Problem Set 5

EE419: Digital Communication Systems

1. Consider the formula for capacity of an ideal band-limited AWGN channel

$$C(P, W, N_0) = W \log_2\left(1 + \frac{P}{N_0 W}\right) \approx \begin{cases} W \log_2\left(\frac{P}{N_0 W}\right), & \frac{P}{N_0 W} >> 1, \\ \frac{P \log_2 e}{N_0}, & \frac{P}{N_0 W} << 1, \end{cases},$$

where P is the signal power, W is the bandwidth and N_0 denotes noise spectral level. The two approximations are for two regimes of SNR defined as $\frac{P}{N_0 W}$.

   (a) Find W' such that $C(P, W', N_0) = 2C(P, W, N_0)$ in both regimes.

   (b) Find P' such that $C(P', W, N_0) = 2C(P, W, N_0)$ at low SNRs and $C(P', W, N_0) = W + C(P, W, N_0)$ at high SNRs.

   (c) Find N_0' such that $C(P, W, N_0') = 2C(P, W, N_0)$ at low SNRs and $C(P', W, N_0) = W + C(P, W, N_0)$ at high SNRs.

2. Capacity in bits per dimension (or spectral efficiency) expressed using SNR is given by

$$\nu_C = \frac{C}{2W} = \frac{1}{2}\log_2(1 + \text{SNR}) \approx \begin{cases} \frac{1}{2}\log_2(\text{SNR}), & \text{SNR} >> 1 \\ \frac{1}{2}(\text{SNR})(\log_2 e), & \text{SNR} << 1. \end{cases}$$

Determine the increase in SNR needed for an addition of 1 bit per dimension at high SNRs. Plot ν_C versus SNR in dB.

3. Consider the 8-PAM real baseband constellation $\{\pm 1, \pm 3, \pm 5, \pm 7\}$.

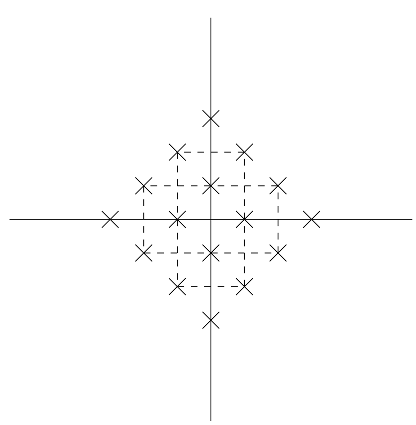
   (a) Write down the probability of symbol error in terms of Q functions.

   (b) Assign bits using Gray labeling. Find expressions for LLR of each bit, when a value r is received. Find probability of error expressions for each bit.

   (c) Assign bits using binary sequential labeling (000 to 111 from left to right). Find expressions for LLR of each bit, when a value r is received. Find probability of error expressions for each bit.

   (d) Compare the error probabilities obtained in the above two cases using suitable approximations.

4. Consider the constellation shown in the figure below.



The points on the x-axis and y-axis are at the locations ± 1 and ± 3.

(a) Assign four bits per symbol using Gray labeling.

(b) Find the decision regions and estimate the probability of symbol error.

(c) Given r_x and r_y are the values received on the I and Q channel, find an expression for the LLR of each bit.

(d) Write an expression for the probability of error of the left-most bit in your labeling.

5. Consider the channel model

$$Y = AX + N,$$

where $X \in \{\pm 1\}$ is the transmitted symbol and $N \sim N(0, \sigma^2)$ is AWGN. Sketch the received constellation, design an optimal detector and find probability of error expressions for the following cases.

(a) A is a known constant.

(b) $A \in \{\pm 1\}$ is a discrete random variable (independent of X and N) with $p = \Pr\{A = 1\}$.

(c) A is a Rayleigh-distributed continuous random variable with $f_A(a) = \frac{a}{\rho^2} e^{-a^2/(2\rho^2)}$ for $a \geq 0$.

6. In the above problem, design a soft detector, i.e. find LLR, in each case.

7. Consider the channel model

$$Y = AX + N,$$

where $X \in \{\pm 1\}$ is the transmitted symbol and $N \sim N(0, \sigma^2)$ is AWGN. Suppose that A is an unknown positive constant. How will you estimate A from observations of Y?