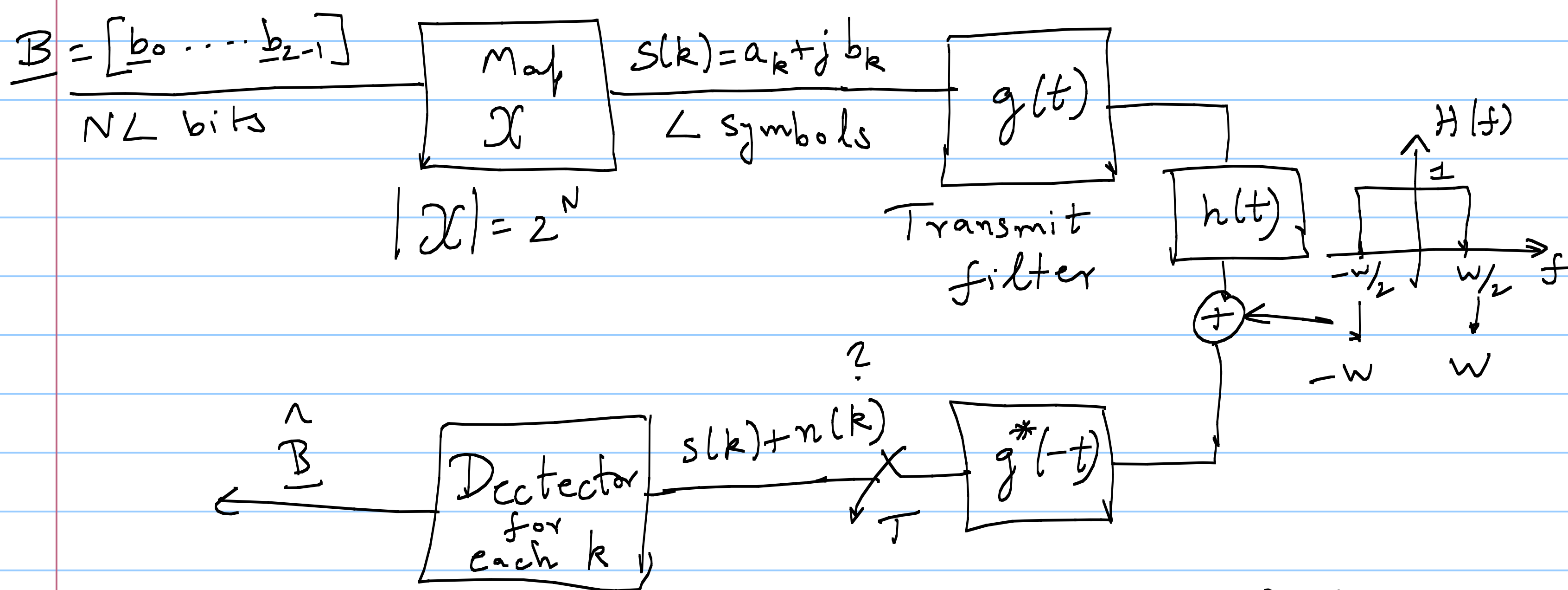
Lecture 16

Note Title 8/25/2008

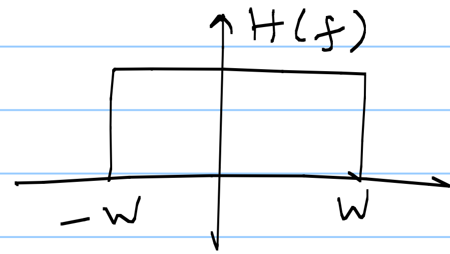


$$g(t) = \frac{4\beta}{\pi\sqrt{T}} \; \frac{\cos\frac{(1+\beta)\pi t}{T} + T\left(\frac{\sin\frac{(1-\beta)\pi t}{T}}{4\beta t}\right)}{1-\left(\frac{4\beta t}{T}\right)^2}$$

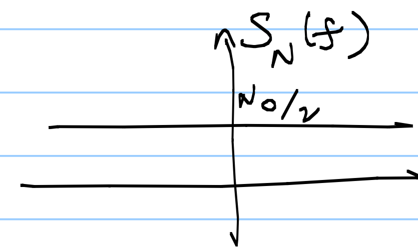
$$\boxed{\frac{1}{T} = \frac{W}{1+\beta}} \qquad 0 \le \beta \le 1$$

Capacity, SNR and E_b/N_0:

$$y(t) = x(t) + n(t)$$



$BW(x(t)) \le W.$



P = Power of $x(t)$.

