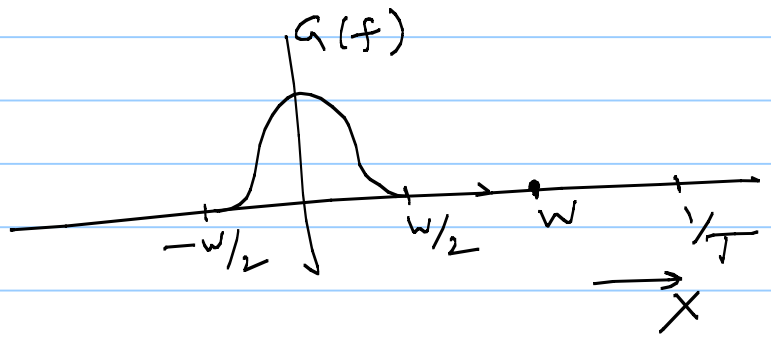
Lecture 15

Note Title 8/21/2008

Nyquist Criteria:

$g(t)$:

band-limited



$\{g(t-kT)\}$: orthonormal

k: integer

$$C(f) = |G(f)|^2$$

$$\frac{1}{T}\sum_{m=-\infty}^{\infty} C\left(f-\frac{m}{T}\right) = 1$$

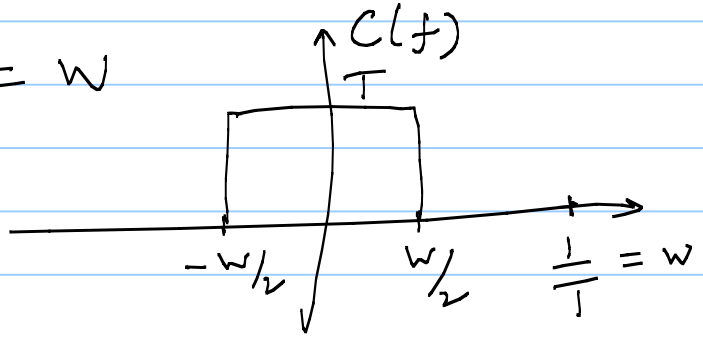
$\frac{1}{T} \leq W$ for Nyquist criteria to be satisfied

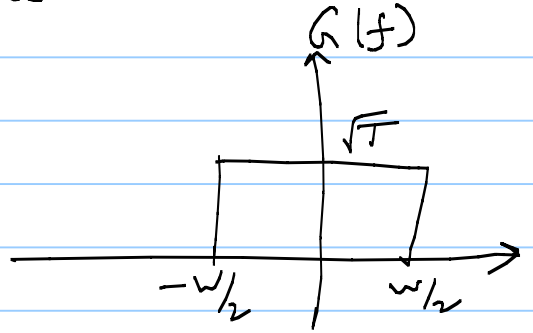
$\rightarrow$ Baseband BW $= \frac{W}{2} \Rightarrow$ W : Nyquist symbol rate

