- Discuss Figures 1(a), 1(b), 2, 3(a), 3(b).

- How to achieve linear increase in capacity with 1-D codecs?
  * Question addressed in more detail in following papers

- Foschini (1996)

"Layered Space-Time Architecture for Wireless Communication in a Fading Environment when Using Multi-element Antennas".

- Figures 3(a)-(d). Outage capacity $\propto$ no. of antennas. ($n_T = n_R$).

- Space-time layering in reception
  Figure 6. (SIC+ZF), Fig. 8.

- Referred to as D-BLAST now.
  $\downarrow$
  Diagonal Bell Labs Layered Space Time approach.

- No plots/simulations: just a structure proposed here.

- Wolniansky P.W, G.J. Foschini, G.D. Golden, R.A. Valenzuela (1998)

"V-BLAST: An architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless channel".

- V-BLAST: Vertical BLAST.
  * Simpler than D-BLAST
  * Each layer thro' only one tx. antenna.
  * Results from experiments
    Figures 2 and 3.
    8×12 system + 16-QAM.
    25.9 bps/Hz
    over a 30 kHz channel.
  * Will discuss this in detail in the next lecture.

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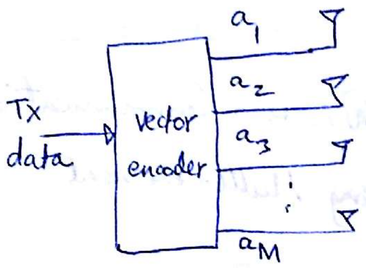
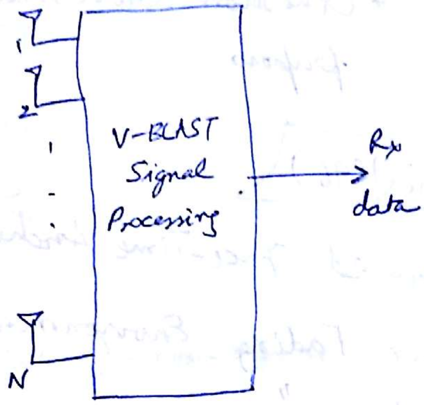
V-BLAST

Wolniansky et al. 1998, Golden et al 1999

System model:





$$\underline{a} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{pmatrix}$$

$\underline{r}_1$ received vector.

Assume $M \leq N$

$a_i \in$ QAM constellation

$\rightarrow$ $\underline{r}_1 = \underbrace{H}_{N \times M \text{ matrix}}\underline{a} + \underline{v}$ $\rightarrow$ i.i.d elements.

$\rightarrow$ Tx power from each antenna $\propto \frac{1}{M}$

Total tx power = constant independent of M.

$\rightarrow$ Transmissions in bursts of L symbols & channel quasi-static in this duration.

$\rightarrow$ Assume uncoded independent data symbols.

$\rightarrow$ H known at receiver.

V-BLAST detection

$\rightarrow$ Choose an Order in which symbols in $\underline{a}$ are detected.

Let $S = \{k_1, k_2, .., k_M\}$ an ordered set specify this order (S is a permutation of $\{1, 2, .., M\}$)

$\rightarrow$ Detection algorithm operates on $\underline{r}_1$ and progressively computes $y_{k_1}, y_{k_2}, \cdots, y_{k_M}$.

→ $\hat{a}_{k_1}, \hat{a}_{k_2}, \ldots, \hat{a}_{k_M}$ are then obtained by quantizing $y_{k_1}, y_{k_2}, \ldots, y_{k_M}$ to the nearest symbols in the QAM constellation.

Step 1

→ Compute y_{k_1} as

$$y_{k_1} = \underline{w}_{k_1}^T \underline{r}_1$$

$\underline{w}_{k_1}$ chosen such that ($(H)_{k_j}$ is the k_j^{th} column of H.)

$$\underline{w}_{k_1}^T (H)_{k_j} = \begin{cases} 0 & j > 1 \\ 1 & j = 1 \end{cases} \quad \text{—(1)}$$

(orthogonality with columns of H for users to be decoded later)

Step 2

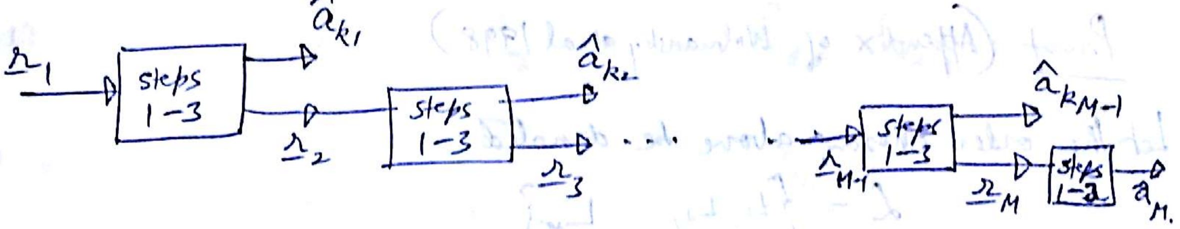
$$\hat{a}_{k_1} = Q(y_{k_1})$$ (Slicing / quantization / decision)

Step 3

$$\underline{r}_2 = \underline{r}_1 - \hat{a}_{k_1} (H)_{k_1}$$

(Remove contribution of k_1^{th} symbol in $\underline{r}_1$ to get $\underline{r}_2$).

Repeat the above steps for components $k_2, k_3, \ldots, k_M$ with progressively modified received vectors $\underline{r}_2, \underline{r}_3, \ldots, \underline{r}_M$.



→ In (1), $\underline{w}$'s are chosen as nulling vectors using ZF criterion. (MMSE criterion is another possibility).

Easy to see that $\underline{w}_{k_1}$ is the k_1^{th} row of the pseudoinverse H^+ of H.

Let $H_{\overline{k_I}}$ denote the matrix obtained by zeroing columns $k_1, k_2, \ldots, k_I$ of H. Then,

$$\boxed{\underline{w}_{k_i} \text{ is the } k_i^{th} \text{ row of } H^+_{\overline{k_{i-1}}}}.$$

(Assuming the interference cancellation is correct).

→ What is the best order $k_1, k_2, \ldots, k_M$?
(ZF case)

Post-detection SNR for k_i^{th} component

$$\rho_{k_i} = \frac{E[|a_{k_i}|^2]}{\sigma^2 \|\underline{w}_{k_i}\|^2}.$$

Need to choose k_i with least $\|\underline{w}_{k_i}\|^2$

⇒ Choose the k_i corresponding to the row of $H^+_{\overline{k_{i-1}}}$ with smallest norm.

---

k_1 → index of row with min. norm in H^+

k_2 → —— " —— $H^+_{\overline{k_1}}$

k_3 → —— " —— $H^+_{\overline{k_2}}$

⋮

k_{M-1} → —— " —— $H^+_{\overline{k_{M-2}}}$

k_M.

Proof (Appendix of Wolniansky et al 1998)

Let the order chosen above be denoted

$$\mathcal{L} = \{L_1, L_2, \ldots, L_M\}.$$

Consider any other ordering

$$Q = \{Q_1, Q_2, \ldots, Q_M\}$$

Show that post-detection SNR for Q is worse than that for $\mathcal{L}$.

Constraint set of an element of an ordering Q:

for Q_i it is $\underbrace{\{Q_{i+1}, \ldots, Q_M\}}_{\text{components of } \underline{a} \text{ that have not been detected and cancelled.}}$

Use these 2 results:

(1) ~~Let~~ A & B be 2 distinct orderings. If $A_k = B_k$ and the constraint sets of A_k and B_k consist of identical elements (regardless of their order) then $\rho_{A_k} = \rho_{B_k}$.

(2) Let A & B be 2 distinct orderings. If $A_k = B_k$ and the constraint set of $A_k \subset B_k$, then $\rho_{A_k} \geq \rho_{B_k}$. (Using Cauchy-Schwarz inequality).

$$\mathcal{L} = \{L_1, L_2, \ldots, L_d, \ldots, L_M\}$$

$$Q = \{Q_1, Q_2, \ldots, Q_d, \ldots Q_r, Q_M\}$$

$r = L_d$.

Let this be the first index at which $\mathcal{L}$ and Q are different.

Find r such that $Q_r = L_d$.

$$\text{Let } Q' = \{Q_1, Q_2, \ldots, Q_{d-1}, Q_r, Q_d, \ldots, Q_M\}$$

insert Q_r here & remove from original location.

From results above,

$$(1) \Rightarrow \quad \rho_{Q_1} = \rho_{Q'_1},\ \rho_{Q_2} = \rho_{Q'_2}, \ldots, \rho_{Q_{d-1}} = \rho_{Q'_{d-1}}$$

$$(1) \text{ or } (2) \Rightarrow \quad \rho_{Q_{d+1}} \leq \rho_{Q'_{d+1}}, \ldots, \rho_{Q_M} \leq \rho_{Q'_M}.$$

if $Q_k = Q'_k$ & constraint sets are equal.

if constraint sets are unequal.

$\rho_{Q_d} \leq \rho_{Q'_d}$ since $\rho_{Q'_d} = \rho_{L_d}$ and ρ_{L_d} is the locally optimum choice.

All this $\Rightarrow$ we can repeat this process to get $\mathcal{L}$ as the best ordering.

Algorithm.