$N_0 W$ = Power of noise

$$SNR = \frac{P}{N_0 W}$$ "waveform"

$$\frac{1}{T} = 2W \text{ sym/sec.}$$

$\left\{\text{Maximum rate with arbitrarily low error rates}\right\}$ $C = W \log_2\left(1 + \frac{P}{N_0 W}\right)$ bps.

Discrete-time model:

$$Y = X + N$$

Energy in signal, $E_s = E[X^2]$

$$P = E_s / T$$

"Waveform" $SNR = \dfrac{P}{N_0 W} = \dfrac{E_s}{N_0 \cdot T W} = \dfrac{E_s}{(N_0/2)}$

E_s : Energy every T seconds.

$\frac{N_0}{2}$: Noise energy per signal-space dimension.

Capacity in bits per dimension $\quad C = \frac{1}{2} \log(1 + SNR)$ bits/dim (or) bits/sec/Hz

W

$\underline{E_b/N_0}$:

E_b : Energy per information bit

Constellation $\mathcal{X}$.

$R =$ rate in bits/symbol.

$\left(= \log_2 |\mathcal{X}| \quad \text{"uncoded"} \right)$

$$E_b = \frac{E_s}{R}$$

$$SNR = \frac{E_s}{\sigma^2} = \frac{E_b \cdot R}{\frac{N_0}{2}}$$

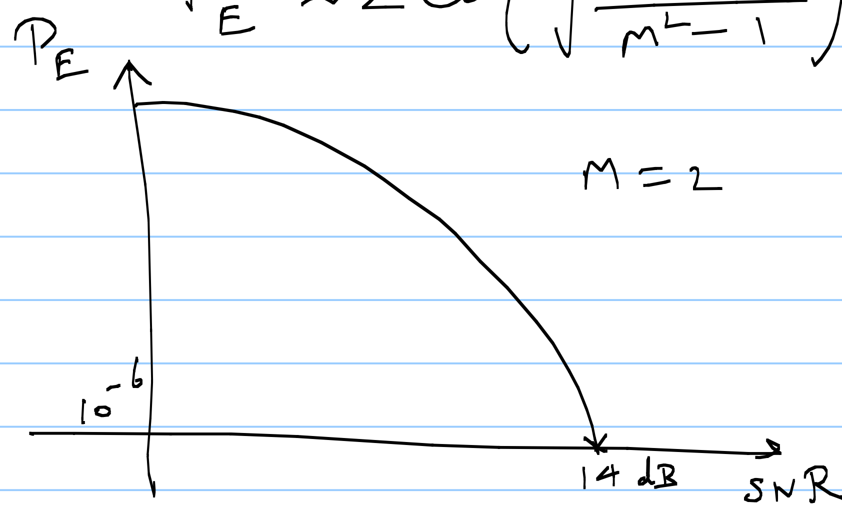
$$\boxed{\frac{E_b}{N_0} = \frac{SNR}{2R}} \longrightarrow \text{rate-normalized SNR.}$$

M-PAM: "Uncoded"

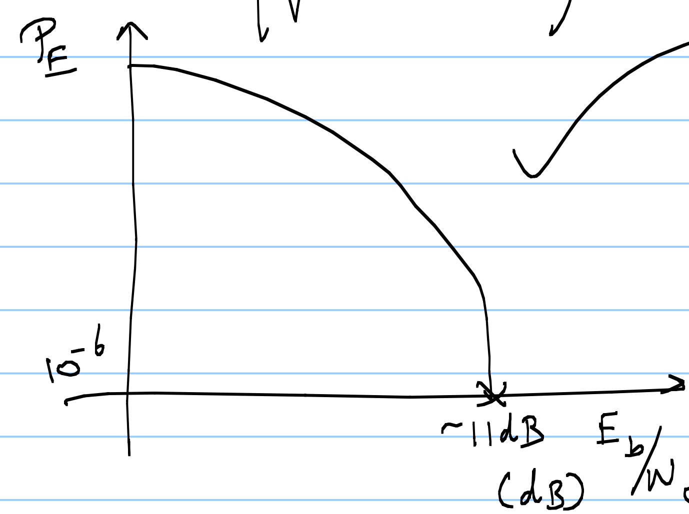
$$\frac{E_b}{N_0} = \frac{(M^2-1)\,d^2/12}{N_0/2} \cdot \frac{1}{2\log_2 M}$$

