EE613: Estimation Theory
Problem Set 6

1. Consider the observed data set

$$x[n] = A + w[n] \qquad n = 0, 1, \cdots, N-1$$

where A is an unknown level, which is assumed to be positive ($A > 0$) and $w[n]$ is WGN with unknown variance A. Find the variance of the sample mean estimator and compare it to the CRLB. Does the sample mean estimator attain the CRLB for finite N? How about if $N \to \infty$? Is the MLE or the sample mean a better estimator ?

2. We observe N IID samples from the following PDF:
Exponential :

$$p(x;\lambda) = \begin{cases} \lambda \exp\{-\lambda x\} & x > 0 \\ 0 & x < 0 \end{cases} \tag{1}$$

Find the MLE of the unknown parameter and be sure to verify that it indeed maximizes the likelihood function. Does the estimator make sense?

3. A formal definition of the consistency of an estimator is given as follows. An estimator $\hat{\theta}$ is consistent if, given any $\epsilon > 0$,

$$\lim_{N\to\infty} Pr\{|\hat{\theta} - \theta| > \epsilon\} = 0$$

Prove that the sample mean is a consistent estimator for the problem of estimating a DC level A in white Gaussian noise of known variance. Hint : Use Chebychev's inequality: For a random variable X with mean μ and variance σ^2, $P(|X - \mu| \geq \epsilon) \leq \dfrac{\sigma^2}{\epsilon^2}$.

4. If we observe N IID samples from a Bernoulli experiment (coin toss) with the probabilities

$$Pr\{x[n] = 1\} = p$$
$$Pr\{x[n] = 0\} = 1 - p$$

find the MLE of p.

5. For N IID observations from a $\mathcal{U}[0, \theta]$ PDF find the MLE of θ.

6. For N IID observations from a $\mathcal{N}(0, \frac{1}{\theta})$ PDF, where $\theta > 0$, find the MLE of θ and its asymptotic PDF.

7. For N IID observations from the PDF $\mathcal{N}(A, \sigma^2)$, where A and σ^2 are both unknown, find the MLE of the SNR $\alpha = \frac{A^2}{\sigma^2}$.