Lecture 13

Note Title 8/19/2008

M-PAM:

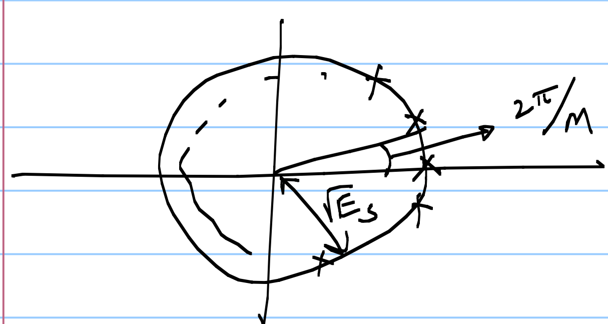
$$\Pr\{\text{Error}\} \simeq 2\,Q\left(\frac{d/2}{\sqrt{N_0/2}}\right) = 2\,Q\left(\sqrt{\frac{3\,\text{SNR}}{M^2-1}}\right)$$

M^2-QAM:

$$\Pr\{\text{Error}\} \simeq 4\,Q\left(\sqrt{\frac{3\,(\text{SNR})}{2(M^2-1)}}\right)$$

M-PSK:

$$\Pr\{\text{Error}\} \simeq 2\,Q\left(\sqrt{\sin^2\frac{\pi}{M}\,(\text{SNR})}\right)$$



$$E_N = \frac{N_0}{2}$$

M^2-PSK versus M^2-QAM

$$P_E(M^2\text{-QAM}) = \cancel{4}\, Q\left(\sqrt{\frac{3}{2(M^2-1)}(SNR)}\right)$$

$$P_E(M^2\text{-PSK}) = \cancel{2}\, Q\left(\sqrt{\sin^2\left(\frac{\pi}{M^2}\right)(SNR)}\right)$$

Suppose $P_E(M^2\text{-QAM}) = P_E(M^2\text{-PSK})$

at SNR_Q at SNR_P

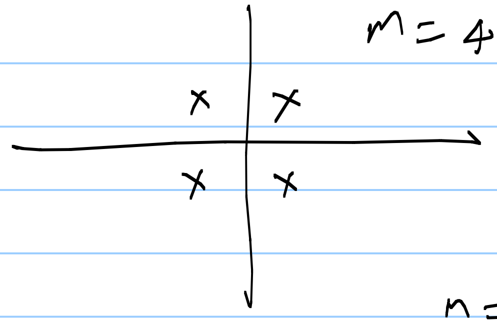
$$\frac{3}{2(M^2-1)} SNR_Q = \sin^2\left(\frac{\pi}{M^2}\right)(SNR_P)$$

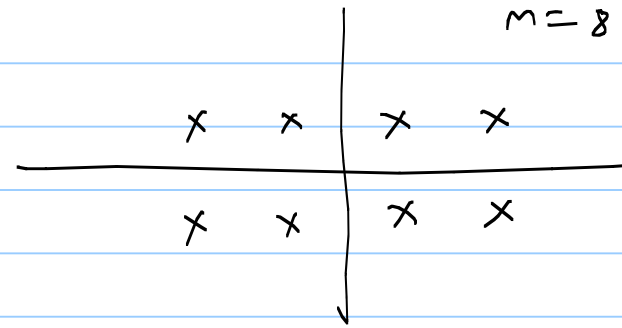
$$\frac{SNR_P}{SNR_Q} \approx \frac{3M^2}{2\pi^2}$$

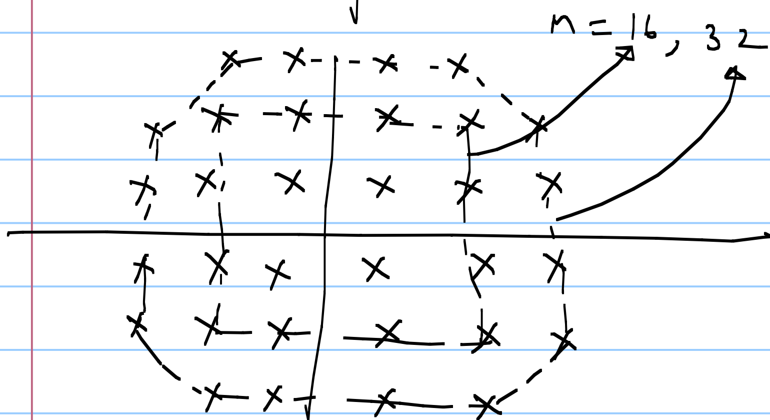
$M = 2, 4, 8$

$2 \downarrow 0\,dB$, $4 \downarrow 4.2\,dB$, $8 \downarrow 10\,dB$

M - QAM (M : power of 2)







