EE6340: Information Theory
Problem Set 7

1. *Channels with memory have higher capacity.* Consider a binary symmetric channel with $Y_i = X_i \oplus Z_i$, where $\oplus$ is mod 2 addition, and $X_i, Y_i \in \{0, 1\}$.
Suppose that $\{Z_i\}$ has constant marginal probabilities $Pr\{Z_i = 1\} = p = 1 - Pr\{Z_i = 0\}$, but that $Z_1, Z_2, ...Z_n$ are not necessarily independent. Assume that Z^n is independent of the input X^n. Let $C = 1 - H(p, 1-p)$. Show that

$$\max_{p(x_1,x_2,...x_n)} I(X_1, X_2, ..., X_n; Y_1, Y_2, ..., Y_n) \geq nC.$$

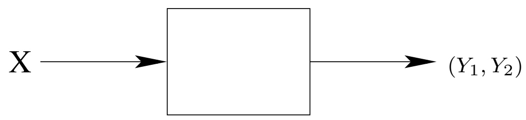
2. *Time-varying channels.* Consider a time-varying discrete *memoryless* channel.
Let $Y_1, Y_2, ...Y_n$ be conditionally independent given $X_1, X_2, ...X_n$, with conditional distribution given by $p(\mathbf{y}|\mathbf{x}) = \prod_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{p_i(y_i|x_i)}$. Let $\mathbf{X} = (\mathbf{X_1}, \mathbf{X_2}, ...\mathbf{X_n}), \mathbf{Y} = (\mathbf{Y_1}, \mathbf{Y_2}, ..., \mathbf{Y_n})$.
Find $\max_{p(x)} I(\mathbf{X}; \mathbf{Y})$.

3. *Can signal alternatives lower capacity?* Show that adding a row to a channel transition matrix does not decrease capacity.

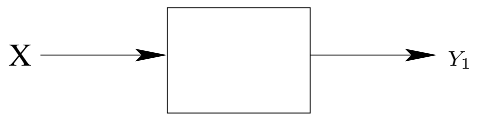
4. *A channel with two independent looks at Y.* Let Y_1 and Y_2 be conditionally independent and identically distributed given X.

   (a) Show that $I(X; Y_1, Y_2) = 2I(X; Y_1) - I(Y_1; Y_2)$.

   (b) Conclude that the capacity of the channel (1) is less than twice the capacity of the channel (2).



(1)



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