Three cases: (1) $\frac{1}{T} > W$

$\rightarrow$ ISI-free communication is not possible

(2) $\frac{1}{T} = W$



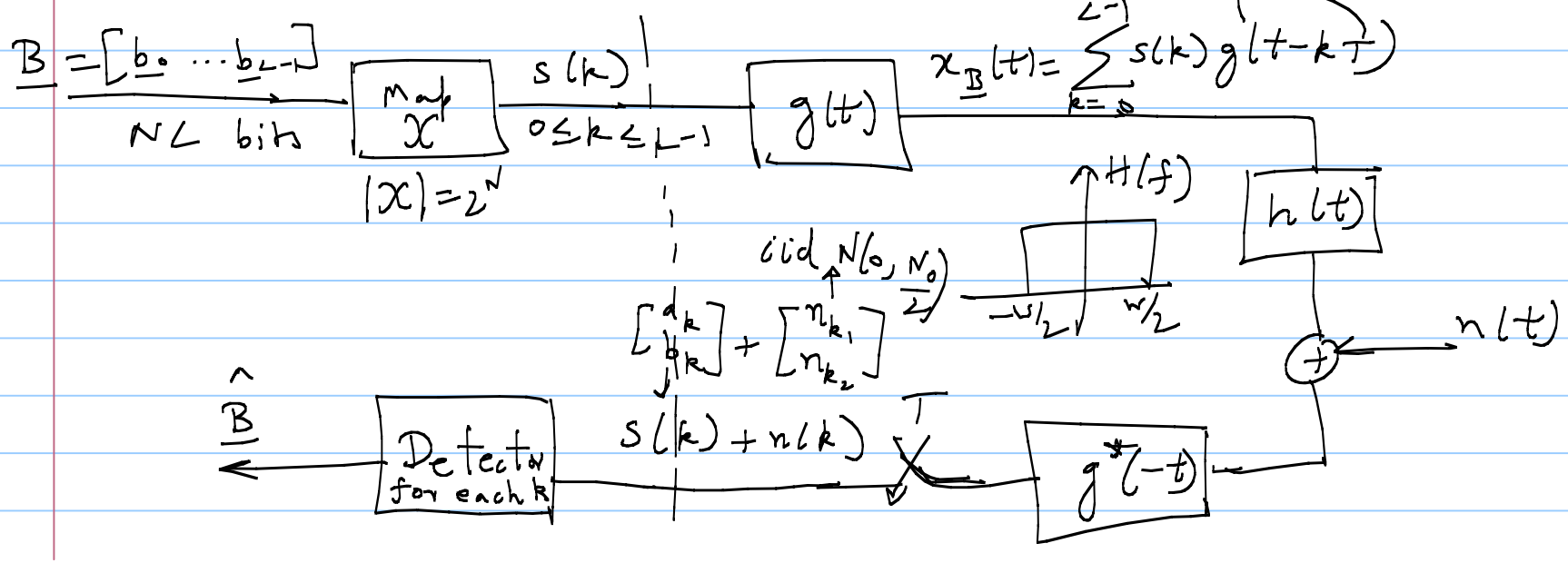


$$C(f) = T \operatorname{rect}_{[-W/2,\, W/2]}(f) \quad \longleftrightarrow \quad c(t) = \operatorname{Sinc}\left(\frac{t}{T}\right) = \frac{\sin \frac{\pi t}{T}}{\frac{\pi t}{T}}$$

$$g(t) = \frac{1}{\sqrt{T}} \operatorname{Sinc}\left(\frac{t}{T}\right)$$

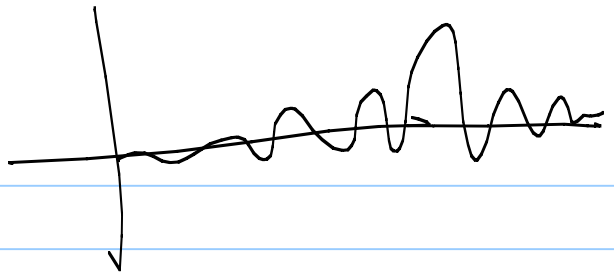
"unique"

$T = 1/W$



Issues $T = 1/W$ $g(t)$: sinc

1) $g(t)$: non-causal



→ overcome by delaying

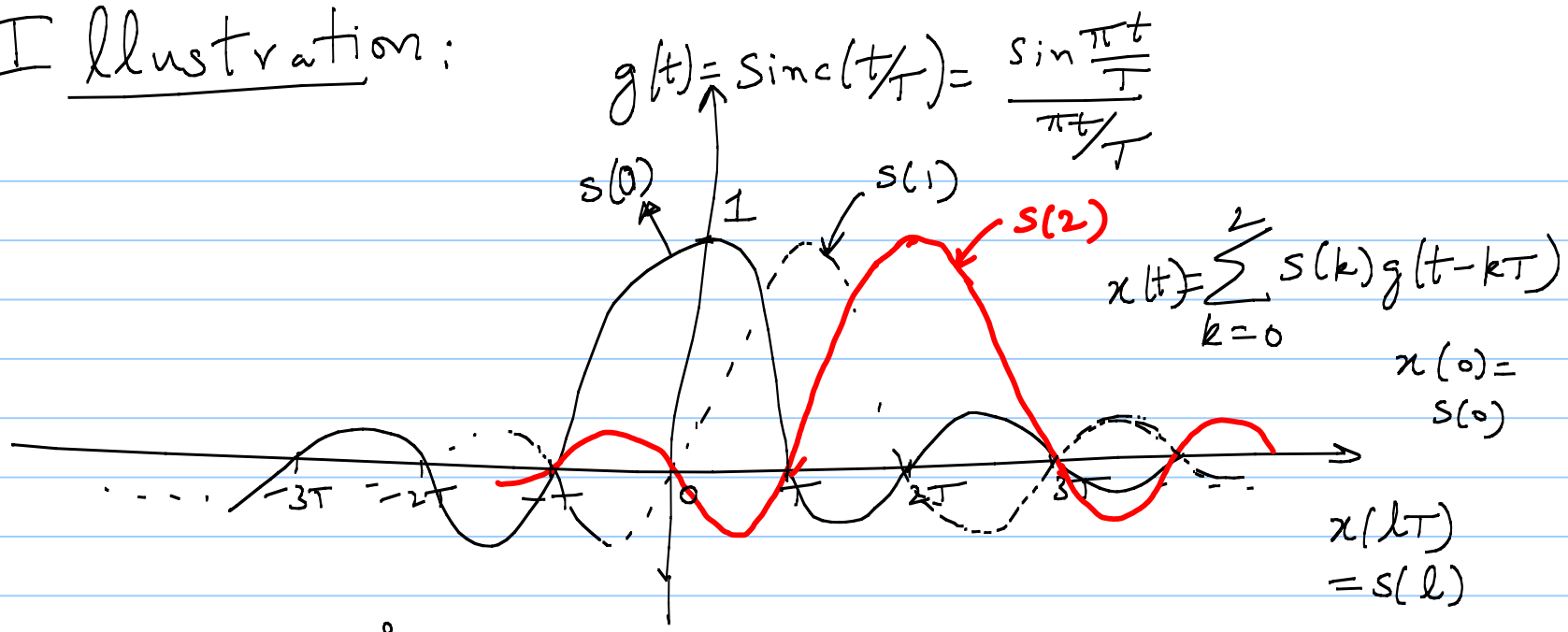
$g^*(-t)$: non-causal

→ delay can solve this.