Initialization: $G_1 = H^{+}$

$i = 1$

Recursion:

$$\left\{\begin{array}{l}
k_i = \arg\min\limits_{j \notin \{k_1, \ldots, k_{i-1}\}} \|(G_i)_j\|^2 \quad \leftarrow j^{th} \text{ row of } G_i \\[2ex]
\underline{w}_{k_i}^T = (G_i)_{k_i} \leftarrow k_i^{th} \text{ row of } G_i \\[2ex]
y_{k_i} = \underline{w}_{k_i}^T \underline{r}_i \\[2ex]
\hat{a}_{k_i} = Q(y_{k_i}) \\[2ex]
\underline{r}_{i+1} = \underline{r}_i - \hat{a}_{k_i}(H)_{k_i} \\[2ex]
G_{i+1} = H_{\overline{k_i}}^{+} \\[2ex]
i = i+1.
\end{array}\right.$$

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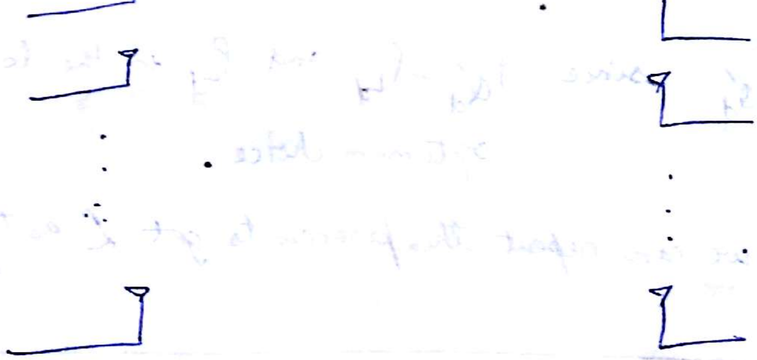
"Space-Time Codes for High Data Rate Wireless Communication: Performance criterion and Code Construction"

V. Tarokh, N. Seshadri, A.R. Calderbank, Trans. I.T., Mar 1998.



n antennas

m antennas.

c_t^i : Transmit signal from i^{th} tx. antenna during t^{th} slot

d_t^j : Received signal at j^{th} rx antenna during t^{th} slot.

$\alpha_{i,j}$: Fading coefficient for channel from i^{th} tx. antenna to j^{th} rx. antenna.

$$d_t^j = \sum_{i=1}^{n} \alpha_{i,j}\, c_t^i \sqrt{E_s} + \eta_t^j \qquad \text{for } j=1,\ldots,m.$$

$\eta_t^j \hookrightarrow CN(0, N_0)$ i.i.d.

Average energy of constellation (from which c_t^i is chosen) = 1.

$\boxed{\alpha_{i,j} : \text{constant for one frame and varies from frame to frame (quasi-static flat fading)}}$

1 (Quasistatic fading)

In SISO case, we talked about reliable message transmission using codewords of length n. Now, we have 2 dimensions (antennas, time). $\Rightarrow$ We send messages using <u>code matrices</u>.

(code matrix n tx antennas & l time slots)

$$\begin{bmatrix} c_1^1 & c_2^1 & \cdots & c_l^1 \\ c_1^2 & c_2^2 & & \vdots \\ \vdots & \vdots & & \vdots \\ c_1^n & c_2^n & \cdots & c_l^n \end{bmatrix}_{n\times l} \triangleq C$$

$$(d_1^j\ d_2^j \ldots d_l^j) = \underbrace{[\alpha_{1,j}\ \alpha_{2,j} \ldots \alpha_{n,j}]}_{\triangleq \Omega_j} \begin{bmatrix} \\ \text{Code matrix} \\ \\ \end{bmatrix} + (\eta_1^j\ \eta_2^j \ldots \eta_l^j)$$

received vector at j^{th} rx. antenna over l time slots

assumed to be constant during the frame.

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Let $\underline{c} = (c_1^1\ c_1^2 \cdots c_1^n\ c_2^1\ c_2^2 \cdots c_2^n \cdots\ c_l^1\ c_l^2 \cdots c_l^n)$

(Corresponding code matrix C)

$\underline{e} = (e_1^1\ e_1^2 \cdots e_1^n\ e_2^1\ e_2^2 \cdots e_2^n \cdots\ e_l^1\ e_l^2 \cdots e_l^n)$

(corresponding code matrix E).

Assume ML decoding at the receiver.

Assume perfect CSI at the receiver.

Pairwise error probability

$$P(\underline{c} \to \underline{e} / \{\alpha_{i,j}\}) = Q\left(\frac{\sqrt{E_s}\, d(\underline{c}, \underline{e})}{2 \cdot \sqrt{N_0}}\right)$$

$$\leq \exp\left(-\frac{E_s}{4N_0} d^2(\underline{c}, \underline{e})\right).$$

$$d^2(\underline{c}, \underline{e}) = \sum_{j=1}^{m} \| [\Omega_j B(\underline{c}, \underline{e})] \|^2$$ ($m \times l$ coordinates received distance in ml dimension)

Where $B(\underline{c}, \underline{e})$ is the codeword difference matrix.

$$\begin{bmatrix} c_1^1 - e_1^1 & c_2^1 - e_2^1 & \cdots & c_l^1 - e_l^1 \\ \vdots & & & \vdots \\ c_1^n - e_1^n & c_2^n - e_2^n & \cdots & c_l^n - e_l^n \end{bmatrix}_{n \times l}$$

$$d^2(\underline{c}, \underline{e}) = \sum_{j=1}^{m} \Omega_j \underbrace{B(\underline{c}, \underline{e})\, B(\underline{c}, \underline{e})^*}_{\triangleq A(\underline{c}, \underline{e})\ (n \times n \text{ matrix})} \Omega_j^*$$

$$= \sum_{j=1}^{m} \Omega_j A(\underline{c}, \underline{e}) \Omega_j^*$$

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$$P(\underline{c} \to \underline{e} / \{\alpha_{i,j}\}) \leq \exp\left(-\frac{E_s}{4N_0}\sum_{j=1}^{m} \Omega_j A(\underline{c},\underline{e})\Omega_j^*\right)$$

$$= \prod_{j=1}^{m} \exp\left(-\frac{E_s}{4N_0}\Omega_j A(\underline{c},\underline{e})\Omega_j^*\right).$$

$A(\underline{c},\underline{e}) \geq 0 \Rightarrow \quad A(\underline{c},\underline{e}) = V^* D V^*$

where V is unitary $V^* = [v_1 \; v_2 \; \ldots \; v_n]$

$D \geq 0$ diagonal $\to \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$

$$P(\underline{c} \to \underline{e} \,|\, \{\alpha_{i,j}\}) \leq \prod_{j=1}^{m} \exp\left(-\frac{E_s}{4N_0}(\Omega_j V^*) D (V \Omega_j^*)\right)$$

$\left[\text{Let } \Omega_j V^* = (\beta_{1,j} \; \cdots \; \beta_{n,j}) \triangleq \underline{\beta}_j\right]$

$$= \prod_{j=1}^{m} \exp\left(-\frac{E_s}{4N_0}\underline{\beta}_j D \underline{\beta}_j^*\right)$$

$$= \prod_{j=1}^{m} \exp\left(-\frac{E_s}{4N_0}\sum_{i=1}^{n} |\beta_{i,j}|^2 \lambda_i\right)$$

Rayleigh fading case

$\alpha_{i,j}$ i.i.d. $CN(0,1)$

Ricean fading case

$\alpha_{i,j}$ independent $CN(E(\alpha_{i,j}), 1)$

$K^j = [E\alpha_{1,j} \; E\alpha_{2,j} \; \ldots \; E\alpha_{n,j}]$.

$K^j \cdot v_i = E[\beta_{i,j}]$

$K_{i,j} \triangleq |E\beta_{i,j}|^2.$

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$$P(\underline{c} \to \underline{e}) \le E_{\beta}\left[\prod_{j=1}^{m} \exp\left[\left(-\frac{E_s}{4N_0}\right)\sum_{i=1}^{n} \lambda_i |\beta_{i,j}|^2\right]\right]$$

$$= \text{(Rayleigh)} \left[\frac{1}{\prod_{i=1}^{n}\left(1+\lambda_i \frac{E_s}{4N_0}\right)}\right]^m$$

$$\text{(Rician)} \quad \prod_{j=1}^{m}\left[\prod_{i=1}^{n} \frac{1}{\left(1+\lambda_i \frac{E_s}{4N_0}\right)} \exp\left(-\frac{K_{i,j}\frac{E_s}{4N_0}\lambda_i}{1+\frac{E_s}{4N_0}\lambda_i}\right)\right]$$

$$\left(\text{for high SNR} \approx \prod_{j=1}^{m}\left[\prod_{i=1}^{n} \frac{1}{\left(1+\lambda_i \frac{E_s}{4N_0}\right)} \exp(-K_{i,j})\right]\right)$$

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Diversity advantage:

→ Exponent of SNR in the denominator of the expression for $P(\underline{c} \to \underline{e})$.

Div. advantage $= mr$ where r is rank of $A(\underline{c}, \underline{e})$.

In order to maximize diversity advantage, we need to

maximize $\min_{\substack{\underline{c},\underline{e} \\ \underline{c} \ne \underline{e}}} \text{rank}(A(\underline{c}, \underline{e}))$.

(RANK Criterion)

Coding advantage:

→ Measure of gain over an uncoded system operating with the same diversity advantage.

$\left(\prod_{i=1}^{r} \lambda_i\right)^{1/r}$ in the Rayleigh case.

Need to maximize the min of det $A(\underline{c}, \underline{e})$ over $\underline{c}, \underline{e}$ with $\underline{c} \ne \underline{e}$.

(DETERMINANT Criterion)

Rician case : (high SNR)

— Same RANK Criterion as Rayleigh case.

— Same DETERMINANT criterion as Rayleigh case but coding gain is now

$$(\lambda_1 \lambda_2 \cdots \lambda_r)^{1/r} \left[\prod_{j=1}^{m} \prod_{i=1}^{r} \exp(-K_{i,j})\right]^{1/rm}.$$

(2) Correlated fading coefficients case : (Rayleigh) (can also be extended to Rician).

$\alpha_{i,j}$ zero-mean CN with variance 0.5 per dim but could be dependent.

Define $\Omega = (\Omega_1 \ \Omega_2 \ \cdots \ \Omega_m)_{1 \times mn}$

$$Y(\underline{c},\underline{e}) = \begin{bmatrix} A(\underline{c},\underline{e}) & 0 & & 0 \\ 0 & A(\underline{c},\underline{e}) & & \\ & & \ddots & 0 \\ 0 & & 0 & A(\underline{c},\underline{e}) \end{bmatrix}_{mn \times mn}$$

$$P(\underline{c} \to \underline{e} \mid \Omega) \leq \exp\left(-\Omega Y(\underline{c},\underline{e}) \Omega^* \frac{E_s}{4N_0}\right).$$

Let $E[\Omega^* \Omega] \triangleq \Theta$ (correlation matrix of fading coefficients).

