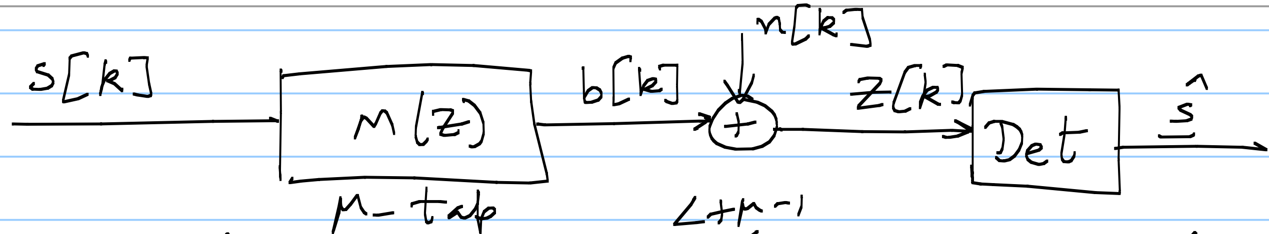
Lecture 22

Note Title 9/11/2008



$$\hat{\underline{s}} = \arg\min_{\underline{a}\in\mathcal{X}^L} \sum_{k=0}^{L+\mu-1} \left|z[k]-b_{\underline{a}}[k]\right|^2$$

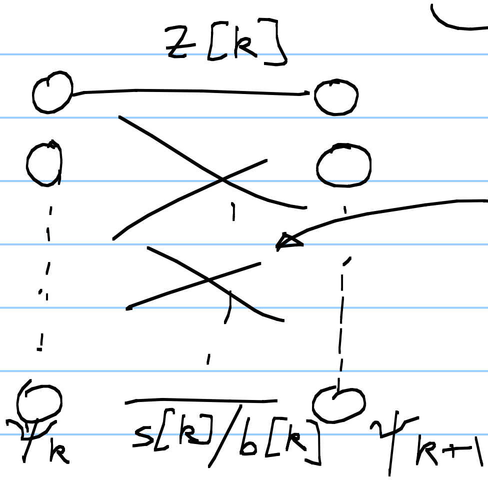
$\left(b_{\underline{a}}[k] = a[k] * m[k]\right)$

$\downarrow$

complexity $|\mathcal{X}|^{\mu} L$

$$\underline{s} = [\,s[0] \ldots s[L-1]\ \underbrace{+1\ +1 \ldots +1}_{\mu\text{-times}}\,]$$

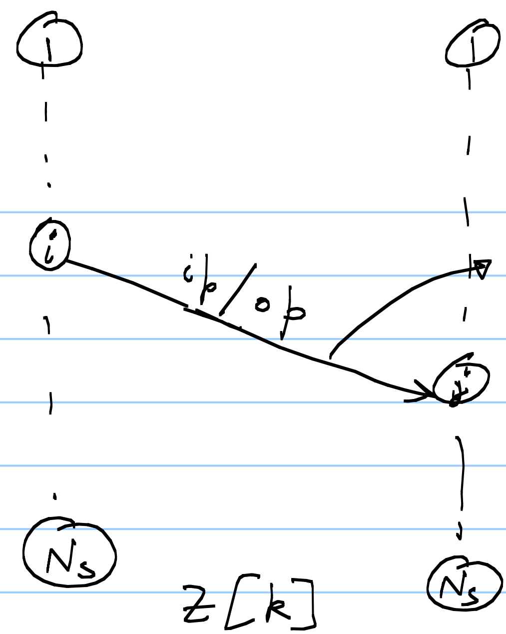
(BPSK)



$\left|z[k]-b_{\underline{a}}[k]\right|^2$

$\downarrow$

?



$|Z[k] - o_p|^2 =$ branch metric
$(BM_k(i \to j))$

$(N_s = |\mathcal{X}|^{\mu})$

→ Branch metrics can be computed at Rx.

Paths in the trellis $\longleftrightarrow$ One $\underline{a} \in \mathcal{X}^L$

$\parallel$

{sequence of states} $= \text{Path}(\underline{a}) = \{\psi_0, \psi_1(\underline{a}), \psi_2(\underline{a}), \ldots, \psi_{\mu+L}\}$

