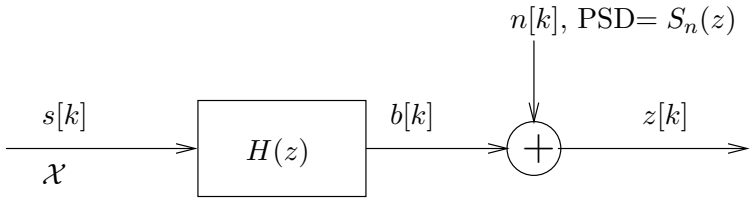
Problem Set 6

EE419: Digital Communication Systems

Consider the general ISI model shown below for all problems in this assignment.



1. Derive and compare the MSEs of ZF-LE, ZF-DFE, MMSE-LE and MMSE-DFE for the following cases with $\mathcal{X} = \{1, -1\}$. Compare with MLSD and matched-filter bound wherever possible. Constants c, d are complex. Constant N_0 is real and positive. Consider $|c| > 1$ and $|c| < 1$ separately, and assume $|d| < 1$. Assume suitable spectral factorizations, wherever necessary. Omit cases where there are more than 2 zeros or poles.

   (a) $H(z) = 1 - cz^{-1}$, $S_n(z) = N_0$.

   (b) $H(z) = 1 - cz^{-1}$, $S_n(z) = N_0(1 - cz^{-1})(1 - c^*z)$.

   (c) $H(z) = 1 - cz^{-1}$, $S_n(z) = N_0(1 - dz^{-1})(1 - d^*z)$.

   (d) $H(z) = 1 - cz^{-1}$, $S_n(z) = N_0 \dfrac{1}{1 - cz^{-1}} \dfrac{1}{1 - c^*z}$.

   (e) $H(z) = 1 - cz^{-1}$, $S_n(z) = N_0 \dfrac{1}{1 - dz^{-1}} \dfrac{1}{1 - d^*z}$.

   (f) $H(z) = \dfrac{1}{1 - cz^{-1}}$, $S_n(z) = N_0$.

   (g) $H(z) = \dfrac{1}{1 - cz^{-1}}$, $S_n(z) = N_0 \dfrac{1}{1 - cz^{-1}} \dfrac{1}{1 - c^*z}$.

   (h) $H(z) = \dfrac{1}{1 - cz^{-1}}$, $S_n(z) = N_0 \dfrac{1}{1 - dz^{-1}} \dfrac{1}{1 - d^*z}$.

   (i) $H(z) = \dfrac{1}{1 - cz^{-1}}$, $S_n(z) = N_0(1 - cz^{-1})(1 - c^*z)$.

   (j) $H(z) = \dfrac{1}{1 - cz^{-1}}$, $S_n(z) = N_0(1 - dz^{-1})(1 - d^*z)$.

2. Derive and compare the MSEs of ZF-LE, ZF-DFE, MMSE-LE and MMSE-DFE for the following cases with $\mathcal{X} = \{1, -1\}$ and $S_n(z) = N_0 > 0$. Constants c, d are complex. Consider $|c| > 1$ and $|c| < 1$ separately, and assume $|d| < 1$. Assume suitable spectral factorizations, wherever necessary. Omit cases where there are more than 2 zeros or poles.

   (a) $H(z) = \dfrac{1 - cz^{-1}}{1 - dz^{-1}}$.

   (b) $H(z) = (1 - cz^{-1})(1 - dz^{-1})$.

(c) $H(z) = \dfrac{1}{(1 - cz^{-1})(1 - dz^{-1})}$. Assume $|c| < 1$.

3. Based on the above two problems, make a table of observations (practicality, error etc.) on the various equalizers depending on the locations of poles/zeros and spectrum of noise.

4. Compute the figure of merit of MLSD and matched-filter bound for the following cases. Assume $S_n(z) = N_0 > 0$.

   (a) $\mathcal{X} = \{-1, 1\}$, $H(z) = 1 + 0.5z^{-1}$.

   (b) $\mathcal{X} = \{-1, 1\}$, $H(z) = (1 + 0.5z^{-1})(1 + 0.25z^{-1})$.

   (c) $\mathcal{X} = \{-1, 1, -j, j\}$, $H(z) = 1 + 0.5z^{-1}$.

   (d) $\mathcal{X} = \{-1, 1\}$, $H(z) = 1 - \sqrt{2}z^{-1} + z^{-2}$.