We can write $\Omega = \underline{\nu} C^*$ where $\underline{\nu}$ indep CN $(\underline{0}, I)$ & $CC^* = \Theta$.

$$P(\underline{c} \to \underline{e} / \Omega) \leq \exp\left(-\underline{\nu} \, \underbrace{C^* Y(\underline{c},\underline{e}) C} \, \underline{\nu} \frac{E_s}{4N_0}\right).$$

→ Need to apply rank and determinant criterion to $C^* Y(\underline{c},\underline{e}) C$ instead of $A(\underline{c},\underline{e})$.

If C is full rank (i.e. Θ is full rank) rank$(C^* Y(\underline{c},\underline{e}) C)$ is maximized by maximizing rank$(Y(\underline{c},\underline{e}))$ which is nothing but m rank$(A(\underline{c},\underline{e}))$

$\Rightarrow$ Same as RANK criterion for i.i.d case.

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→ Coding gain

$\det(C^* \Psi(c,e) C)$ must be maximized.

$\Rightarrow \det(\Theta) \cdot \det(X(c,e)) = \det(\Theta) \left[\det(A(c,e))\right]^m$

should be maximized.

$\Rightarrow$ Same determinant criterion.

But coding gain has an extra factor $[\det(\Theta)]^{1/m}$.

③ <u>Rapid fading case:</u>

<u>Model:</u>

$$d_t^j = \sum_{i=1}^{n} \underbrace{\alpha_{i,j}(t)}_{\substack{\text{i.i.d for each } t \text{ as well.} \\ \text{+ Rayleigh model}}} c_t^i \sqrt{E_s} + n_t^j$$

$$P(c \to e \mid \{\alpha_{i,j}(t)\}) \le \exp\left(-d^2(c,e) \cdot \frac{E_s}{4N_0}\right).$$

Let $\Omega_j(t) = [\alpha_{1,j}(t) \;\; \alpha_{2,j}(t) \;\; \ldots \;\; \alpha_{n,j}(t)]$.

$C(t) = [C_{pq}(t)]$ where $C_{pq} = (c_t^p - e_t^p)(c_t^q - e_t^q)^*$.

$$d^2(c,e) = \sum_{j=1}^{m} \sum_{t=1}^{l} \Omega_j(t) \underbrace{C(t)}_{\substack{\text{Hermitian} \\ \text{rank 1 or rank 0.}}} \Omega_j^*(t)$$

$$\Omega_j(t) C(t) \Omega_j^*(t) = \sum_{i=1}^{n} |\beta_{i,j}(t)|^2 D_{ii}(t)$$

$$\Rightarrow P(c \to e) \le \prod_{i,t} \left(1 + D_{ii}(t) \frac{E_s}{4N_0}\right)^{-m}$$

$$D_{ii}(t) = \begin{cases} 0 & \text{for } n-1 \text{ elements} \\ \|c_t - e_t\|^2 & \text{for the } n^{th} \text{ element.} \end{cases}$$

$$P(c \to e) \leq \prod_{t=1}^{l} \left(1 + \|c_t - e_t\|^2 \frac{E_s}{4N_0}\right)^{-m}.$$

$$= \prod_{t \in \mathcal{V}(c,e)} \left(1 + \|c_t - e_t\|^2 \frac{E_s}{4N_0}\right)^{-m}$$

where $\mathcal{V}(c,e)$ denotes the set of time instances where $\|c_t - e_t\| \neq 0$.

$\Rightarrow$ Div. gain $= |\mathcal{V}(c,e)|\, m$.

Distance criterion:

for div. gain vm, any 2 codewords c, and e strings $c_t^1 c_t^2 \cdots c_t^n$ & $e_t^1 e_t^2 \cdots e_t^n$ should differ for at least v values of t.

Product criterion:

Maximize the $\min\limits_{\substack{c,e \\ c \neq e}} \prod\limits_{t \in \mathcal{V}(c,e)} \|c_t - e_t\|^2$.

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Note that we need $\boxed{l \geq n}$ in the quasistatic case to get $A(c,e)$ to be rank n.

Sec III

→ Code design: Space-time trellis codes

→ Comparison with outage probability and outage capacity.

Discuss. Figs. 10–13.

and compare with Figs. 14+15.

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→ How to read the trellis? Fig 4.

→ r-space time trellis codes have div. advantage r.

→ Trade-off between rate, diversity, constellation size, trellis complexity.

1. Given $n \times m$ system, Rician model, rm div. advantage, Constellation Q with 2^b elements,

$$R \leq \frac{\log_2 [A_{2^b}(n, r)]}{l}.$$

2. Div. adv $= mn \Rightarrow$ rate at most b b/s/Hz.

3. Constraint length of an r-space-time trellis code at least $r-1$

4. Trellis complexity at least $2^{b(r-1)}$.

   (b : transmission rate
   r : div. advantage).

→ Can be complex (no. of states exponential in b and r).

→ Smart-Greedy codes.

* Achieves max. div. for quasistatic fading
* Also achieves higher gain for fast-fading case.
* See Figs. 23–26.

→ Discussion on block vs trellis codes. (brief).

$\boxed{\text{"A Simple Transmit Diversity Technique for Wireless Communications", S. M. Alamouti JSAC Oct 1998.}}$

→ Space-time block code for 2×1 case

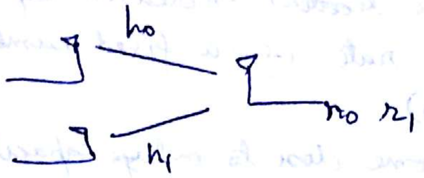
→ Can also be easily used for $2 \times M$ case.

→ Achieves $2M$ div. advantage.

→ Decoding is simple.

* Linear in number of symbols (since symbols decouple)

→



$$s_0, s_1 \longrightarrow \begin{bmatrix} s_0 & -s_1^* \\ s_1 & s_0^* \end{bmatrix}$$

$$r_0 = h_0 s_0 + h_1 s_1 + n_0$$

$$r_1 = -h_0 s_1^* + h_1 s_0^* + n_1$$

$$\Rightarrow r_1^* = -h_0^* s_1 + h_1^* s_0 + n_1^*$$

$$\begin{bmatrix} r_0 \\ r_1^* \end{bmatrix} = \underbrace{\begin{bmatrix} h_0 & h_1 \\ h_1^* & -h_0^* \end{bmatrix}}_{H} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + \underbrace{\begin{bmatrix} n_0 \\ n_1^* \end{bmatrix}}_{\text{same dist. as } \begin{bmatrix} n_0 \\ n_1 \end{bmatrix}}$$

orthogonal columns.

$$H^H \begin{bmatrix} r_0 \\ r_1^* \end{bmatrix} = \begin{bmatrix} \diagdown \end{bmatrix} \begin{bmatrix} s_0 \\ s_1 \end{bmatrix} + H^H \begin{bmatrix} n_0 \\ n_1^* \end{bmatrix}$$

s_0 & s_1 can now be <u>independently decoded</u>.

→ Extends to $2 \times M$ case easily

Combine across M antennas for each symbol s_0, s_1).

→ Rest of paper as self-study.

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"Space-Time Block Codes from Orthogonal Designs"

V. Tarokh, H. Jafarkhani, A. R. Calderbank, Trans-IT July 1999

→ Space-time trellis codes

* Decoding complexity (measured by the number of trellis states in the decoder) increases exponentially with transmission rate (for a fixed number of transmit antennas).
* Perform well, can come close to outage capacity

→ Alamouti code

* Much less decoding complexity
* Orthogonal structure ⇒ linear processing at decoder

→ Can we get orthogonal designs for more than two transmit antennas?

In this paper

* Real orthogonal designs
  Linear Processing orthogonal designs
  Generalized real orthogonal designs

* Complex orthogonal designs
  Complex linear processing orthogonal designs
  Generalized complex orthogonal designs.

Real orthogonal designs

– A real orthogonal design of size n is an $n \times n$ orthogonal matrix with entries the indeterminates $\pm x_1, \pm x_2, \ldots, \pm x_n$.

– An orthogonal design exists if and only if $n = 2, 4$ or 8. (reference [5])

— Given an orthogonal design $\mathcal{O}$, one can negate certain columns of $\mathcal{O}$ to arrive at another orthogonal design where all the entries of the first row have +ve signs.

— By permuting the columns, we can make sure that the first row of $\mathcal{O}$ is $x_1, x_2, \ldots, x_n$.

$\Rightarrow$ We may assume w.l.o.g. that $\mathcal{O}$ has this property.

$\underline{n=2}$

$$\begin{pmatrix} x_1 & x_2 \\ -x_2 & x_1 \end{pmatrix}$$

$\underline{n=4}$

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{pmatrix}$$

$\underline{n=8}$

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ -x_2 & x_1 & x_4 & -x_3 & x_6 & -x_5 & -x_8 & x_7 \\ -x_3 & -x_4 & x_1 & x_2 & x_7 & x_8 & -x_5 & -x_6 \\ -x_4 & x_3 & -x_2 & x_1 & x_8 & -x_7 & -x_6 & -x_5 \\ -x_5 & -x_6 & -x_7 & -x_8 & x_1 & x_2 & x_3 & x_4 \\ -x_6 & x_5 & -x_8 & +x_7 & -x_2 & x_1 & -x_4 & x_3 \\ -x_7 & x_8 & x_5 & -x_6 & -x_3 & x_4 & x_1 & -x_2 \\ -x_8 & -x_7 & x_6 & x_5 & -x_4 & -x_3 & x_2 & x_1 \end{bmatrix}$$

Coding scheme

→ Real signal constellation A with 2^b elements

Diversity order target $= mn$.

From [Tarokh 1998], max. transmission rate $= 6$ bps/Hz.

→ Use an $n \times n$ orthogonal design

→ In slot 1, nb bits arrive at encoder and select constellation signals. $s_1, s_2, \ldots, s_n$.