$$\text{Path}(\underline{a}) = \{\text{sequence of branches}\}$$

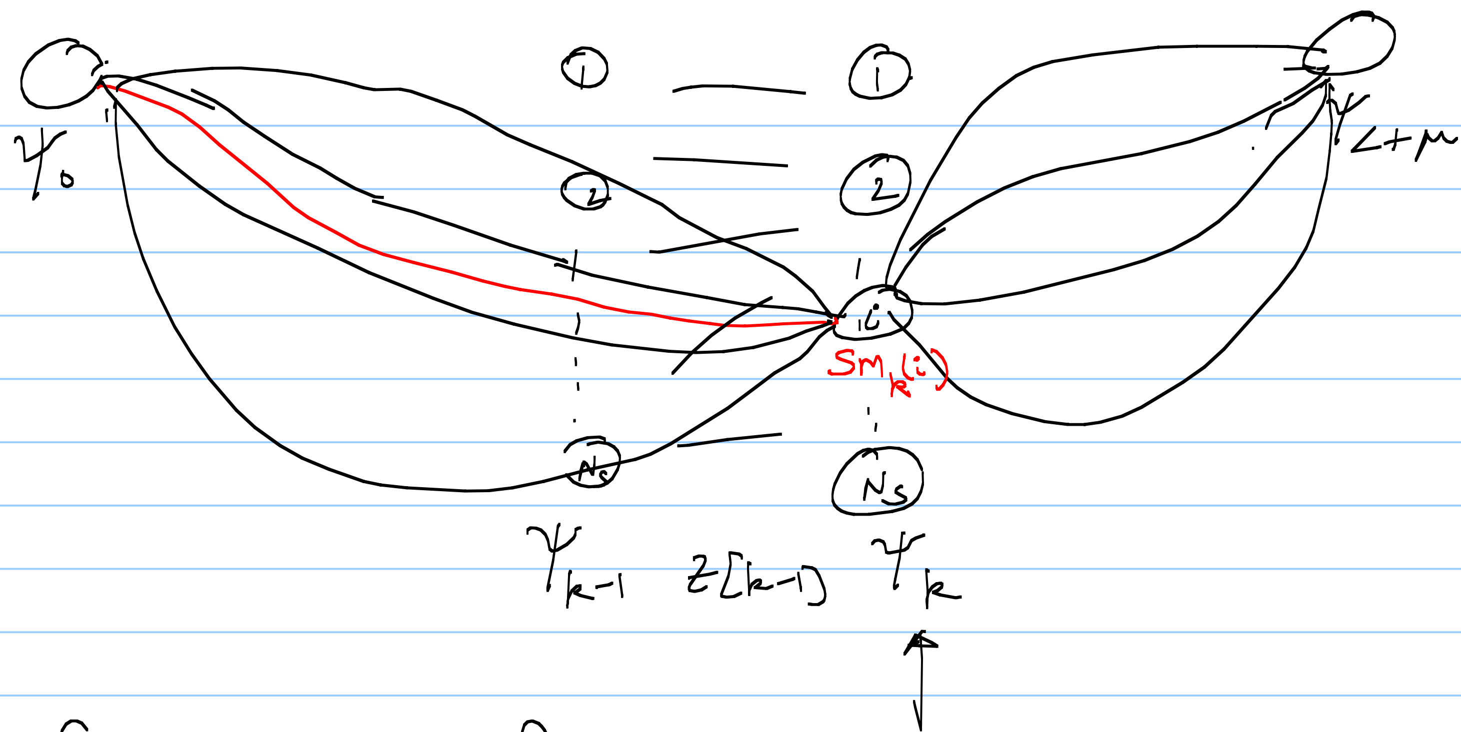
$$\text{Path metric}(\underline{a}) = \sum_{\text{branch} \in \text{Path}(\underline{a})} (\text{Branch Metric})$$

$$= \sum_{k=0}^{L+\mu-1} |z[k] - b_{\underline{a}}[k]|^2$$

$$\text{Path}(\hat{\underline{s}}) = \arg\min_{\text{path} \in \text{Trellis}} (\text{Path Metric})$$

$$\downarrow$$

$$|x|^2$$



$$\{\text{Paths} : \psi_k = i\} = P_{ki}$$

$$\text{path}^* = \arg\min_{\text{path} \in P_{ki}} (\text{Path metric}) \longrightarrow ?$$

? $\uparrow$

$$\text{path}^*\Big|_0^{k-1} = \arg\min_{\text{path} \in P_{ki}|_0^{k-1}} (\text{Path metric}) \longrightarrow \text{Prove by contradiction}$$

Survivor Path

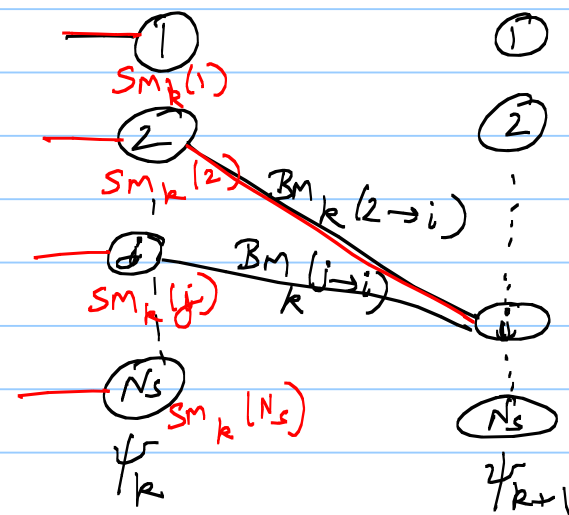
$$SP_k(i) = \arg \min_{\text{paths} \in P_{ki}|_0^{k-1}} (\text{Path metric})$$

$$1 \le i \le N_s. \qquad (\text{for each } i)$$

State metric

$$SM_k(i) = \text{Pathmetric}(SP_k(i))$$

(for each i)



Finding $SP_{k+1}(i)$ and $SM_{k+1}(i)$

$$\hat{i} = \arg \min_{j : j \to i \text{ exists}} \left(SM_k(j) + BM_k(j \to i)\right)$$

$$SP_{k+1}(i) = [SP_k(\hat{i}) \quad i] \quad (\forall i)$$

Viterbi Algorithm:

Given $z[k]$, $0 \le k \le L+\mu-1$

Given Trellis, ψ_0, $\psi_{L+\mu}$.

Compute branch metrics for all branches.

Set $SP_0(\psi_0) = [\psi_0]$, $SM_0(\psi_0) = 0$

For $k = 0$ to $L+\mu-1$

For $i = 1$ to N_s

$$\hat{i} = \underset{\substack{j:\, j\to i \\ \text{exists in stage } k}}{\arg\min} \; SM_k(j) + BM_k(j \to i)$$

$$SP_{k+1}(i) = [SP_k(\hat{i}) \;\; i]$$

$$SM_{k+1}(i) = SM_k(\hat{i}) + BM_k(\hat{i} \to i)$$

end i

end k

Output: $SP_{L+\mu}(\psi_{L+\mu}) = \text{Path}(\hat{\underline{s}})$