2) $g(t)$: decays at $1/t$

↳ Rx is sensitive to errors in T

I llustration:



$$\int_{-\infty}^{\infty} \mathrm{sinc}\left(\frac{t}{T}\right) \mathrm{sinc}\left(\frac{t-T}{T}\right) dt = 0$$

$y(t) = x(t) + n(t) \longrightarrow$ LPF at $-W/2, W/2$

Sample $y(t)$ at lT ?

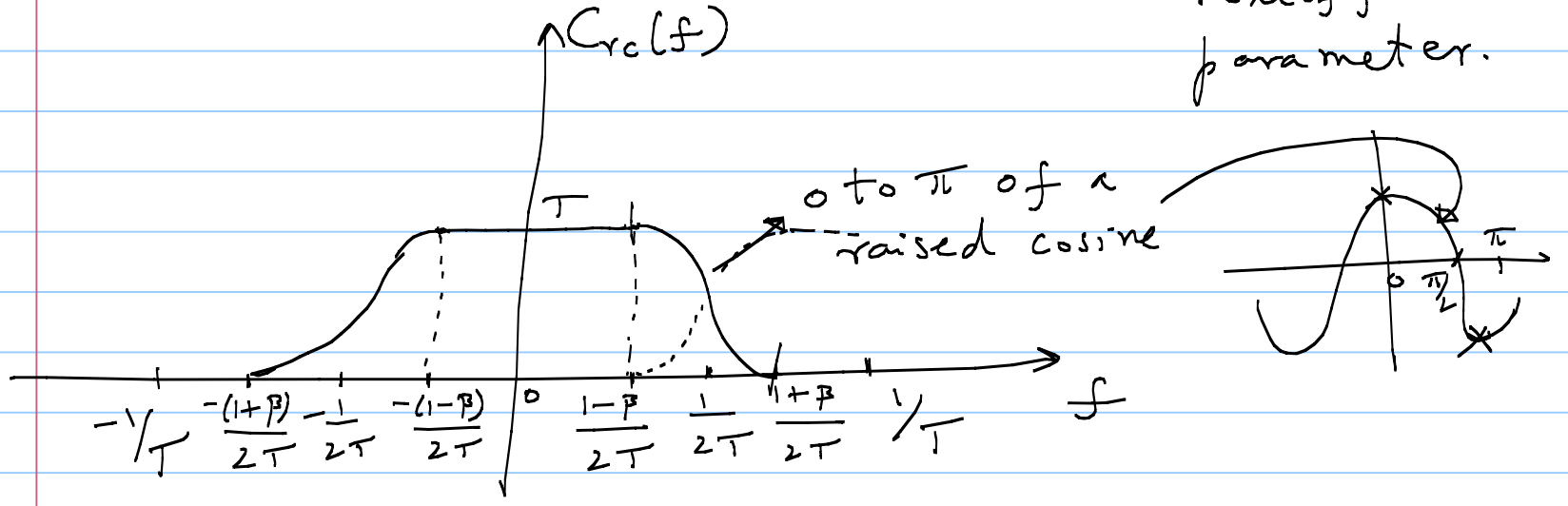
(3) $\frac{1}{T} < W$ (or) $T > \frac{1}{W}$

$\longrightarrow$ many choices for $g(t)$ & $c(t)$

"raised cosine" $C_{rc}(f)$:

$T = \frac{1+\beta}{W}$ (or) $\frac{1}{T} = \frac{W}{1+\beta}$ $\quad 0 \le \beta \le 1$

β $\downarrow$ rolloff parameter.



$$C_{rc}(f) = \begin{cases} T, & 0 \le |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\left\{1 + \cos\left[\frac{\pi T}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right]\right\}, & \frac{1-\beta}{2T} < |f| < \frac{1+\beta}{2T} \\ 0, & |f| > \frac{1+\beta}{2T} \end{cases}$$

$\frac{1+\beta}{2T} = \frac{W}{2}$

$\Downarrow FT$

$$c(t) = \mathrm{Sinc}\left(\frac{t}{T}\right) \cdot \frac{\cos\left(\beta\pi t/T\right)}{1 - \left(\frac{2\beta t}{T}\right)^2}$$

Square-root raised cosine pulse:

$$g(t) = \frac{4\beta}{\pi\sqrt{T}} \frac{\cos\frac{(1+\beta)\pi t}{T} + T\left(\frac{\sin\frac{(1-\beta)\pi t}{T}}{4\beta t}\right)}{1 - \left(\frac{4\beta t}{T}\right)^2}$$