→ matrix $\mathcal{C} = \mathcal{O}(s_1, s_2, \ldots, s_n)$ setting $x_i = s_i$.

→ C_{ti}, $i = 1, 2, \ldots, n$ are transmitted simultaneously from transmit antennas $1, 2, \ldots, n$ in time slot t.

Thm 3.2.1 Div. order $= mn$.

Pf: $\mathcal{O}(\tilde{s}_1, \ldots, \tilde{s}_n) - \mathcal{O}(s_1, \ldots, s_n)$ should be full rank for any $(\tilde{s}_1, \ldots, \tilde{s}_n) \neq (s_1, \ldots, s_n)$. [RANK criterion].

$$\mathcal{O}(\tilde{s}_1, \ldots, \tilde{s}_n) - \mathcal{O}(s_1, \ldots, s_n) = \mathcal{O}(\tilde{s}_1 - s_1, \ldots, \tilde{s}_n - s_n)$$

$$\det(\mathcal{O}\mathcal{O}^T)^{1/2} = \left[\sum_i x_i^2\right]^{n/2}$$

$$\det\left(\mathcal{O}(\tilde{s}_1 - s_1, \ldots, \tilde{s}_n - s_n)\right) = \left[\sum_i |\tilde{s}_i - s_i|^2\right]^{n/2} \neq 0.$$

$\Rightarrow$ div. order $= mn$.

Decoding algorithm:

→ Rows of $\mathcal{O}$ are all permutations of the first row of $\mathcal{O}$ with possibly different signs.

Let $\epsilon_1, \ldots, \epsilon_n$ denote the permutations corresponding to these rows

Let $\delta_k(i)$ denote the sign of x_i in the k^{th} row of $\mathcal{O}$.

Then $\epsilon_k(p) = q$ means that x_p is the $(k, q)^{th}$ element of $\mathcal{O}$ upto a sign change.

Scanned by CamScanner

ML
$$\min \sum_{t=1}^{l} \sum_{j=1}^{m} \left| r_t^j - \sum_{i=1}^{n} \alpha_{i,j} c_t^i \right|^2$$

Here $l = n$. ($n \times n$ code)

Orthogonal design $\Rightarrow$

equivalent to min $\sum_{i=1}^{n} S_i$

where
$$S_i = \left| \sum_{t=1}^{n} \sum_{j=1}^{m} r_t^j \alpha^*_{\epsilon_t(i),j} \delta_t(i) - s_i \right|^2 + \left(-1 + \sum_{k,l} |\alpha_{k,l}|^2 \right) |s_i|^2$$

Each S_i depends only on s_i, $\{r_t^j\}$, $\{\alpha_{i,j}\}$ & the structure of $\mathcal{O}$.

$$\hat{s}_i = \arg\min_{s \in A} |R_i - s|^2 + \left(-1 + \sum_{k,l} |\alpha_{k,l}|^2 \right) |s|^2$$

where $R_i = \sum_{t=1}^{n} \sum_{j=1}^{m} r_t^j \underbrace{\alpha^*_{\epsilon_t(i),j}}_{\text{Channel coeff } t^{th} \text{ tx antenna} \to j^{th} \text{ receive antenna } (s_i \text{ sent from } t^{th} \text{ antenna})} \underbrace{\delta_t(i)}_{\text{sign of } x_i \text{ in } t^{th} \text{ row}}$

$\Rightarrow$ <u>Linear combining + individual decoding of s_i's</u>

Linear Processing Orthogonal Designs

Orthogonal designs $\to$ Max possible rate at full diversity
$\to$ Simple ML decoding

Both these properties are preserved even if linear processing is allowed at transmitter.

An $n \times n$ matrix $\mathcal{E}$ in variables $x_1, x_2, \ldots, x_n$ such that:
(1) entries are real linear combinations of variables $x_1, x_2, \ldots, x_n$
(2) $\mathcal{E}^T \mathcal{E} = D$ where D is diagonal with

$$D_{ii} = l_1^i x_1^2 + l_2^i x_2^2 + \cdots + l_n^i x_n^2$$

where $l_1^i = l_2^i = \cdots = l_n^i > 0$.

Thm 3.4.1

A linear processing orthogonal design $\mathcal{E}$ in variables $x_1, x_2, \ldots, x_n$ exists if and only if there exists a linear processing orthogonal design $\mathcal{L}$ such that

$$\mathcal{L}\mathcal{L}^T = \mathcal{L}^T\mathcal{L} = (x_1^2 + x_2^2 + \cdots + x_n^2)I.$$

Pf:

Let $\mathcal{E} = x_1 A_1 + \cdots + x_n A_n$ be a linear processing orthogonal design.

and let $\mathcal{E}^T\mathcal{E} = x_1^2 D_1 + \cdots + x_n^2 D_n$

where D_i are diagonal, full-rank.

$$\mathcal{E}^T\mathcal{E} = \left(\sum_{i=1}^{n} x_i A_i\right)^T \left(\sum_{j=1}^{n} x_j A_j\right)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j A_i^T A_j$$

$$= \sum_{i=1}^{n} x_i^2 A_i^T A_i + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} x_i x_j A_i^T A_j$$

$$\Rightarrow A_i^T A_i = D_i \quad i = 1, 2, \ldots, n$$

$$A_i^T A_j = -A_j^T A_i \quad 1 \le i < j \le n.$$

Define $U_i = A_i D_i^{-1/2}$

Then, $U_i^T U_i = I \quad i = 1, \ldots, n$

$U_i^T U_j = -U_j^T U_i \quad 1 \le i < j \le n$

$\Rightarrow \mathcal{L} = x_1 U_1 + \cdots + x_n U_n$ is a linear processing orthogonal array

with $\mathcal{L}\mathcal{L}^T = \mathcal{L}^T\mathcal{L} = (x_1^2 + x_2^2 + \cdots + x_n^2)I$

Using this thm., we restrict ourselves to linear processing orthogonal designs $\mathcal{L}$ that satisfy $\mathcal{L}\mathcal{L}^T = \mathcal{L}^T\mathcal{L} = (x_1^2 + \cdots + x_n^2) I$.

Hurwitz-Radon Theory:

- A set of $n \times n$ real matrices $B_1, B_2, \ldots, B_k$ is called a size k Hurwitz-Radon family of matrices if

$$B_i^T B_i = I$$
$$B_i^T = -B_i \qquad i = 1, \ldots, k$$
$$\& \quad B_i B_j = -B_j B_i \qquad 1 \le i < j \le k.$$

- Thm: Let $n = 2^a b$ where b is odd and $a = 4c + d$ with $0 \le d < 4$ and $0 \le c$.
  Any Hurwitz-Radon family of $n \times n$ matrices contains strictly less than $\rho(n) = 8c + 2^d$ matrices. Furthermore $\rho(n) \le n$. A Hurwitz-Radon family containing $n-1$ matrices exists if and only if $n = 2, 4$ and 8.

- $A = [a_{ij}]_{p \times q}$ $\quad A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \cdots & a_{1q}B \\ \vdots & & & \vdots \\ a_{p1}B & \cdots & & a_{pq}B \end{bmatrix}$

- Integer matrix: if all its entries $\in \{-1, 0, 1\}$.

- Lemma: For any n there exists a Hurwitz-Radon family of matrices of size $\rho(n) - 1$ whose members are integer matrices.

- Theorem: A linear processing orthogonal design of size $n \ge 2$ exists if and only if $n = 2, 4, 8$.

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Generalized Real Orthogonal Designs
(non-square matrices)

→ A generalized orthogonal design $\mathcal{G}$ of size n is a $p \times n$ matrix with entries $0, \pm x_1, \pm x_2, \ldots, \pm x_k$ such that $\mathcal{G}^T \mathcal{G} = D = (x_1^2 + x_2^2 + \cdots + x_k^2) I_n$.

(need $p \geq n$).

→ Constellation : $\mathcal{A}$ with 2^b symbols

→ $\frac{kb}{p}$ throughput

→ Rate $R = \frac{k}{p}$ $(R = 1$ will be called full rate)

Goal: construct high-rate linear processing orthogonal designs with low decoding complexity and full diversity order.

(Given R and n, minimize p.) for memory requirements/delay.

→ For a given R, n, define $A(R,n)$ to be the minimum number p such that there exists a $p \times n$ generalized orthogonal design with rate at least R.

* If no such design exists, $A(R,n) = \infty$.
* If a design attains $A(R,n)$, it is delay-optimal.

Fundamental question: value of $A(R,n)$?

specifically, $A(1,n)$.

→ Construction I:

Let $X = (x_1, x_2, \ldots, x_p)$

and $n \leq \rho(p)$.

$\xrightarrow{\text{Lemma 3.5.1}}$ $\{A_1, A_2, \ldots, A_{\rho(p)-1}\}$ $p \times p$ Hurwitz-Radon integer matrices.

Let $A_0 = I$

j^{th} column $= A_{j-1} X^T$

$\mathcal{G} = \left[\quad \right]_{p \times n}$ with p indeterminate variables

i.e. $\mathcal{G} = [A_0 X^T \;\; A_1 X^T \;\; \cdots \;\; A_{n-1} X^T] = (g_0 \; g_1 \cdots g_{n-1})$

$$(\mathcal{G}^T \mathcal{G})_{ij} = X A_i^T A_j X^T = \begin{cases} X X^T = \sum_{i=1}^{p} x_i^2 & i = j \\ X A_i^T A_j X^T = 0 & i \neq j \end{cases}$$

$\Rightarrow \mathcal{G}^T \mathcal{G}$ is a full-rate $\left(R = \frac{k}{p} = \frac{p}{p} = 1\right)$ real space-time block code. (orthogonal design).

→ Thm: $\boxed{A(1,n)}$ = smallest p such that $n \le \rho(p)$.
i.e $\boxed{\min_{n \le \rho(p)} p}$

→ Corollary: For any R, $A(R,n) < \infty$.

→ Corollary: $A(1,n) = \min \left(2^{4c+d}\right)$ ← power of 2.
where minimization is over the set $\{c, d \mid 0 \le c, \; 0 \le d < 4, \; 8c + 2^d \geq n\}$.

$p = 2^a b$, b odd
$a = 4c + d$
$0 \le d < 4$
$0 \le c$.
$\rho(p) = 8c + 2^d$.

$A(1,2) = 2$
$A(1,3) = 4$
$A(1,4) = 4$
$A(1,5) = 8$
$A(1,6) = 8$
$A(1,7) = 8$
$A(1,8) = 8$

$\Rightarrow$ Orthogonal designs are delay optimal.

Thm 4.1.3: Designs $\mathcal{G}_3, \mathcal{G}_5, \mathcal{G}_6, \mathcal{G}_7$ (Full rate & delay optimal).

→ Removing x columns of an orthogonal design results in another orthogonal design that can be used to design an STBC with x transmit antennas lesser. If the original design is delay optimal, the new design is also delay optimal.

Separate decodability of symbols of a STBC from generalized orthogonal designs (Jafarkhani book 2005) p75-76

in (48) $\mathcal{G} = \sum_{k=1}^{k} x_k E_k$ — $p\times n$ matrices.

$$\mathcal{G}^T\mathcal{G} = \left(\sum_{i=1}^{k} x_i E_i^T\right)\left(\sum_{j=1}^{k} x_j E_j\right)$$

$$= \sum_{i=1}^{k}\sum_{j=1}^{k} (E_i^T E_j)(x_i x_j)$$

$$= \sum_{i=1}^{k} x_i^2 I_n$$

$$\Rightarrow \left.\begin{array}{l} E_i^T E_i = I_n \\ \& \quad E_i^T E_j + E_j^T E_i = 0 \quad j\neq i \end{array}\right\} \text{ — } (*)$$

→ Received vector at antenna j:

$$\underbrace{(r_1^j \; r_2^j \; \ldots \; r_p^j)}_{r_j} = \underbrace{(\alpha_{1,j} \; \alpha_{2,j} \; \ldots \; \alpha_{n,j})}_{h_j\,(1\times n)} \left(\mathcal{G}^T \right) + \underbrace{(\eta_{1,j} \; \eta_{2,j} \; \ldots \; \eta_{p,j})}_{\text{additive noise } \eta_j}$$

channel coeff. vector (constant for the whole block: quasi-static)

$n\times p$ matrix codeword.

→ Received matrix:

$$R_{m\times p} = \underset{m\times n}{H}\cdot \underset{n\times p}{\mathcal{G}^T} + \underset{m\times p}{N}$$

$$\cancel{R} = H\sum_{i=1}^{k} x_i E_i^T + N$$

$$\cancel{R = \ldots}$$

$$\cancel{= \sum_{i=1}^{k} s_i \Omega_i + N}$$

$$\underline{r}_j = \underline{h}_j \mathcal{G}^T + \underline{n}_j$$

$$= \sum_{i=1}^{k} s_i \underbrace{(\underline{h}_j E_i^T)}_{1\times p} + \underline{n}_j$$

$$= \begin{bmatrix} s_1 & s_2 & \cdots & s_k \end{bmatrix} \underbrace{\begin{bmatrix} \overbrace{- \; \underline{h}_j E_1^T \; -}^{\triangleq \Omega_1} \\ - \; \underline{h}_j E_2^T \; - \\ \vdots \\ - \; \underline{h}_j E_k^T \; - \end{bmatrix}}_{\triangleq \Omega} + \underline{n}_j$$

$$= \begin{bmatrix} s_1 & s_2 & \cdots & s_k \end{bmatrix} \begin{bmatrix} - \; \Omega_1 \; - \\ - \; \Omega_2 \; - \\ \vdots \\ - \; \Omega_k \; - \end{bmatrix} + \underline{n}_j$$

$$(\Omega \cdot \Omega^T)_{p,q} = \underline{h}_j E_p^T \cdot (\underline{h}_j E_q^T)^T = \underline{h}_j \underbrace{E_p^T E_q}_{} \underline{h}_j^T$$

$(p,q)^{th}$ element of $\Omega \cdot \Omega^T$

We know from $(*)$.

$$E_p^T E_q = \begin{cases} I_n & p = q \\ 0 & p \neq q. \end{cases}$$

$$\Rightarrow \Omega \cdot \Omega^T = \begin{bmatrix} \underline{h}_j \underline{h}_j^T & & & 0 \\ & \underline{h}_j \underline{h}_j^T & & \\ & & \ddots & \\ 0 & & & \underline{h}_j \underline{h}_j^T \end{bmatrix}$$

$$\underline{r}_j \cdot \Omega^T = \begin{bmatrix} s_1 \underline{h}_j \underline{h}_j^T & s_2 \underline{h}_j \underline{h}_j^T & \cdots & s_k \underline{h}_j \underline{h}_j^T \end{bmatrix} + \underline{n}_j \Omega^T.$$

$$\underline{h}_j \underline{h}_j^T = \sum_{i=1}^{n} |\alpha_{i,j}|^2$$

Doing the same for each received antenna, results in decoupling of s_i's and also in full diversity for each s_i.

$$\underline{r}_j = \underline{h}_j \mathcal{G}^T + \underline{n}_j$$

$$= \sum_{i=1}^{k} s_i \underbrace{\left(\underline{h}_j E_i^T\right)}_{1\times p \;\triangleq\; \Omega_i} + \underline{n}_j$$

$$= \begin{bmatrix} s_1 & s_2 & \cdots & s_k \end{bmatrix} \underbrace{\begin{bmatrix} - & \underline{h}_j E_1^T & - \\ - & \underline{h}_j E_2^T & - \\ & \vdots & \\ - & \underline{h}_j E_k^T & - \end{bmatrix}}_{\triangleq\; \Omega} + \underline{n}_j$$

$$= \begin{bmatrix} s_1 & s_2 & \cdots & s_k \end{bmatrix} \begin{bmatrix} - & \Omega_1 & - \\ - & \Omega_2 & - \\ & \vdots & \\ - & \Omega_k & - \end{bmatrix} + \underline{n}_j$$

$$\left(\Omega . \Omega^T\right)_{p,q} = \underline{h}_j E_p^T . \left(\underline{h}_j E_q^T\right)^T = \underline{h}_j \underbrace{E_p^T E_q} \underline{h}_j^T$$

$(p,q)^{th}$ element of $\Omega . \Omega^T$

We know from (*)

$$E_p^T E_q = \begin{cases} I_n & p = q \\ 0 & p \neq q . \end{cases}$$

$$\Rightarrow \Omega . \Omega^T = \begin{bmatrix} \underline{h}_j \underline{h}_j^T & & & 0 \\ & \underline{h}_j \underline{h}_j^T & & \\ & & \ddots & \\ 0 & & & \underline{h}_j \underline{h}_j^T \end{bmatrix}$$

$$\underline{r}_j . \Omega^T = \begin{bmatrix} s_1 \underline{h}_j \underline{h}_j^T & s_2 \underline{h}_j \underline{h}_j^T & \cdots & s_k \underline{h}_j \underline{h}_j^T \end{bmatrix} + \underline{n}_j \Omega^T .$$

$$\underline{h}_j \underline{h}_j^T = \sum_{i=1}^{n} |\alpha_{i,j}|^2$$

Doing the same for each received antenna results in decoupling of s_i's and also in full diversity for each s_i.

Complex Orthogonal Designs:

$\mathcal{O}_c$ of size n : An $n \times n$ orthogonal matrix with entries

$\pm x_1, \pm x_2, \dots, \pm x_n$.

$\pm x_1^*, \pm x_2^*, \dots, \pm x_n^*$

or multiples of these by $\pm i$

$n=2$

$$\begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}$$

Construction II:

Given a complex orthogonal design $\mathcal{O}_c$ of size n, we can replace each complex variable $x_i = x_i^1 + x_i^2 i$ by the 2×2 real matrix

$$\begin{pmatrix} x_i^1 & x_i^2 \\ -x_i^2 & x_i^1 \end{pmatrix}$$

to get a real orthogonal design of size $2n$.

$\Rightarrow \mathcal{O}_c$ can exists only for $n = 2$ or 4. (Thm: 5.3.1).

In fact $\mathcal{O}_c$ cannot exist even for $n=4$ (Thm. 5.4.2)

$\Rightarrow \mathcal{O}_c$ can exist only for $n=2$. (Alamouti design)

Complex linear processing orthogonal designs.

Each entry of $\mathcal{E}_c$ is a complex linear combination of variables $x_1, \dots, x_n$ and $x_1^*, \dots, x_n^*$.

$$\mathcal{E}_c^* \mathcal{E}_c = \underset{\text{diagonal}}{D}$$

Generalized complex Orthogonal Designs

$\mathcal{G}_c$ $p \times n$ matrix entries $0, \pm x_1, \pm x_1^*, \pm x_2, \dots, \pm x_k, \pm x_k^*$
or their product with $i = \sqrt{-1}$.

· $\mathcal{G}_c^* \mathcal{G}_c$ = diagonal.

→ Rate $R = \frac{k}{p}$ as before.

→ $A_c(R, n)$: min p such that there exists a generalized complex linear processing orthogonal design of size $p \times n$ with rate $\geq R$.
If no such design exists, $A_c(R,n) = \infty$.

Thm 5.5.2

(1) For any R, $A(R, 2n) \leq 2A_c(R, n)$

Pf: Using construction II & known result for generalized real orthogonal designs.

(2) For $R \leq 0.5$, $A_c(R, n) \leq 2A(2R, n)$

Pf: From a real design of size $p \times n$ $\mathcal{G}$
Construct $\mathcal{G}_c = \begin{pmatrix} \mathcal{G} \\ \mathcal{G}^* \end{pmatrix}$.

Show $\mathcal{G}_c$ is a generalized complex orth. design $2p \times n$
(or) $2A(2R,n) \times n$.
$\Rightarrow A_c(R, n) \leq 2A(2R, n)$

→ So, $R = \frac{1}{2}$ designs exists for any n.

→ For $n = 3, 4$, $R = \frac{3}{4}$ designs are proposed in Sec. V.F.

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(Separate decodability to be done first p. 29)
& then min rank $= 2$.

