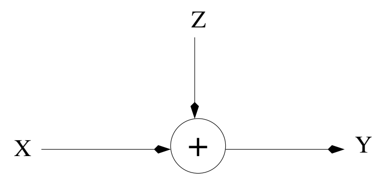
EE6340: Information Theory
Problem Set 6

1. *Preprocessing the output.* One is given a communication channel with transition probabilities $p(y|x)$ and channel capacity $C = \max_{p(x)} I(X;Y)$. A helpful statistician preprocesses the output by forming $\tilde{Y} = g(Y)$. He claims that this will strictly improve the capacity.

   (a) Show that he is wrong.

   (b) Under what conditions does he not strictly decrease the capacity?

2. *An additive noise channel.* Find the channel capacity of the following discrete memoryless channel, where $\Pr\{Z = 0\} = \Pr\{Z = a\} = \frac{1}{2}$. The alphabet for X is $\mathcal{X} = \{0, 1\}$. Assume

   that Z is independent of X.

3. *Channel capacity.* Consider the discrete memoryless channel $Y = X + Z \pmod{11}$, where

$$Z = \begin{pmatrix} 1, & 2, & 3 \\ 1/3, & 1/3, & 1/3 \end{pmatrix}$$

   and $X \in \{0, 1, ...10\}$. Assume that Z is independent of X.

   (a) Find the capacity.

   (b) What is the maximizing $p^*(x)$?

4. *Using two channels at once.* Consider two discrete memoryless channels $(X_1, p(y_1|x_1), Y_1)$ and $(X_2, p(y_2|x_2), Y_2)$ with capacities C_1 and C_2 respectively. A new channel $(X_1 \times X_2, p(y_1|x_1) \times p(y_2|x_2), Y_1 \times Y_2)$ is formed in which $x_1 \in X_1$ and $x_2 \in X_2$, are simultaneously sent, resulting in y_1, y_2. Find the capacity of this channel.

5. *The Z channel.* The Z-channel has binary input and output alphabets and transition probabilities $p(y|x)$ given by the following matrix:

$$Q = \begin{bmatrix} 1 & 0 \\ 1/2 & 1/2 \end{bmatrix} \qquad x, y \in \{0, 1\}$$

   Find the capcity of the Z-channel and the maximizing input probability distribution.