Lecture 10

Note Title 8/13/2008

Errata: M^2-QAM

$$E[|X|^2] = \frac{2(M^2-1)}{3}$$

$$a+jb \longleftrightarrow x_{ab}(t) = a\sqrt{\frac{2}{T}}\cos 2\pi f_0 t + b\sqrt{\frac{2}{T}}\sin 2\pi f_0 t$$

$0 \le t \le T$

"real, passband" $\Updownarrow$

2-D signal space over $\mathbb{R}$

Basis: $\cos 2\pi f_0 t$, $\sin 2\pi f_0 t$

$0 \le t \le T$

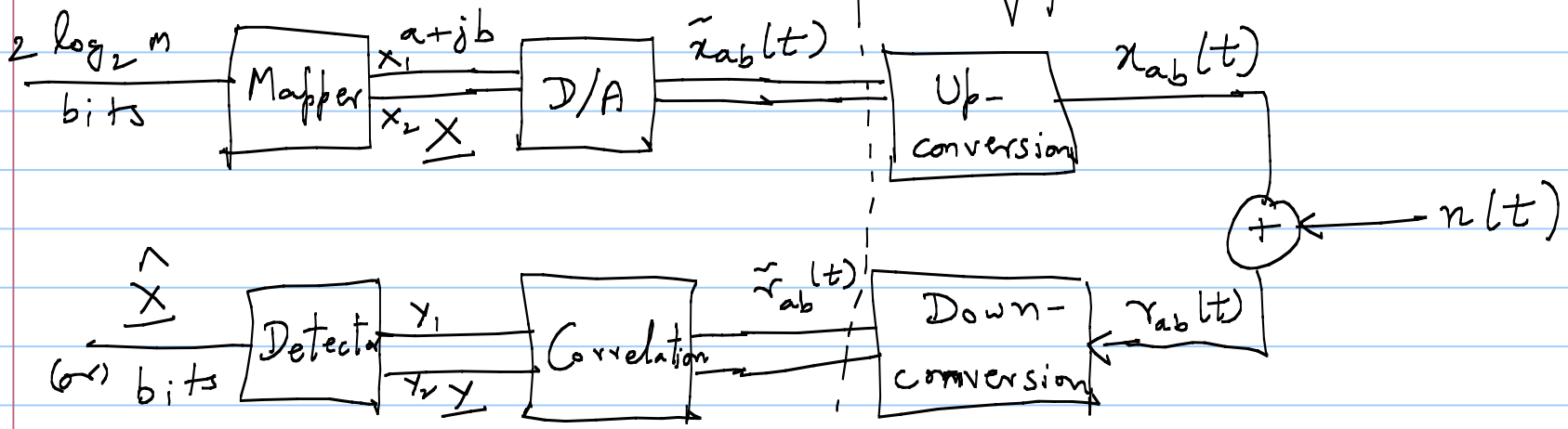
$x_{ab}(t)$ (real, passband)

Down-conversion

$$\tilde{x}_{ab}(t) = (a - jb)\sqrt{\frac{1}{T}}, \quad 0 \le t \le T$$

1-D signal space over $\mathbb{C}$

Basis: $\sqrt{\frac{1}{T}}$, $0 \le t \le T$.



Detection:

Model: $\underline{Y} = \underline{X} + \underline{N}$ (n-Dimensional)

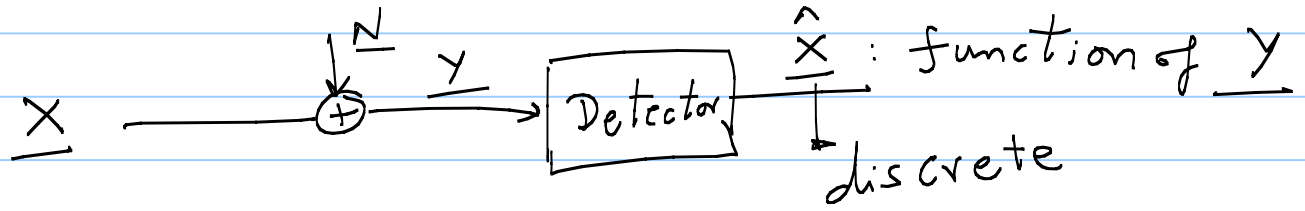
$f_{\underline{X}}(\underline{x})$: given (discrete)

$\underline{x} \in \mathcal{X}$ (signal constellation)

independent

$f_{\underline{N}}(\underline{n})$: given n-D pdf (continuous)

Problem: Given $\underline{Y} = \underline{y}$, what is the "optimal" choice for $\underline{X}$?



Prob. of correct decision $P(C) = P_r(\hat{\underline{X}} = \underline{X})$

$$P(C) = \int_{\underline{y}} P_r(\hat{\underline{X}} = \underline{X} \mid \underline{Y} = \underline{y}) f_{\underline{Y}}(\underline{y}) \, d\underline{y}$$

$\max P(C) \iff \max P_r(\hat{\underline{X}} = \underline{X} \mid \underline{Y} = \underline{y})$ for each $\underline{y}$.

For each $\underline{y}$, we pick $\hat{\underline{x}}$ s.t. $P_r(\hat{\underline{X}} = \underline{X} \mid \underline{Y} = \underline{y})$ is maximized.

"MAP" $\hat{\underline{X}} = \arg \max_{\underline{x} \in \mathcal{X}} P_r(\underline{x} = \underline{X} \mid \underline{Y} = \underline{y})$

$$P_r(\underline{X} = \underline{x} \mid \underline{Y} = \underline{y}) = \frac{f_{\underline{y}|\underline{x}}(\underline{y}|\underline{x}) \cdot P_r(\underline{X} = \underline{x})}{f_{\underline{y}}(\underline{y})}$$

"MAP"
Maximum a posteriori

$$\Longleftrightarrow \quad \max_{\underline{x} \in \mathcal{X}} \; P_r(\underline{X} = \underline{x}) \, f_{\underline{y}|\underline{x}}(\underline{y}|\underline{x})$$

Uniform

"ML"
Maximum Likelihood

$$\hat{\underline{x}} = \arg\max_{\underline{x} \in \mathcal{X}} \; f_{\underline{y}|\underline{x}}(\underline{y}|\underline{x})$$

AWGN $\quad \underline{Y} = \underline{X} + \underline{N}$

$$f_{\underline{Y}|\underline{X}}(\underline{y}|\underline{x}) = f_{\underline{N}}(\underline{y}-\underline{x})$$

$$= \frac{1}{(\sqrt{\pi N_0})^m} e^{\frac{-\|\underline{y}-\underline{x}\|^2}{N_0}}$$

$$ML \iff \hat{\underline{x}} = \arg\min_{\underline{x} \in \mathcal{X}} \|\underline{y}-\underline{x}\|^2$$

"Closest symbol"

Example:

① BPSK: