Assignment on Miscellaneous Topics

EE512: Error Control Coding

Questions marked (Q) or (F) are questions from previous quizzes or final exams, respectively.

1. Simulate syndrome, ML, and bitwise-MAP decoding for the (7,4) Hamming code and the (3,1) Repetition code. Use MATLAB, SCILAB, C or C++. Find the coding gain or loss in E_b/N_0 (dB) at a BER of 10^{-3} for each case. Compare with theoretical values wherever possible.

2. Show that the soft ML and bitwise-MAP decoders for the $(n, 1)$ repetition code are identical over an AWGN channel under BPSK modulation. Find an exact expression for probability of bit and block error at the output of either decoder. Is there any coding gain in terms of E_b/N_0?

3. Consider a code with parity-check matrix
$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}.$$
   (a) Design soft ML and bitwise-MAP decoders over an AWGN channel under BPSK modulation.
   (b) Design a hard-decision syndrome decoder after converting the above channel to a BSC.
   (c) Is there a received word for which the above three decoders provide three different outputs?

4. Consider a code with generator matrix
$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}.$$
   (a) Design soft ML and bitwise-MAP decoders over an AWGN channel under BPSK modulation.
   (b) Design a hard-decision syndrome decoder after converting the above channel to a BSC.

5. Consider the $(n, n-1)$ even-weight code being used over an AWGN channel with BPSK modulation with all codewords equally likely *a priori*. Let $\mathbf{c} = [c_1\ c_2 \cdots c_n]$ and $\mathbf{r} = [r_1\ r_2 \cdots r_n]$ denote the transmitted codeword and the received word, respectively. Let $\lambda_i = \log \dfrac{\Pr\{c_i = 0|\mathbf{r}\}}{\Pr\{c_i = 1|\mathbf{r}\}}$ and $\gamma_i = \log \dfrac{\Pr\{c_i = 0|r_i\}}{\Pr\{c_i = 1|r_i\}}$ for $1 \le i \le n$. Show that
$$\lambda_i = \gamma_i + \left(\prod_{j=1, j \ne i}^{n} \operatorname{sign}(\gamma_j) \right) f\left[\sum_{j=1, j\ne i}^{n} f(|\gamma_j|) \right],$$
where $f(x) = \log \dfrac{e^x + 1}{e^x - 1}$.

6. (Q) The code $\{0000, 0101, 1010, 1111\}$ is being used with BPSK modulation $(0 \to +1; 1 \to -1)$ over an AWGN channel. Write down an algorithm to perform maximum-likelihood decoding minimizing the number of real-number additions (N_a) and real-number comparisons (N_c) $(10-N_a-N_c$ Marks).

7. (F) Consider a convolutional code with the generator matrix
$$G(D) = \begin{bmatrix} 1 + D^3 & 1 + D + D^2 + D^3 \end{bmatrix}.$$
   (a) Draw the circuit for a non-systematic encoder for the code.
   (b) Draw the circuit for a systematic encoder for the code.

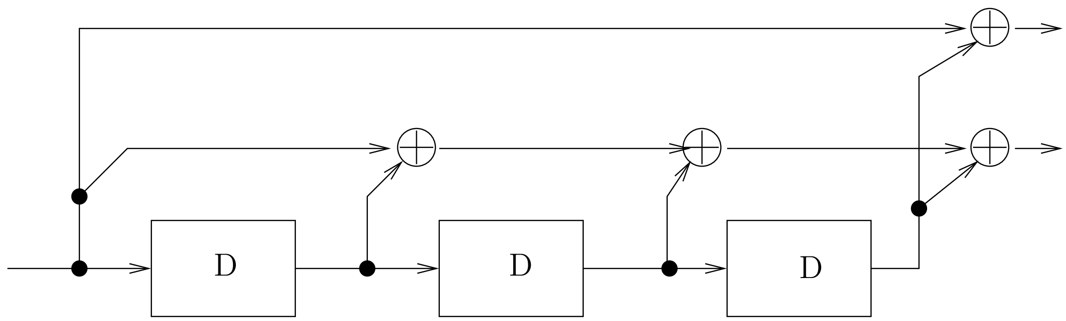

Figure 1: Encoder for Problem 8

(c) Draw one stage of the trellis for the systematic encoder.

8. (F) Consider the convolutional encoder shown in Fig. 1.

   (a) Draw one stage of the trellis for the encoder.

   (b) Write down the transfer function matrix for the convolutional code.

   (c) Encode the infinite message sequence (11111111.......) (all 1s).

9. Consider the convolutional encoder shown in Fig. 2.

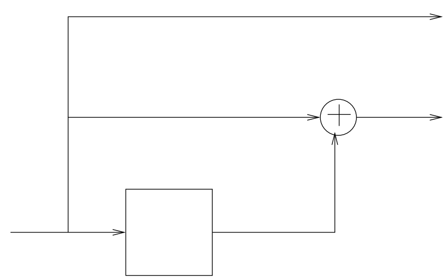

Figure 2: Encoder for Problem 9

   (a) Draw the trellis corresponding to four information digits.

   (b) Find the number of codewords represented in the trellis (or calculate the number of paths in the trellis).

   (c) Decode the received sequence $\mathbf{r} = [11; 01; 10; 00; 10]$ over a BSC.

   (d) Decode the received sequence $\mathbf{r} = [1.5, -1.1; +0.8, -2.5; -0.4, +0.4; -1.2, +0.7; +2.8, -0.9]$ assuming BPSK modulation over an AWGN channel.

10. Consider a convolutional code with generator matrix

$$G(D) = \begin{bmatrix} 1 & D & 1+D \\ 0 & 1+D+D^2+D^3 & 1+D^2+D^3 \end{bmatrix}.$$

   (a) What is the designed rate of the encoder?

   (b) Draw observer and controller canonical forms for a nonsystematic feedforward encoder.

   (c) Draw observer and controller canonical forms for a systematic encoder.

11. Let C_R be the t-error-correcting narrow-sense RS code over GF(2^m) with $n = 2^m - 1$. Let C_B be a t-error-correcting narrow-sense binary BCH code with blocklength n.

   (a) Provide parity-check matrices for C_R and C_B with elements from GF(2^m).

   (b) Write down precise expressions for probability of block error under bounded-distance decoding for both the codes over a BSC with transition probability p.

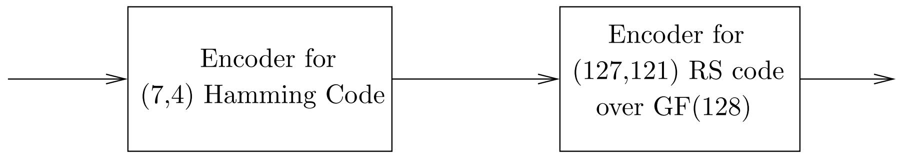

Figure 3: Encoder for Problem 12

12. (F) Consider the concatenated encoder shown in Fig. 3. Every set of 4 bits entering the encoder is first encoded using the (7,4) Hamming code. Each 7-bit codeword of the Hamming code is treated as a symbol over GF(128). Every set of 121 symbols is further encoded using the (127,121) RS code.

   (a) Determine the block-length and dimension of the overall binary code.

   (b) Consider the decoder shown in Fig. 4 over a BSC. The RS decoder is a bounded distance decoder. Syndrome decoding is used for the Hamming code. What is the error-correcting capability of the overall code under the above decoder?

Figure 4: Decoder for Problem 12b

13. Let C be a t-error-correcting RS code of length $n = 2^m - 1$ over GF(2^m).

   (a) Determine the exact burst-error-correcting capability of C in bits.

   (b) Let M codewords of C be symbol-interleaved by a row-column interleaver. Determine the burst-error-correcting capability after interleaving.

14. Let a 2-error-correcting $[8, 4]$ RS code over GF(8) be shortened to a $[5, 1]$ code over GF(8).

   (a) Write down the generator and parity-check matrices for the shortened code. Is the shortened code cyclic?

   (b) What is the minimum distance of the shortened code?

   (c) Write down the blocklength and messagelength of the binary expanded version of the shortened code. Are there higher-rate 2-error-correcting binary codes of the same blocklength?

15. Let a 2-error-correcting $[8, 4]$ RS code over GF(8) be punctured to a $[6, 4]$ code over GF(8).

   (a) Write down a systematic generator matrix for the $[8, 4]$ RS code.

   (b) Puncture any two parity symbols and write down a generator and parity-check matrices for the punctured code.

   (c) Can the punctured code be made 1-error-correcting?