"A Quasi-Orthogonal Space-Time Block Code"
H. Jafarkhani, TCOM, Jan 2001.

→ So far: orthogonal designs
- full diversity, simple single symbol decoding
- Rate ≤ 1 only $\frac{1}{2}$ or $\frac{3}{4}$ in complex case for more than 2 antennas.

→ Now: Quasi-orthogonal design
- double-symbol decoding
Transmission matrix columns are divided into groups (pairs in this paper). Different groups are orthogonal to each other. Within groups, columns are not orthogonal.
- Higher transmission rate, full diversity not achieved.

→ $R=1$, diversity $= \frac{1}{2}$ (maximum possible diversity)
double-symbol decoding.

Alamouti transmission matrix $\quad A_{12} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}$

Proposed design for 4 antennas

$$A = \begin{pmatrix} A_{12} & A_{34} \\ -A_{34}^* & A_{12}^* \end{pmatrix} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{pmatrix}$$
$$\qquad\qquad\qquad\qquad\qquad\quad \uparrow \quad\; \uparrow \quad\; \uparrow \quad\; \uparrow$$
$$\qquad\qquad\qquad\qquad\qquad\quad \nu_1 \quad \nu_2 \quad \nu_3 \quad \nu_4$$

$\langle \nu_1, \nu_2 \rangle = 0$

$\langle \nu_1, \nu_3 \rangle = 0 \qquad \{\nu_1, \nu_4\}$ orth. to $\{\nu_2, \nu_3\}$

$\langle \nu_2, \nu_4 \rangle = 0$

$\langle \nu_3, \nu_4 \rangle = 0$

$$(r_1^j\ r_2^j\ r_3^j\ r_4^j) = (\alpha_{1,j}\ \alpha_{2,j}\ \alpha_{3,j}\ \alpha_{4,j})\left(A^T\right) + (n_{1,j}\ n_{2,j}\ n_{3,j}\ n_{4,j})$$

$$r_1^j = \alpha_{1,j}x_1 - \alpha_{2,j}x_2^* - \alpha_{3,j}x_3^* + \alpha_{4,j}x_4 + n_{1,j}$$

$$\left(r_2^j = \alpha_{1,j}x_2 + \alpha_{2,j}x_1^* - \alpha_{3,j}x_4^* - \alpha_{4,j}x_3 + n_{2,j}\right)^*$$

$$\left(r_3^j = \alpha_{1,j}x_3 - \alpha_{2,j}x_4^* + \alpha_{3,j}x_1^* - \alpha_{4,j}x_2 + n_{3,j}\right)^*$$

$$r_4^j = \alpha_{1,j}x_4 + \alpha_{2,j}x_3^* + \alpha_{3,j}x_2^* + \alpha_{4,j}x_1 + n_{4,j}$$

$$\underline{r}\begin{pmatrix} r_1^j \\ r_2^{j*} \\ r_3^{j*} \\ r_4^j \end{pmatrix} = \overset{H}{\begin{pmatrix} \alpha_{1,j} & -\alpha_{2,j} & -\alpha_{3,j} & \alpha_{4,j} \\ \alpha_{2,j}^* & \alpha_{1,j}^* & -\alpha_{4,j}^* & -\alpha_{3,j}^* \\ \alpha_{3,j}^* & -\alpha_{4,j}^* & \alpha_{1,j}^* & -\alpha_{2,j}^* \\ \alpha_{4,j} & \alpha_{3,j} & \alpha_{2,j} & \alpha_{1,j} \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2^* \\ x_3^* \\ x_4 \end{pmatrix} + \begin{pmatrix} n_{1,j} \\ n_{2,j}^* \\ n_{3,j}^* \\ n_{4,j} \end{pmatrix}$$

$$\uparrow\ \underline{h}_1 \qquad \uparrow\ \underline{h}_2 \qquad \uparrow\ \underline{h}_3 \qquad \uparrow\ \underline{h}_4$$

Need to rewrite. Used A instead of A^T. See (32)

$$\underline{h}_1^*\underline{h}_2 = 0 = -\alpha_{1,j}^*\alpha_{2,j} + \alpha_{2,j}\alpha_{1,j}^* - \alpha_{3,j}\alpha_{4,j}^* + \alpha_{4,j}^*\alpha_{3,j}$$

$$\underline{h}_1^*\underline{h}_3 = 0 = -\alpha_{1,j}^*\alpha_{3,j} - \alpha_{2,j}\alpha_{4,j}^* + \alpha_{3,j}\alpha_{1,j}^* + \alpha_{4,j}^*\alpha_{2,j}$$

$$\underline{h}_2^*\underline{h}_4 = 0 = -\alpha_{2,j}^*\alpha_{4,j} - \alpha_{1,j}\alpha_{3,j}^* + \alpha_{4,j}\alpha_{2,j}^* + \alpha_{3,j}^*\alpha_{1,j}$$

$$\underline{h}_3^*\underline{h}_4 = 0 = -\alpha_{3,j}^*\alpha_{4,j} + \alpha_{4,j}\alpha_{3,j}^* - \alpha_{1,j}\alpha_{2,j}^* + \alpha_{2,j}^*\alpha_{1,j}$$

$$H^H\underline{r} = \begin{pmatrix} \times & 0 & 0 & \times \\ 0 & \times & \times & 0 \\ 0 & \times & \times & 0 \\ \times & 0 & 0 & \times \end{pmatrix}\begin{pmatrix} x_1 \\ x_2^* \\ x_3^* \\ x_4 \end{pmatrix} + \underline{n}$$

diag entries: $|\alpha_{1,j}|^2 + |\alpha_{2,j}|^2 + |\alpha_{3,j}|^2 + |\alpha_{4,j}|^2$

$$\underline{h}_1^*\underline{h}_4 = \alpha_{1,j}^*\alpha_{4,j} - \alpha_{2,j}\alpha_{3,j}^* - \alpha_{3,j}\alpha_{2,j}^* + \alpha_{4,j}^*\alpha_{1,j}$$
$$= 2\mathrm{Re}(\alpha_{1,j}^*\alpha_{4,j}) - 2\mathrm{Re}(\alpha_{2,j}\alpha_{3,j}^*)$$

$$\underline{h}_2^*\underline{h}_3 = \alpha_{2,j}^*\alpha_{3,j} - \alpha_{1,j}\alpha_{4,j}^* - \alpha_{4,j}\alpha_{1,j}^* + \alpha_{3,j}^*\alpha_{2,j}$$
$$= 2\mathrm{Re}(\alpha_{2,j}\alpha_{3,j}^* - \alpha_{1,j}\alpha_{4,j}^*)$$

Decode x_1, x_4 jointly, and x_2, x_3 jointly.

$$A\left(s_1-\tilde{s}_1,\ s_2-\tilde{s}_2,\ s_3-\tilde{s}_3,\ s_4-\tilde{s}_4\right)=$$

$$\begin{bmatrix} s_1-\tilde{s}_1 & s_2-\tilde{s}_2 & s_3-\tilde{s}_3 & s_4-\tilde{s}_4 \\ -(s_2-\tilde{s}_2)^* & (s_1-\tilde{s}_1)^* & -(s_4-\tilde{s}_4)^* & (s_3-\tilde{s}_3)^* \\ -(s_3-\tilde{s}_3)^* & -(s_4-\tilde{s}_4)^* & (s_1-\tilde{s}_1)^* & (s_2-\tilde{s}_2)^* \\ s_4-\tilde{s}_4 & -(s_3-\tilde{s}_3) & -(s_2-\tilde{s}_2) & (s_1-\tilde{s}_1) \end{bmatrix}$$

→ Suppose $s_1-\tilde{s}_1 = (s_3-\tilde{s}_3)^*$

and $s_2-\tilde{s}_2 = 0$

$s_4-\tilde{s}_4 = 0.$

$$\begin{pmatrix} s_1-\tilde{s}_1 & 0 & s_3-\tilde{s}_3 & 0 \\ 0 & (s_1-\tilde{s}_1)^* & 0 & (s_3-\tilde{s}_3)^* \\ -(s_3-\tilde{s}_3)^* & 0 & (s_1-\tilde{s}_1)^* & 0 \\ 0 & -(s_3-\tilde{s}_3) & 0 & (s_1-\tilde{s}_1) \end{pmatrix}$$

→ Set $s_1-\tilde{s}_1 = s_4-\tilde{s}_4 = a$

$s_2-\tilde{s}_2 = s_3-\tilde{s}_3 = 0$

$$\begin{pmatrix} a & 0 & 0 & a \\ 0 & a^* & -a^* & 0 \\ 0 & -a^* & a^* & 0 \\ a & 0 & 0 & a \end{pmatrix}$$

rank = 2

$\times(-1)$

$=$

Min. rank = 2 $\Rightarrow$ Div. gain = 2M

L19

4/9/12

→ P(31) Double-symbol decodability

→ Plots in paper

→ Generalizations for 8 antennas, etc.

→ Comparison with using an OSTBC with only half the tx. antenna

5/9/12 Q1

7/9/12 L20

"Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels" Zheng & Tse 2003. IT Trans. May.

Sec. I, II, III.A.

→ Model & Problem Formulation.

* $\underbrace{Y}_{n\times l} = \sqrt{\frac{SNR}{m}}\, \underbrace{H}_{n\times m}\, \underbrace{X}_{m\times l} + \underbrace{W}_{n\times l}$.

* Codebook $\mathcal{C}$, R: Rate in bits/s/Hz.

* $|\mathcal{C}| = \lfloor 2^{Rl} \rfloor$ codewords $X(1), X(2), \ldots, X(|\mathcal{C}|)$

* $\frac{1}{|\mathcal{C}|}\sum_{i=1}^{|\mathcal{C}|} \|X(i)\|_F^2 \le ml$.

→ Scheme: Family of codes $\{\mathcal{C}(SNR)\}$

Scheme achieves spatial multiplexing gain r if

$$\lim_{SNR\to\infty} \frac{R(SNR)}{\log SNR} = r$$

Scheme achieves div. gain d if

$$\lim_{SNR\to\infty} \frac{\log P_e(SNR)}{\log SNR} = -d$$

$d^*(r)$: supremum of the div. gain achieved over all schemes.