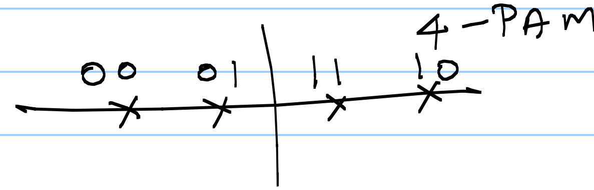
Other signaling schemes: MSK, Offset QPSK

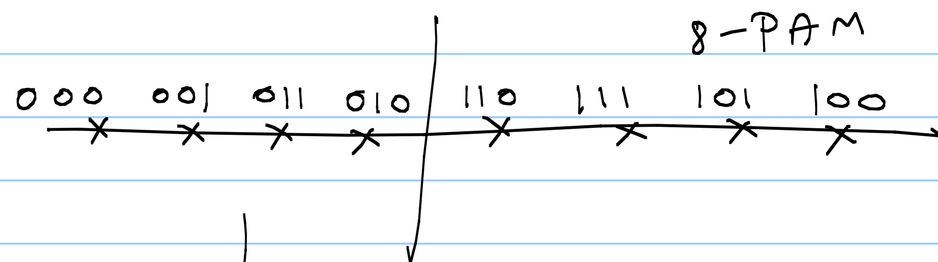
(1) Continuous phase modulation (CPM)

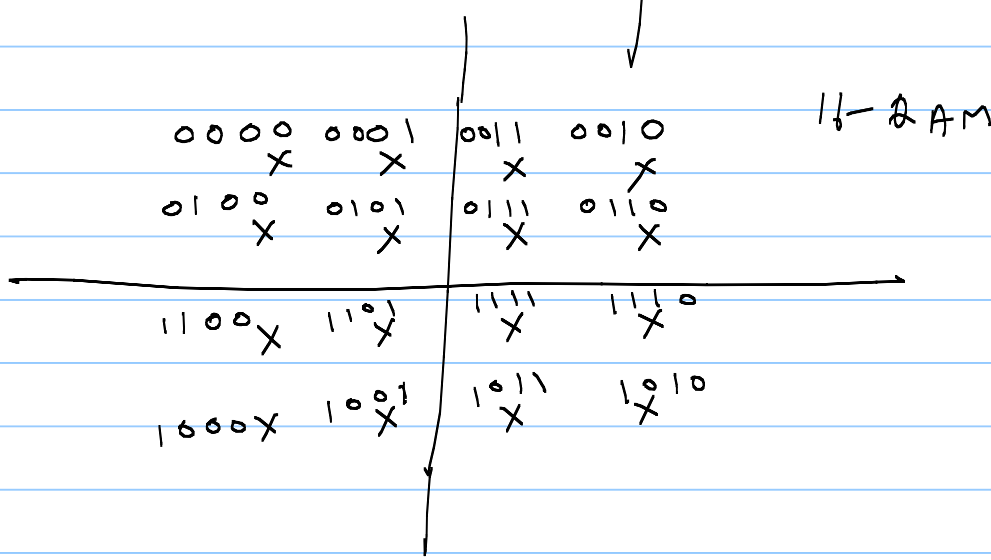
(2) FSK $\rightarrow$ CPFSK

Labeling points with bits:

Gray coding: adjacent points differ in 1 bit







Symbol errors versus bit errors

M-QAM

1 symbol error

$$\text{1 bit error} \quad - - - \quad \log_2 M \text{ bit errors}$$

N symbols $\longrightarrow$ N_s symbol errors

$N \log_2 M$ bits $\longrightarrow$ N_s bit errors

$\vdots$

$N_s \log_2 M$ bit errors

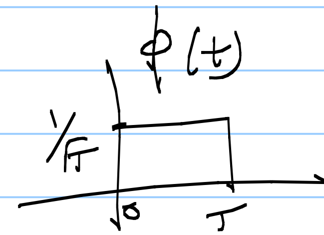
$$\text{Symbol error rate} \simeq \frac{N_s}{N}$$

