} \qquad = \left\langle \frac{S_n}{|H|^2 + \frac{S_n}{E_s}} \right\rangle_G$$

$$DW = \frac{E_s H^*}{\gamma_z^2 M_z M_z^*} \cdot \frac{M_z}{M_n} = \frac{E_s}{\gamma_z^2} \frac{H^*}{M_z^*} \cdot M_n^{-1}$$



$$S_z = E_s |H|^2 + S_n$$
$$= \gamma_z^2 M_z M_z^*$$

$$\overline{M_{MSE\text{-}DFE}}$$

$$MSE = \frac{E_s \gamma_n^2}{\gamma_z^2} = \left\langle \frac{S_n}{|H|^2 + \frac{S_n}{E_s}} \right\rangle_a$$

Special case:

(1) $H(z) = M(z)$, minimum phase

(2) $S_n = N_0$, white

ZF-DFE: Precursor $= 1$

$$W(z) = M(z)$$

$$MSE = \left\langle \frac{N_0}{|M|^2} \right\rangle_G$$

MMSE-DFE: Precursor $= \dfrac{E_s}{N_0 \gamma_z^2} \cdot \dfrac{M^*}{M_z^*}$

$$W(z) = M_z(z)$$

$$MSE = \left\langle \frac{N_0}{|M|^2 + \frac{N_0}{E_s}} \right\rangle_G$$