→ Main result (Thm 2):

$$d^*(k) = (m-k)(n-k)$$

$d^*(r)$ is given by the piecewise linear fn. connecting the points $(k, d^*(k))$, $k = 1, 2, \ldots, \min(m,n)$

$$d^*_{max} = mn, \quad r^*_{max} = \min(m,n).$$

→ Discuss Figures 1, 2, 3

→ Difference between div. gain defn. here and in STC design.

→ Discuss Figure 8.

L21

10/9/12 (Secs. III.B., III.C, Appendix)

Outage Formulation:

→ $I(x_t ; y_t / H = H) = \log\det\left(I + \frac{SNR}{m} H Q H^H\right)$

→ $P_{out}(R) = \inf\limits_{\substack{Q \geq 0 \\ tr(Q) \leq m}} P\left[\log\det\left(I + \frac{SNR}{m} H Q H^H\right) < R\right]$

→ Show upper and lower bounds have same diversity:

$$P\left[\log\det\left(I + \frac{SNR}{m} HH^H\right) < R\right] \underset{\text{Set } Q = I}{\geq} P_{out}(R) \underset{\text{Use } mI - Q \geq 0}{\geq} P_r\left[\log\det\left(I + SNR\, HH^H\right) < R\right]$$

(Removes Q from analysis)

→ Now, analyze $P_r\left[\log\det\left(I + SNR\, HH^H\right) < R\right]$ to get diversity of DMT.

→ Single-antenna channel :

$$d^*(r) = 1-r.$$

→ MIMO case. (consider $m \geq n$, can easily write for $m<n$ as well)

$$P_{out}(R) \doteq P\left[\log\det\left(I + SNR\, HH^H\right) < (R)\right]$$

$$= P\left[\prod_{i=1}^{n}(1+SNR\lambda_i) < SNR^{r}\right]$$

→ Joint PDF of λ_i's from random matrix theory

→ Define $\lambda_i = SNR^{-\alpha_i}$.

Find joint PDF of α_i's.

→ $P_{out}(R) \doteq P\left[\prod_i SNR^{(1-\alpha_i)^+} < SNR^{r}\right]$

$$= P\Big[\underbrace{\sum_i (1-\alpha_i)^+ < r}_{\text{event } A}\Big]$$

(Note
$1 + SNR \cdot SNR^{-\alpha_i} = 1+ SNR^{(1-\alpha_i)}$

if $1-\alpha_i > 0$, $1+SNR^{(1-\alpha_i)} \approx SNR^{(1-\alpha_i)}$ for large SNR.

if $1-\alpha_i < 0$, $1+SNR^{(1-\alpha_i)} \approx 1$ for large SNR.)

→ $P_{out}(r\log SNR) \doteq \int_A f_{\underline{\alpha}}(\underline{\alpha})\, d\underline{\alpha}$.

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11/9/12

$$f_{\underline{\lambda}}(\lambda_1, \ldots, \lambda_n) = K_{m,n}^{-1}\prod_{i=1}^{n}\lambda_i^{m-n}\prod_{i<j}(\lambda_i-\lambda_j)^2 e^{-\sum_i \lambda_i}$$

$K_{m,n}$: a normalizing constant

$$\underline{\alpha} = [\alpha_1, \dots, \alpha_n]$$

$$\alpha_i = \frac{-\log \lambda_i}{\log SNR} \quad (\text{or}) \quad \lambda_i = SNR^{-\alpha_i}.$$

$$f_{\underline{\alpha}}(\underline{\alpha}) = K_{m,n}^{-1} \prod_{i=1}^{n} SNR^{-\alpha_i(m-n)} \prod_{i<j} \left(SNR^{-\alpha_i} - SNR^{-\alpha_j}\right)^2 e^{-\sum_i SNR^{-\alpha_i}}$$

$$\cdot \left| J(\underline{\alpha}, \underline{\lambda}) \right|.$$

$$\frac{\partial \lambda_i}{\partial \alpha_i} = \cancel{}(-SNR^{-\alpha_i})(\log SNR).$$

$$|J(\underline{\alpha}, \underline{\lambda})| = (\log SNR)^n \prod_{i=1}^{n} SNR^{-\alpha_i}$$

$$\Rightarrow f_{\underline{\alpha}}(\underline{\alpha}) = K_{m,n}^{-1} \prod_{i=1}^{n} SNR^{-\alpha_i(m-n+1)} \prod_{i<j} \left(SNR^{-\alpha_i} - SNR^{-\alpha_j}\right)^2 e^{-\sum_i SNR^{-\alpha_i}}.$$
$$(\log SNR)^n$$

→ We are only interested in

$$\lim_{SNR \to \infty} \frac{\log P_{out}(r \log SNR)}{\log SNR}.$$

We can neglect some terms.

→ 1

$$\int_A f_{\underline{\alpha}}(\underline{\alpha})\, d\underline{\alpha} = K_{m,n}^{-1} (\log SNR)^n \cdot \int_A \prod_{i=1}^{n} SNR^{-\alpha_i(m-n+1)} \prod_{i<j} \left(SNR^{-\alpha_i} - SNR^{-\alpha_j}\right)^2 e^{-\sum_i SNR^{-\alpha_i}} d\underline{\alpha}$$

$$\log\left(\int_A f_{\underline{\alpha}}(\underline{\alpha})\, d\underline{\alpha}\right) = \log\left(K_{m,n}^{-1} (\log SNR)^n\right) + \log\left(\quad \sim \quad \right)$$

$$\frac{\log\left[K_{m,n}^{-1}(\log SNR)^n\right]}{\log SNR} \longrightarrow 0 \quad \text{as } SNR \to \infty.$$

$\Rightarrow$ We can ignore the $\log\left[K_{m,n}^{-1}(\log SNR)^n\right]$ term in $\log\left[P_{out}(r \log SNR)\right]$.

$\Rightarrow$ We can consider

$$\int_A \prod_{i=1}^{n} SNR^{-\alpha_i(m-n+1)} \prod_{i<j} \left(SNR^{-\alpha_i} - SNR^{-\alpha_j}\right)^2 e^{-\sum_i SNR^{-\alpha_i}} d\underline{\alpha}$$

② $A = \left\{ \underline{\alpha} : \sum_i (1-\alpha_i)^+ < r \right\}$

If any $\alpha_i < 0$, $e^{-\sum SNR^{-\alpha_i}}$ decays exponentially with SNR.

$\Rightarrow$ at high SNR, we can ignore the integral over the range with any $\alpha_i < 0$.

$\Rightarrow$ We can replace A by $A' = A \cap \underbrace{\mathbb{R}^{n+}}_{\text{set of real } n\text{-vectors with non-ve elements}}$

$\left(\text{Split } \int_A (\;) = \int_{A'} (\;) + \underbrace{\int_{A \setminus A'} (\;)}_{\text{neglect}} .\right)$

For $\alpha_i > 0$, $e^{-SNR^{-\alpha_i}} \longrightarrow 1$
For $\alpha_i = 0$, $e^{-SNR^{-\alpha_i}} = e^{-1}$
$\Big\}$ both constants ($\Rightarrow$ do not affect diversity)

$\Rightarrow$ We can just consider

$$\int_{A'} \prod_{i=1}^{n} SNR^{-\alpha_i(m-n+1)} \prod_{i<j} \left(SNR^{-\alpha_i} - SNR^{-\alpha_j}\right)^2 d\underline{\alpha}$$

Now

→ By defn., $\alpha_i \geq \alpha_j$ for any $i < j$ (ordered eigen values).

→ If α_i's are not distinct, $\left(SNR^{-\alpha_i} - SNR^{-\alpha_j}\right) = 0$ for some i, j

$\Rightarrow$ integrand is zero & does not contribute to the integral.

$\Rightarrow$ Consider only α_i's to be distinct.

If $i<j$, $\alpha_i \geq \alpha_j \Rightarrow SNR^{-\alpha_j}$ dominates $SNR^{-\alpha_i}$.

$$\Rightarrow \prod_{i=1}^{n} SNR^{-(m-n+1)\alpha_i} \prod_{i<j}\left(SNR^{-\alpha_i} - SNR^{-\alpha_j}\right)^2$$

$$\doteq \prod_{i=1}^{n} SNR^{-(m-n+1)\alpha_i} \prod_{i<j} SNR^{-2\alpha_j}$$

$$= \prod_{i=1}^{n} SNR^{-(m-n+1)\alpha_i} \cdot SNR^{-2(i-1)\alpha_i}$$

$$= \prod_{i=1}^{n} SNR^{-[(m-n+1)\alpha_i + (2i-2)\alpha_i]}$$

$$= \prod_{i=1}^{n} SNR^{-(m-n+2i-1)\alpha_i}$$

$$P_{out}(r \log SNR) \doteq \int_{A'} \prod_{i=1}^{n} SNR^{-(m-n+2i-1)\alpha_i} \, d\underline{\alpha}. \quad \text{— } A$$

→ As $SNR \to \infty$, this integral is dominated by the term with largest SNR exponent.

i.e.

$$P_{out}(r \log SNR) \doteq SNR^{-d_{out}(r)}$$

where $d_{out}(r) = \inf_{\alpha \in A'} \sum_{i=1}^{n} (m-n+2i-1)\alpha_i$

& $A' = \left\{ \underline{\alpha} \in \mathcal{R}^{n+} : \alpha_1 \geq \cdots \geq \alpha_n \geq 0, \sum_i (1-\alpha_i)^+ < r \right\}$.

(For general m, n:

$$\boxed{d_{out}(r) = \inf_{\underline{\alpha} \in A'} \sum_{i=1}^{\min(m,n)} \left(2i - 1 + |m-n| \alpha_i\right)}$$

and $A' = \left\{ \alpha \in R^{\min(m,n)+} : \alpha_1 \geq \cdots \geq \alpha_{\min(m,n)} \geq 0, \sum_i (1-\alpha_i)^+ < r \right\}$.)