$$\frac{N_s}{N \log_2 M} \leq \text{Bit error rate} \leq \frac{N_s}{N}$$

Summary: Ideal, large BW, AWGN channel

→ real passband $\rightleftarrows$ complex baseband

→ BW $\gg \frac{1}{T}$: signaling is simple

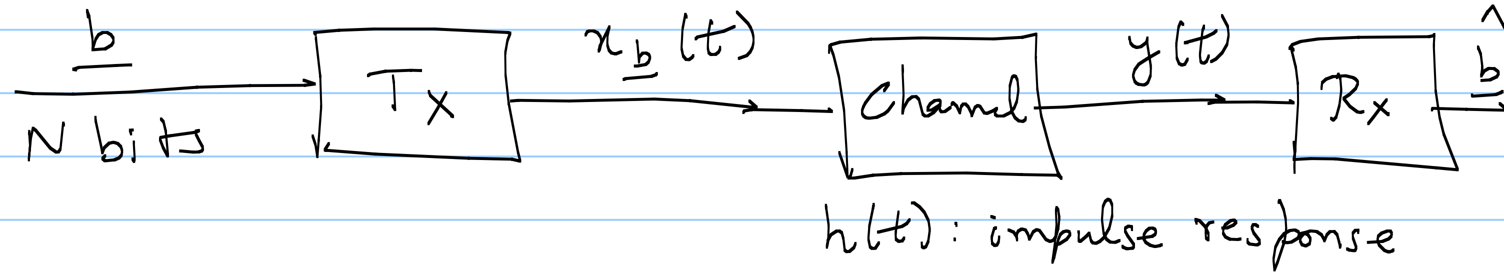


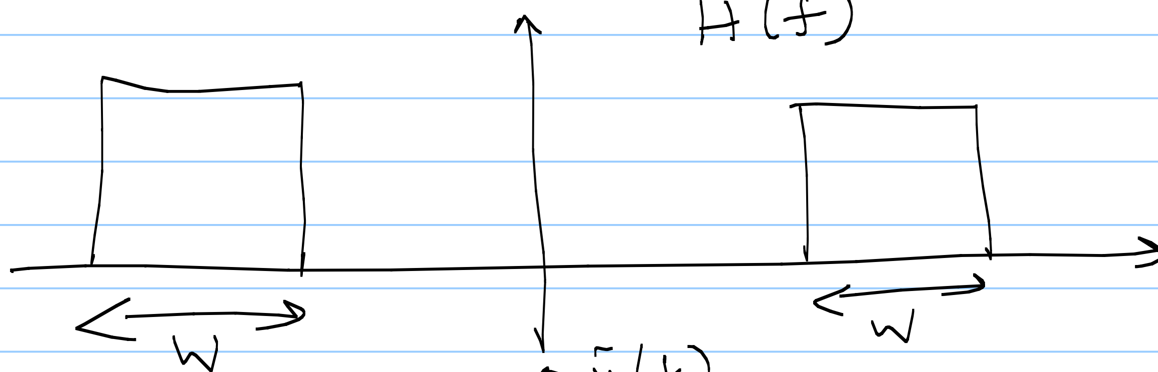
Complex-baseband QAM

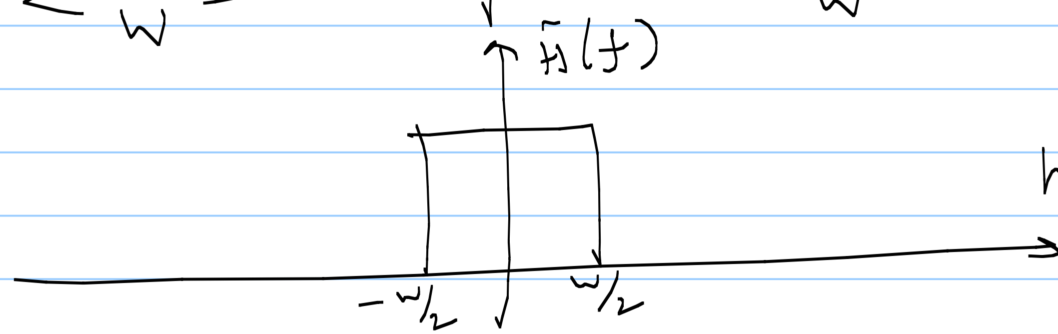
$\underline{b}$ = $2\log_2 M$ bits → $a+jb$ point in Constellation → $(a+jb)\phi(t)$ $\xrightarrow{\text{up}}$ $x_{ab}(t)$ → $\oplus$ ← $n(t)$ (PSD $\frac{N_0}{2}$)

$\oplus$ $\xrightarrow{\text{down}}$ "Correlation" integrate & dump → $a+jb$ + 2-D Noise iid $N(0, \frac{N_0}{2})$ → Detection (MAP/ML Min-dist.) → $\hat{\underline{b}}$

Ideal, Band-limited, AWGN







<u>w log</u>

$h(t)$: baseband
real

$$
\begin{tikzcd}
{} \arrow[r, "\underline{b}", "N \text{ bits}"'] & \boxed{T_x} \arrow[r, "x_{\underline{b}}(t)"] & \boxed{\text{Channel}} \arrow[r, "y(t)"] & \boxed{R_x} \arrow[r, "\hat{\underline{b}}"] & {}
\end{tikzcd}
$$

$h(t)$: impulse response

$\ast$ baseband, real

$\longrightarrow$ $x_{\underline{b}}(t)$: time-limited & band-limited

$[0, T_N]$ $\quad$ $[-W/2, W/2]$

$\longrightarrow$ Continuos operation?