$$= \frac{(M^2-1)\,d^2}{12\,N_0 \log_2 M}$$

$$P_E \simeq 2Q\left(\sqrt{\frac{3 \cdot SNR}{M^2-1}}\right) = 2\,Q\left(\sqrt{\frac{6}{M^2-1}\,\frac{E_b}{N_0}}\right)$$



M = 2



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Capacity in terms of $\frac{E_b}{N_0}$

$$C = \frac{1}{2}\log_2(1+SNR) \text{ bits/Dimension}$$

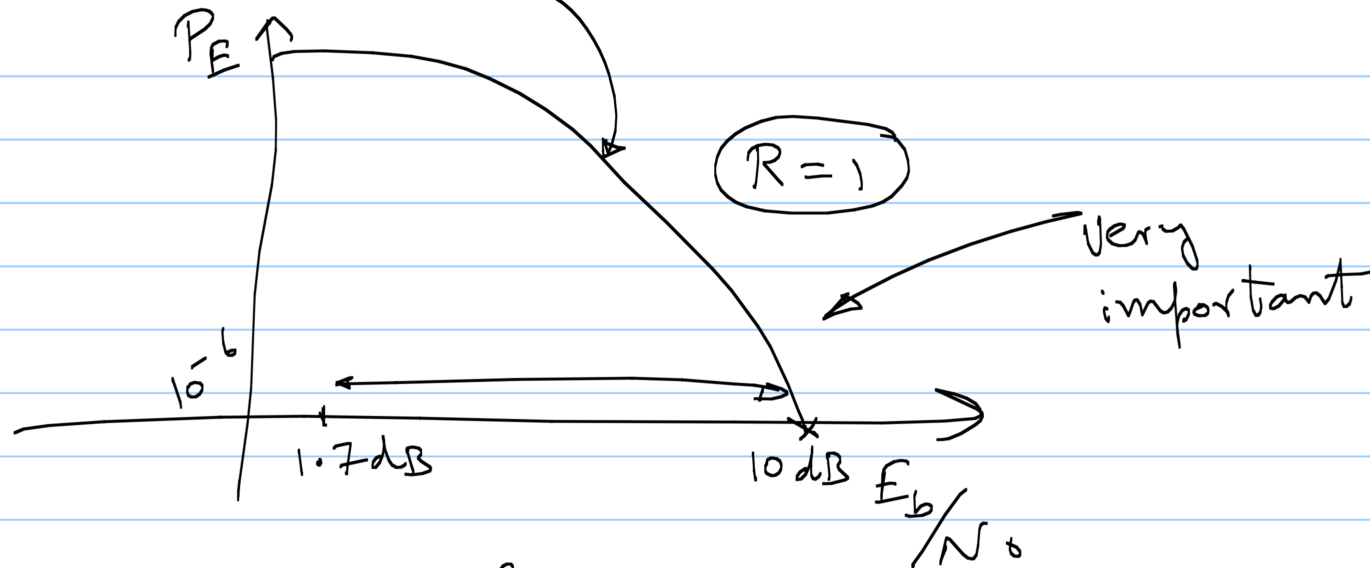
$$\frac{E_b}{N_0} = \frac{SNR}{2R} \qquad R = \# \text{ information bits/symbol.}$$

$$R \leq \frac{1}{2}\log_2(1+SNR).$$

$$SNR \geq 2^{2R} - 1$$

$$\frac{E_b}{N_0} \geq \frac{2^{2R}-1}{2R} \qquad \underline{\underline{R=1}}, \ \frac{E_b}{N_0} \geq 1.7\,dB.$$

Uncoded BPSK



$$\frac{E_b}{N_0} \geq \frac{2^{2R}-1}{2R} \longrightarrow R \to 0$$

$$\text{RHS} \to \log_e 2 = \underline{\underline{-1.59\,\text{dB}}}$$