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12/9/12

→ We can generalize eqn. (A) to any set $\mathcal{B} \subset \mathcal{R}^{n+}$ (instead of A').

$$P(\underline{\alpha} \in \mathcal{B}) \doteq \int_{\mathcal{B}} \prod_{i=1}^{n} SNR^{-(m-n+2i-1)\alpha_i} \, d\underline{\alpha}$$

$$\doteq SNR^{-\min_{\underline{\alpha} \in \mathcal{B}} \sum_i (m-n+2i-1)\alpha_i}$$

For example, consider any $\underline{b} = [b_1\ b_2 \cdots b_n] \in \mathcal{R}^{n+}$.
and the set $\mathcal{B}_b = \{ \underline{\alpha} : \alpha_i \geq b_i \}$.

$$P(\underline{\alpha} \in \mathcal{B}_b) = P\left(\lambda_i \leq SNR^{-b_i}, \forall i\right)$$

$$\doteq SNR^{-\sum_i (m-n+2i-1) b_i}$$

We can also write,

$$-\lim_{\epsilon \to 0} \frac{\log P(\lambda_i \leq \epsilon^{b_i}, \forall i)}{\log \epsilon} = \sum_{i=1}^{n} (m-n+2i-1) b_i$$

(Think of $\epsilon = \frac{1}{SNR}$)

This characterizes the near-singular distribution of the channel matrix H.

$\longrightarrow$ Outage event at multiplexing gain r is

$$\left\{\underline{\alpha} : \sum_i (1-\alpha_i)^+ < r\right\} \ \& \ \alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_n \geq 0.$$

For a given $\underline{\alpha}$, the SNR exponents

$$\sum_i (2i-1+m-n)\alpha_i$$

Now, we need to find $\underline{\alpha}^*$ that gives the min. SNR exponent.

$$\left(\text{Note } 0 \leq (1-\alpha_i)^+ \leq 1 \ ; 0 \leq \sum_i (1-\alpha_i)^+ \leq n\right)$$

$\rightarrow$ Let r be an integer k between 0 & n.

Need. $\displaystyle\sum_{i=1}^{n} (1-\alpha_i)^+ \leq r$

Need to minimize

$$\sum_i (2i-1+m-n)\alpha_i = \alpha_1(1+m-n) + \alpha_2(3+m-n) + \alpha_3(5+m-n) + \cdots + \alpha_n(2n-1+m-n).$$

$\boxed{\underline{\alpha}^* \text{ is given by}}$

$$\alpha_1^* = \alpha_2^* = \cdots = \alpha_{n-k}^* = 1$$

$$\alpha_{n-k+1}^* = \cdots = \alpha_n^* = 0.$$

$$\Rightarrow (1-\alpha_i)^+ = \begin{cases} 0 & i = 1, 2, \ldots, n-k \\ 1 & i = n-k+1, \ldots, n. \end{cases}$$

$$\sum_i (1-\alpha_i)^+ = n-(n-k) = k.$$

$$\sum_i (2i-1+m-n)\alpha_i = \sum_{i=1}^{n-k} (2i-1+m-n)$$

$$= (m-n-1)(n-k) + 2\frac{(n-k)(n-k+1)}{2}$$

$$= (n-k)(m-n-1+n-k+1) = \underline{(n-k)(m-k)}$$

Scanned by CamScanner

→ Suppose $k < r < k+1$

$$\alpha_1^* = \cdots = \alpha_{n-k-1}^* = 1$$

$$\alpha_{n-k}^* = k+1-r$$

$$\alpha_{n-k+1}^* = \cdots = \alpha_n^* = 0.$$

⇒ SNR exponent is linear between integer points.

→ α_1 corresponds to smallest eigenvalue.

Smaller eigenvalues have a higher prob. of being close to zero

Typical outage event: (for $r=k$)

$$\begin{cases} (n-k) \text{ smallest eigenvalues} \\ \quad \lambda_i \doteq SNR^{-1} \\ k \text{ largest eigenvalues} \\ \quad \text{are of order } 1 \; (\lambda_i \doteq 1) \end{cases}$$

$$\boxed{d^*(k) = (n-k)(m-k)}$$

Relating error probability to outage probability

Lemma 5 : For any coding scheme, prob. of detection error is lower bounded by

$$P_e(SNR) \overset{.}{\geq} SNR^{-d_{out}(r)}.$$

where $d_{out}(r)$ is as before (in page 35).

(Proof using Fano's Inequality).

(Conditioned on the channel outage event, it is very likely that a detection error occurs).

→ In the outage formulation, we assume infinite block length. Usually this gives a lower bound on performance on finite block length performance.

→ Random coding: Error can be due to

(1)– channel atypically ill-conditioned

(2)– additive noise atypically large

(3)– some codewords are atypically close

For $l \to \infty$, (2) & (3) are averaged out

error mainly due to (1).

In this paper (theorem 2), it is shown that

for $l \geq m+n-1$

(1) still dominates the error event.

(2) & (3) have SNR exponent not smaller than (1).

Proof: derive an upper bound on $P_e(SNR)$

* Choose a random code from the i.i.d. Gaussian ensemble

$$P_e(SNR) = P_{out}(R)\,P(\text{error}/\text{outage}) + P(\text{error, no outage})$$

$$\leq P_{out}(R) + \underbrace{P(\text{error, no outage})}.$$

Find an upper bound for this & check its SNR exponent.

* $P_e(SNR) \dot{\leq} SNR^{-d_G(r)}$

and $d_G(r) = d_{out}(r)$ for $l \geq m+n-1$.

(For $l < m+n-1$, $d_G(r) \neq d_{out}(r)$).

L24
14/9/12

Space-Time Coding: An Overview

Caire, Elia & Kumar K.R. (2006) Oct

Journal of Communications Software and Systems (Invited Paper)

L25
17/9/12

- Review of SVD transmission
- Possibility of feedback in FDD/TDD, reciprocity, CSIT.
- SVD + water-filling
- SNR gain compared to ergodic capacity with CSIR only (Tse & Viswanath book) (at high SNR)

L26
18/9/12

Sections: II, III, IV

'Limiting Performance of Block-Fading Channels with Multiple Antennas

E. Biglieri, G. Caire, G. Taricco., Trans. IT, May 2001.

→ MIMO with perfect CSIR and perfect CSIT.

→ Model.

→ No Delay Constraints: Ergodic capacity

→ Delay limited capacity defn. & Outage capacity

L27
21/9/12

Sec. V

→ Short-term problem: & soln. ⟺ Dual. Max $I_M(A,\Gamma)$ sub to $P_M(\Gamma) \leq \gamma$.

Explain equations (14) – (18)

SVD + waterfilling + on/off

threshold based on $I_M(\ ) < R$.

→ Long-term problem & soln. $\iff$ Dual

Minimize $P_M(\Gamma)$

sub. to. $I_M(\Lambda, \Gamma) \geq R$

Equations (19)-(24)

SVD + waterfilling + on/off

threshold based on fading statistics.

L28 24/9/2012

Regions:

→ $\mathcal{R}_{off}(R, \gamma)$ and $\mathcal{R}^*_{off}(R, \gamma^*)$ are equivalent

(Proof in Appendix A).

($\mathcal{R}^*$ can be replaced by $\mathcal{R}$, $\gamma^* \geq \gamma$)

→ $P_{out}(R, \gamma) = Pr(\Lambda \in \mathcal{R}_{off}(R, s))$

$s = \gamma$ for s.t.

$s = \gamma^*$ for l.t.

$$\lim_{s \to \infty} P_{out}(R, s) = \lim_{s \to \infty} Pr(\Lambda \in \underbrace{\mathcal{R}_{off}(R, s)}_{\text{region shrinks as } s \text{ increases.}}) = 0.$$

→ If $F(\Lambda)$'s support is the whole orthant $\mathbb{R}_+^{Mm}$,

& we have a short-term constraint,

$P_{out}(R, \gamma) > 0$ for all finite γ

$\Rightarrow C_{delay}(\gamma) = 0$.

If we have a long-term constraint,

$P_{out}(R, \gamma) = 0$ for all $\gamma \geq \gamma(R)$

where $\gamma(R)$ is the minimum average SNR per block

given by $\gamma(R) = E\left[P_M(\Gamma^{lt}(\Lambda, R))\right]$

$R = C_{delay}(\gamma)$ is obtained by inverting $\gamma = \gamma(R)$.

→ Large no. of antennas.

Study $C(\gamma) \triangleq \dfrac{C_{delay}(\gamma)}{m}$ as $m \to \infty$ and $\dfrac{\max\{t,r\}}{m} \to \alpha \geq 0$.

Prop. 8 $C(\gamma) \to \dfrac{C_{CSIR,CSIT}(\gamma)}{m}$ as $m \to \infty$.

Proof: Appendix D.

(Transmitting over many blocks ~~has~~ no effect on capacity)

→ Figures. 2–~~3~~.

→ Conclusions.

L29
25/9/12

→ Summary: Perfect CSIR
Perfect CSIT+CSIR

→ Multiuser MIMO
— Multiple Access
— Broadcast

→ "Iterative Water-filling for Gaussian Vector Multiple-Access Channels"
W. Yu, W. Rhee, S. Boyd, J.M. Cioffi
IT. Trans., Jan 2004.

Sec II.
Sec III. A.
Thm. 1 : Statement alone.

L30
26/9/12

- Proof of Thm 1.
- Algorithm 1
- Fig. 3.
- Thm 2.

— Statement of Thm. 3.

L31 28/9/12

– "On the capacity of Multiuser Wireless Channels with multiple antennas", W. Rhee, J.M. Cioffi, Trans. IT, Oct 2003

Sec 2: Model & definitions

Thm 2 & Proof (Appendix I).

L32 8/10/12

Thm 3

MIMO MAC ergodic capacity.

Sec. V.A. & V.B (Figs. 4 & 5)

L33 9/10/12

MIMO BC capacity.

Model & R_{DPC} from Weingarten, Steinberg, Shamai

IT Trans 2006

L34 10/10/12

Balakrishna's MS thesis

Plots discussed. (DPC, ZF-BF, etc.)

L35 12/10/12

Discussion/Questions?

From 15/10/12 Project presentations